Version DC/3

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 7 June 2017 9 to 12

**Paper 1**

**MECHANICAL ENGINEERING**

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

*Write your candidate number __**not**__ your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Supplementary page: one extra copy of Fig. 7 (Question 9)
Engineering Data Books

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

## SECTION A

1 **(short)** A thin-walled cylinder with cross-sectional area $A$ and mass $m$ is floating with displaced depth $x$ in liquid with density $\rho$ as shown in Fig. 1. By considering the equilibrium air pressure inside the cylinder, or otherwise, find the depth $x$ in terms of $m$, $\rho$ and $A$. [10]

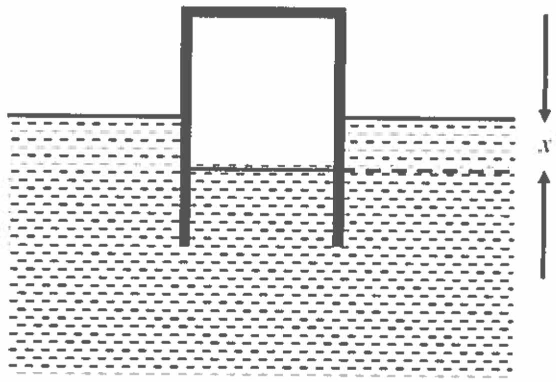



Fig. 1

2 **(short)** The rotor of a pump has diameter $D$, rotates with rotational speed $N$ and delivers volume flow $Q$ with head $H$. Viscous effects may be neglected; the acceleration due to gravity is $g$.

(a) Derive two dimensionless groups to represent the head and flow characteristics of the pump. Sketch a typical characteristic. [5]

(b) At a fixed dimensionless operating point, a particular pump experiences an increase of 10% in its rotational speed. By approximately how much do the head $H$ and flow $Q$ change? [5]

3 **(short)** For polytropic processes, pressure and volume of perfect gases are related by $pV^n = \text{constant}$, where $n$ is the polytropic index.

(a) Plot on a temperature-entropy diagram the transition of a perfect gas undergoing a reversible polytropic expansion from state 1 to two different end states, for polytropic indices:

(i) $n = \gamma$

(ii) $n = 1$ [4]

(b) For a closed system containing a perfect gas undergoing a reversible change from state 1 to 2, the ratio of heat transfer to system work per unit mass is $\delta q / \delta w = K$, where $K$ is a constant. Use conservation of energy to show that the polytropic index is

$$n = K + \gamma - K\gamma$$

Note that $\gamma = c_p / c_v$ and $R = c_p - c_v$. [6]

4 **(long)** Consider an old-fashioned carburettor, as shown in Fig. 2, which supplies an engine with a mixture of fuel and air. Air flows from a duct of area $A_1$ through a 'venturi' with reduced area $A_2$, into a throat of area $A_3 = A_1$. Fuel is drawn from a reservoir into the air stream at 2, through a nozzle of area $A_f$.

(a) Neglecting the volume flow rate of the fuel, write down an expression relating the air velocities *V* and duct areas *A* at stations 1, 2 and 3. [4]

(b) Assuming the flow in the system is steady, loss free and incompressible, derive an expression for the mass flow rate of air to the engine in terms of the static pressure at the venturi, station 2. [8]

(c) Similarly, derive an expression for the mass flow rate of fuel to the engine. Assume the fuel tank is open to the atmosphere and at the same elevation as the venturi. [8]

(d) Hence, for an air-fuel mass ratio of 14 find the ratio of the area of the fuel supply nozzle to the duct throat area, $A_f/A_2$. Assume the densities of air and fuel are 1.2 kg m$^{-3}$ and 720 kg m$^{-3}$ respectively. [10]

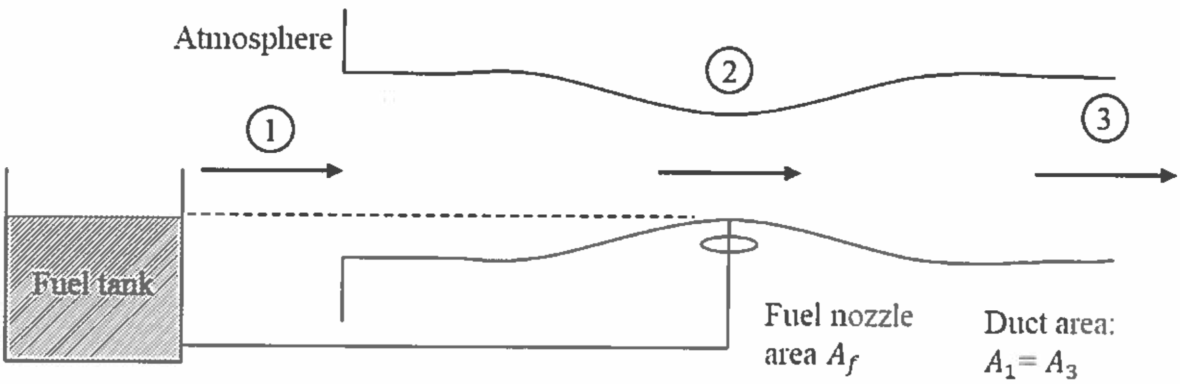



Fig. 2

5. **(short)** Air initially at rest, state 0, undergoes steady adiabatic compression and is mixed with a second stream of air as shown in Fig. 3. At inlet 0 the mass flow rate is $\dot{m}_0 = 0.2$ kg s$^{-1}$ and the temperature and pressure are $T_0 = 300$ K and $p_0 = 10^5$ Pa, respectively. The power supplied to the compressor is $\dot{W}_c = 10$ kW. The exit flow at state 1 has a temperature of $T_1 = 345$ K and unknown velocity, $V_1$, into a pipe with cross sectional area $A_1 = 10$ cm$^2$. The flow at state 1 is mixed with a second flow at state 2 of equal pressure and velocity with a mass flow rate of $\dot{m}_2 = 0.4$ kg s$^{-1}$ and temperature of $T_2 = 600$ K.

(a) By application of the ideal gas equation, show that

$$V_1 = \frac{\dot{m}_0 R T_1}{A_1 p_1}$$ [2]

(b) Accounting for the kinetic energy of the flow at state 1, apply the steady flow energy equation to determine the pressure at the compressor exit, $p_1$. [4]

(c) Calculate the rate of entropy generation resulting from mixing streams 1 and 2. Assume the process is adiabatic with no change in pressure or velocity due to mixing. [4]

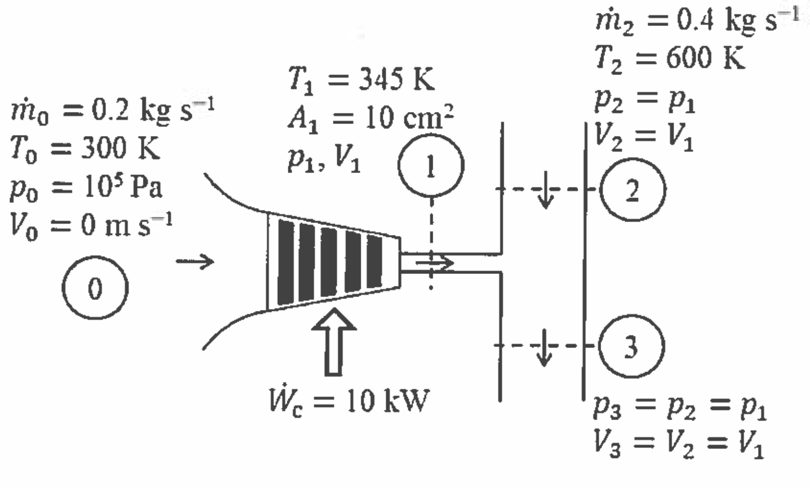



Fig. 3

The properties of air can be taken as $c_p = 1000$ J kg$^{-1}$ K$^{-1}$ and $R = 287$ J kg$^{-1}$ K$^{-1}$.

 (TURN OVER

6. **(long)** An ideal Ericsson cycle operates with maximum and minimum temperatures $T_h$ and $T_l$, respectively, and maximum and minimum pressures $p_h$ and $p_l$, respectively, as shown in Fig. 4. Assume that the cycle fluid is air which can be treated as a perfect gas with gas constant $R$ and specific heat capacity at constant pressure $c_p$.

(a) Sketch the cycle on a temperature-entropy diagram. Label the heat transfer and work done as positive or negative for each process. [5]

(b) Use conservation of energy to determine a relationship for the heat transfer per unit mass of working fluid for the constant temperature compression, $q_{1\to2}$, in terms of $R$, $T_l$, $p_h$ and $p_l$. Relate this quantity to the specific work done, $w_{1\to2}$. [6]

(c) Show that the efficiency of the standard Ericsson cycle is

$$\eta = \frac{\left(1-\hat{T}_R\right)\ln \hat{p}_R}{\frac{5}{2}\left(1-\hat{T}_R\right)+\ln \hat{p}_R}$$

where $\hat{T}_R = T_l/T_h$, $\hat{p}_R = p_h/p_l$ and $c_p = 5R/2$. [7]

(d) For a temperature ratio $\hat{T}_R = 0.2$ sketch a graph of the efficiency of the standard Ericsson cycle, as $\hat{p}_R$ varies from 10 to 100. [4]

(e) Consider an ideal Ericsson cycle operating as a heat engine between constant temperature reservoirs. If all heat is transferred to the cycle at a temperature $T_h$ and rejected to the environment at a temperature $T_l$, show that the total rate of entropy generation is

$$\dot{S}_{gen} = \dot{m}c_p\left(\hat{T}_R-1\right)^2/\hat{T}_R$$

where $\dot{m}$ is the mass flow rate of air within the cycle. Using the Clausius inequality, discuss how the Ericsson cycle can become reversible using an ideal regenerator. [8]

(cont.

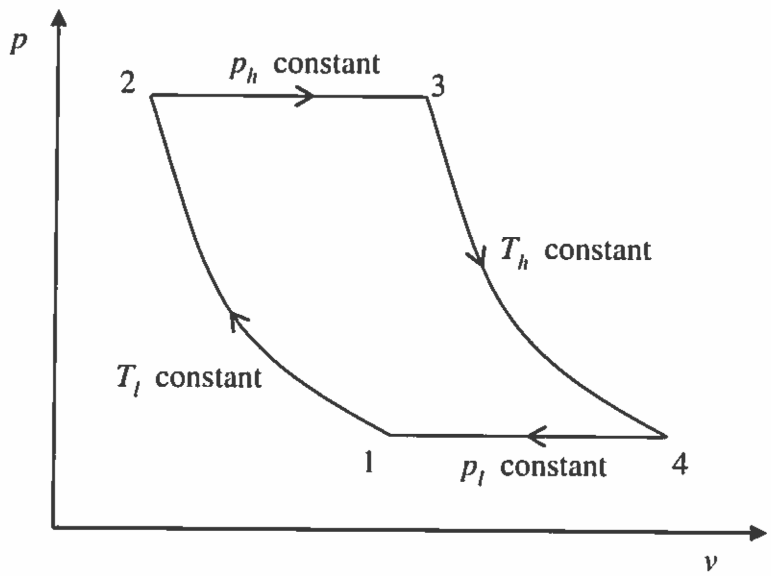



Fig. 4

## SECTION B

7 **(short)** A fairground ride consists of two linked rotating arms, as illustrated in Fig. 5. The arm OC rotates at a constant rate $\dot{\theta}$ about fixed point O and the arm CP rotates about point C at a constant rate $\dot{\phi}$ relative to OC. The ride is designed to accommodate a person at point P. The length OC is $L_1$ and the length of CP is $L_2$. Unit vector $\mathbf{e}_r$ points along the arm OC and unit vector $\mathbf{e}_\theta$ is perpendicular to it, as shown.

(a) Write down the position vector $\mathbf{r}_P$ for point P in terms of $L_1$, $L_2$, $\theta$, $\phi$ and the unit vectors shown. [3]

(b) What is the velocity $\dot{\mathbf{r}}_P$ and the acceleration $\ddot{\mathbf{r}}_P$ of the point P? [7]

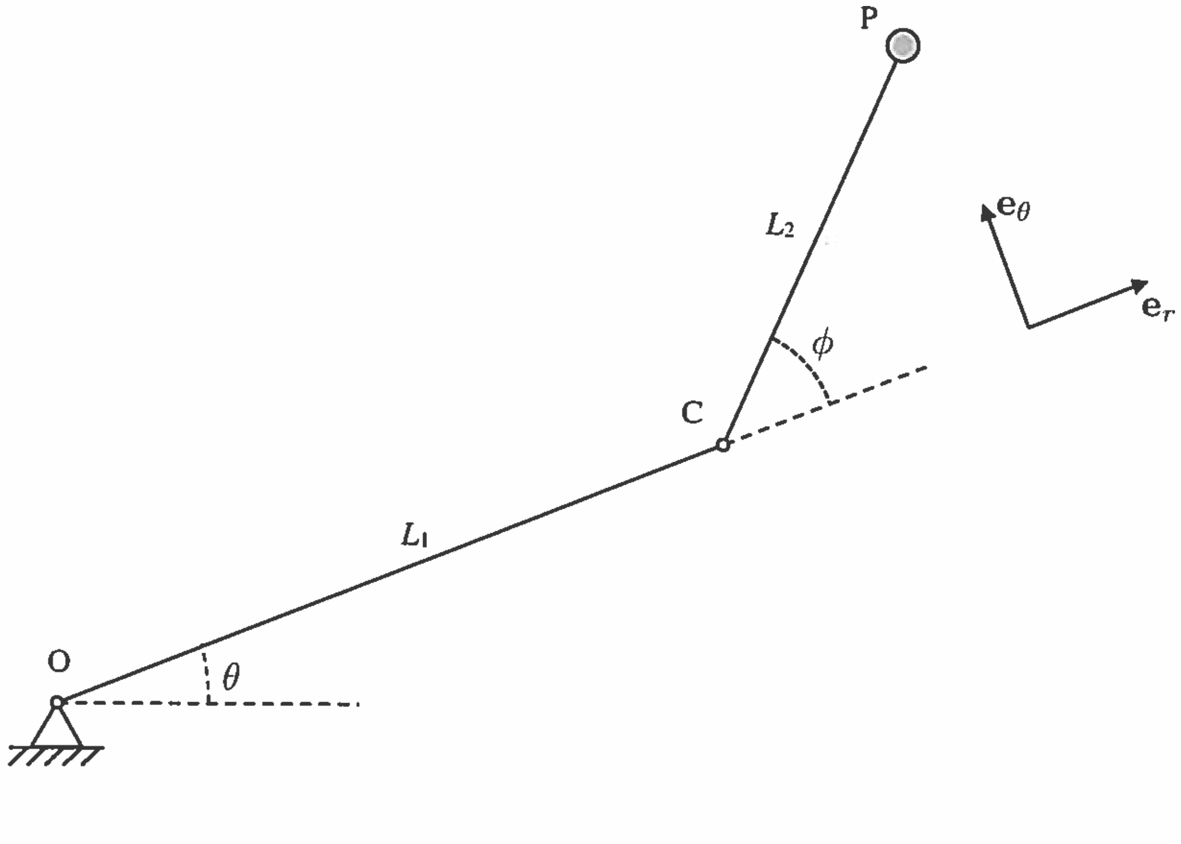



Fig. 5

8 **(short)** Figure 6 illustrates two circular discs with radii $R_1$ and $R_2$ that are free to spin about centres $C_1$ and $C_2$. Both discs have mass per unit area $\sigma$. They are held in contact by a normal preloading force $N$ and the coefficient of friction between the contacting surfaces is $\mu$. A mass $M$ is hung by a light string which is wrapped about the right hand disc.

(a) Derive the polar moment of inertia of a uniform circular disc of radius $R$ and mass per unit area $\sigma$, about its centre. Verify that your answer agrees with the Mechanics databook. [4]

(b) Find expressions for the angular acceleration of the left disc and the acceleration of the mass when there is slipping between the two discs. [6]

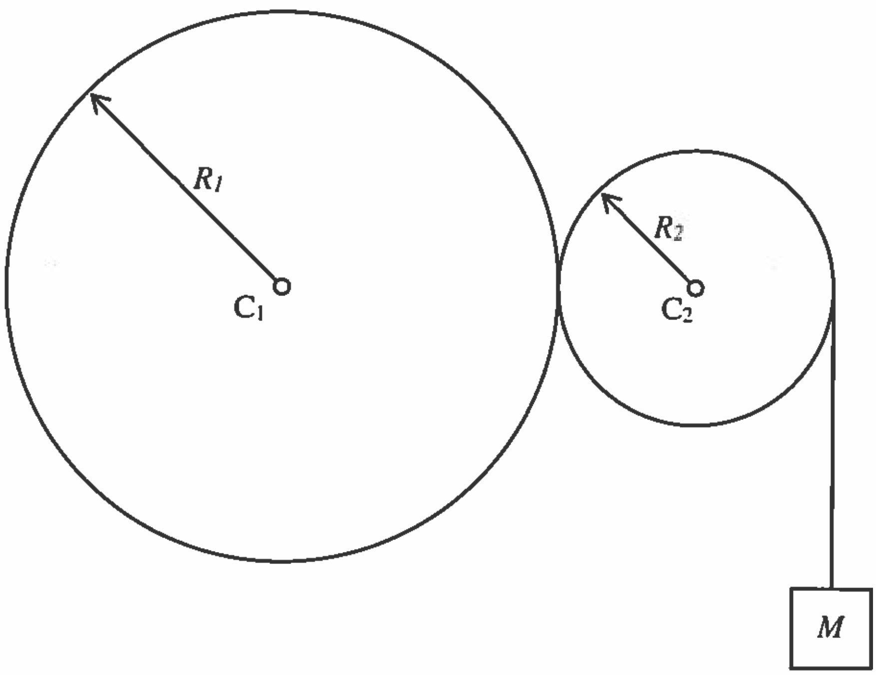



Fig. 6



(TURN OVER

9 **(long)** Figure 7 shows a scale diagram of a proposed design for a new crane. The link QA has a fixed length of 5 m and rotates clockwise at a rate of $\Omega = 0.1$ rad $s^{-1}$, while the piston PB can extend or contract as needed. In the position shown, the length of PB is 4 m.

(a) As a first test of the design, the piston PB contracts at a rate of 0.4 m $s^{-1}$.

(i) Draw a velocity diagram for the system and find the magnitude of the velocity of point C. A scale of 10 cm = 1 m $s^{-1}$ is recommended. *An additional copy of Fig. 7 is attached to the back of this paper. It should be detached and handed in with your answers.* [7]

(ii) At what angle to the horizontal is the velocity of point C? [3]

(b) The piston is now used to ensure that point C travels horizontally to the right.

(i) Does the piston PB need to extend or contract in order to achieve this? [3]

(ii) Using your velocity diagram, determine the rate at which the piston should extend or contract. [10]

(c) A friction torque of 0.3 N m resists the motion of the crane at each joint A, B, P and Q. For the case when the piston PB contracts at a rate of 0.4 m $s^{-1}$, what drive torque is needed at Q? [7]

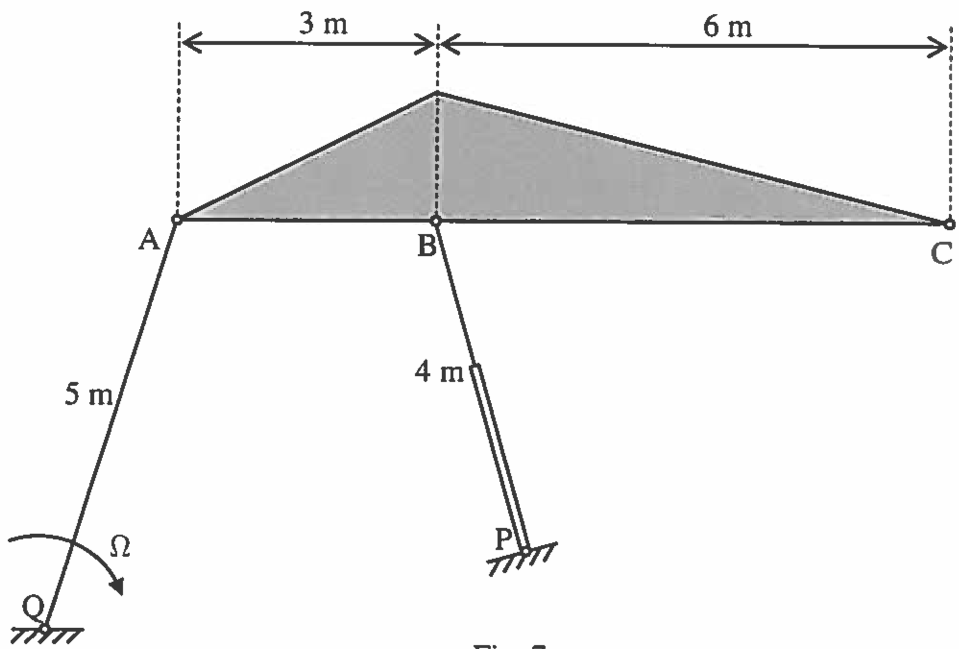



Fig. 7

10 **(short)** The accelerometer shown in Fig. 8 consists of a mass $m$ attached to a rigid housing by a spring of stiffness $k$ and un-stretched length $L$ (not shown). A dashpot with damping $\lambda$ is parallel to the spring. A strain gauge attached to the spring gives a voltage output $V$ proportional to the strain $\varepsilon$ in the spring so that $V = C\varepsilon$, where $C$ is a constant. The housing has small absolute displacement $x$ and the mass has small absolute displacement $y$, both measured relative to their equilibrium positions in the orientation shown.

(a) Find the equation of motion and show that it can be written in the form:

$$\ddot{V} + 2\zeta\omega_n\dot{V} + \omega_n^2 V = -\alpha\ddot{x}$$

Give expressions for the constants $\zeta$, $\omega_n$ and $\alpha$. [4]

(b) The accelerometer is placed on a smooth, horizontal, stationary table with the $x$-axis pointing vertically upward. It is turned over and then put back on the table with the $x$-axis pointing vertically downward. What will be the difference between the steady-state voltage outputs in the two orientations (i.e. after the transients have settled)? [3]

(c) The housing is given a sinusoidal acceleration input in the $x$-direction. Comment on the frequency range over which the accelerometer will give an accurate measurement of the acceleration input, assuming the damping is light. [3]

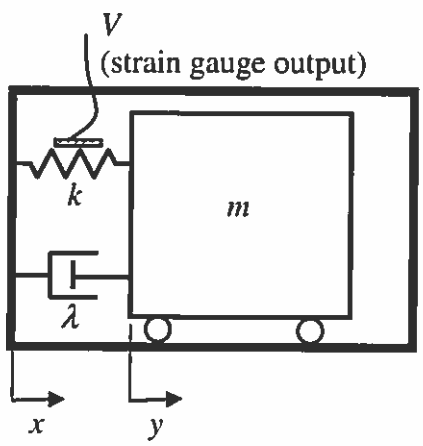



Fig. 8

11 **(short)** (a) Under what circumstances are the following quantities conserved for a particle:

(i) total mechanical energy;

(ii) moment of momentum about a point;

(iii) moment of momentum about an axis? [3]

(b) A conical pendulum consists of a ball of mass *m*, connected by a light, inextensible string of length $L$ to a pivot at point O. The conical pendulum is free to rotate in three dimensions about the pivot under the action of gravity, without friction and with the string remaining taut. The horizontal radius $r$ of the ball is measured from the vertical axis through O, as shown and the depth of the ball below O is $d$.

The ball is given an initial horizontal velocity $V_1$ in a direction perpendicular to horizontal radius $r_1$ at point P, as shown in Fig. 9. In the subsequent motion, the ball reaches its maximum height at point Q (not shown), where it travels with velocity $V_2$, perpendicular to the horizontal radius $r_2$.

(i) Write two equations relating the motion at P to the motion at Q.

(ii) Hence derive (but do not solve) one equation that determines $r_2$. [7]

(cont.

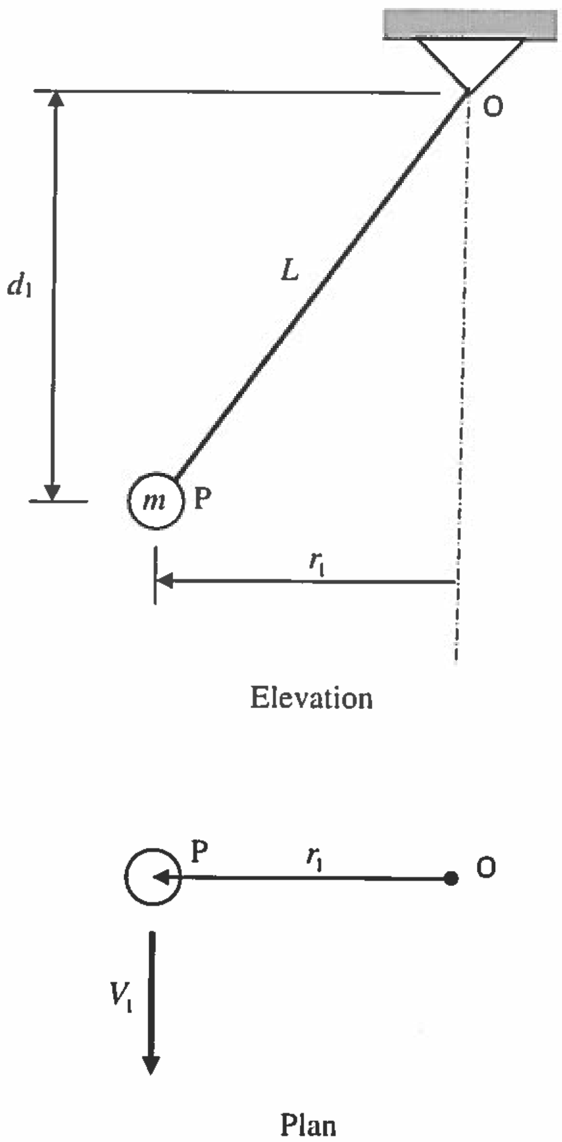



Fig. 9

12 **(long)** Figure 10 shows a vibration model of an engine which consists of two disks with polar moments of inertia $J$ and $2J$, having torsional displacements $\theta_1$ and $\theta_2$, connected by a light shaft with torsional stiffness $k$. The system is supported at each end by a frictionless bearing. A torque $q$ can be applied to the disk with inertia $2J$.

(a) Show that the equations of motion of the engine can be written:

$$\begin{bmatrix} J & 0 \\ 0 & 2J \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ q \end{bmatrix}$$ [6]

(b) Determine the natural frequencies and mode shapes for free vibration of the engine. Sketch the mode shapes. [10]

(c) A sinusoidal torque $q = Q\sin\omega t$ is applied to the larger disk. Calculate the torsional displacements of the two disks. Sketch a graph of these displacements as a function of the input frequency $\omega$, showing salient values. [8]

(d) Describe the nature of the motion of the engine in part (c) at very low frequencies and at very high frequencies. Comment on the phases of the two responses relative to the input torque. [6]

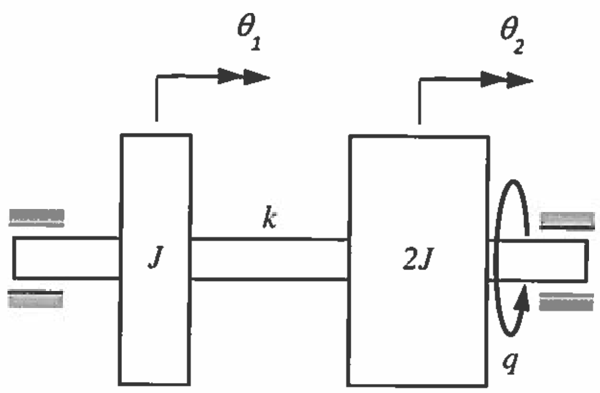
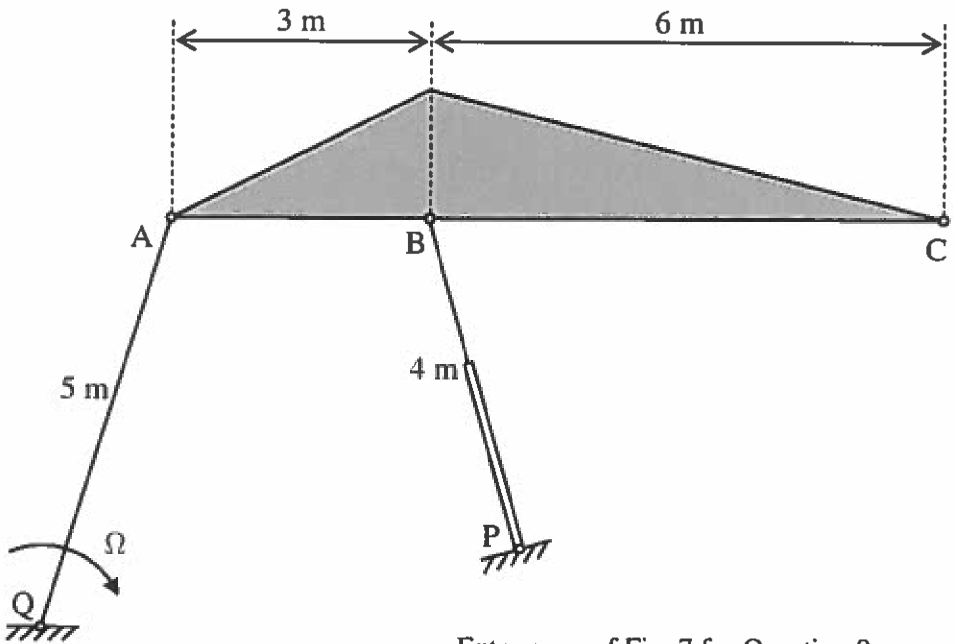



Fig. 10

**END OF PAPER**

Version DC/3 Candidate Number:

EGT0
ENGINEERING TRIPOS PART IA
Wednesday 7 June 2017, Question 9




Extra copy of Fig. 7 for Question 9.