EGT1
ENGINEERING TRIPOS PART IB

Monday 5 June 2017 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section. All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section*

1 Figure 1(a) shows an athlete carrying a slender uniform rigid rod AB of length $2a$ and mass m. The rod is vertical and travelling at velocity u_1 in a horizontal direction. The athlete applies an impulse, I, at A so that the rod rises and rotates in a clockwise direction, Fig. 1(b). Immediately after the impulse is applied the horizontal velocity at A is zero.

(a) Find an expression for the horizontal component of I and show that after the impulse is applied the horizontal velocity, u_2, at the centre of mass G is $3u_1/4$ and the angular velocity of the rod is u_2/a. [6]

(b) Once the centre of mass G has returned to its original height, the rod has rotated clockwise through π rad, Fig. 1(c). Find an expression for the vertical component of I required to achieve this. [13]

(c) If $m = 50\,\mathrm{kg}$, $a = 3\,\mathrm{m}$ and $u_1 = 4\,\mathrm{m\,s^{-1}}$, calculate the magnitude of I. Comment on whether this is realistic for an athlete to generate. [6]

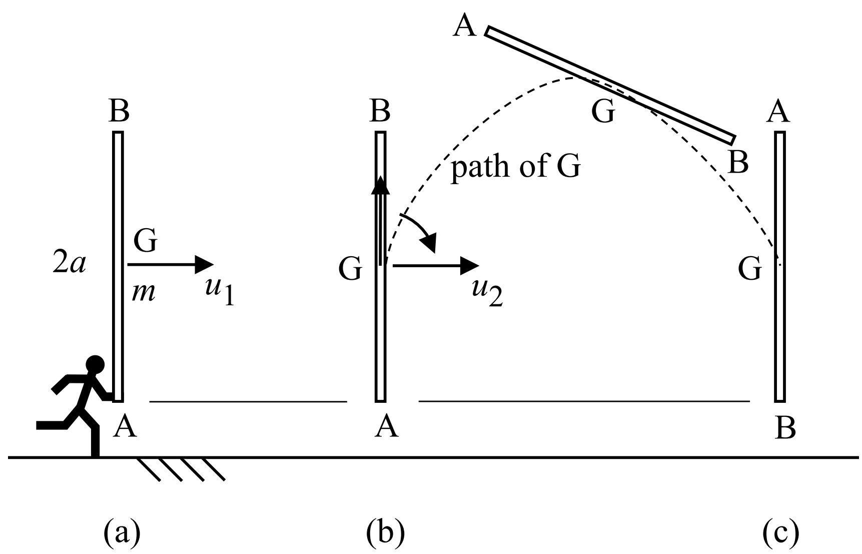

Fig. 1

2 Figure 2 shows a uniform solid cylinder of mass m, radius b and centre G. The cylinder is placed inside a fixed tube of internal radius a and centre O. The axes of the cylinder and the tube are horizontal. When the cylinder is at rest, point A$'$ on the surface of the tube and point A$''$ on the surface of the cylinder are coincident. When the cylinder is displaced the point of contact moves from A$'$ to A defined by the small angle θ. There is no slip between the cylinder and the tube.

(a) Show that the angle of rotation of the cylinder, ψ, is related to the angle θ by the expression $\psi = \theta(a-b)/b$. [6]

(b) Find an expression for the potential energy of the cylinder as a function of θ. [6]

(c) Find an expression for the kinetic energy of the cylinder as a function of $\dot{\theta}$. [6]

(d) The cylinder is now released. Find an expression for the angular frequency of small oscillations. [7]

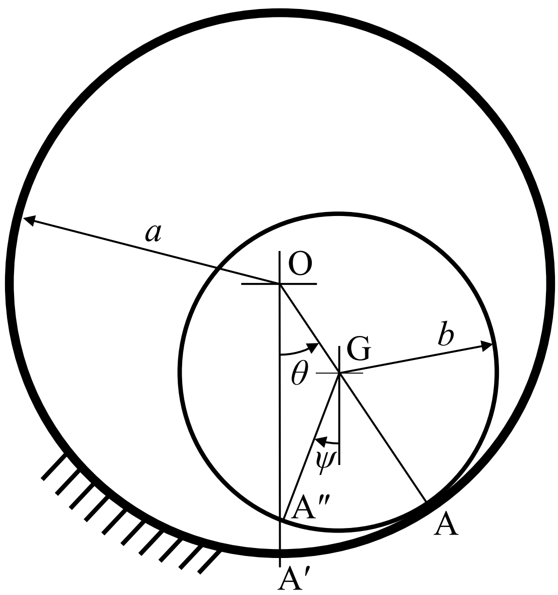

Fig. 2

3 Figure 3 shows a light arm OA of length R that is driven at constant angular velocity Ω about a vertical axis $\mathbf{k}$. Mounted on the arm at A is a thin uniform disc of radius a and mass m. The plane of the disc is vertical and coincident with OA. The disc rotates at constant speed ω about an axis perpendicular to the plane of the disc in the direction shown. At the instant shown, and expressing your answer using the unit vectors shown in Fig. 3, find expressions for:

(a) the velocity and acceleration of A relative to O; [5]

(b) the forces and moments acting on the arm OA at O. [8]

A point mass m is now attached to a point B on the circumference of the disc. Point B is instantaneously above A. At the instant shown, and expressing your answer using the unit vectors shown in Fig. 3, find expressions for:

(c) the velocity and acceleration of B relative to O; [7]

(d) the additional forces and moments acting on the arm OA at O. [5]

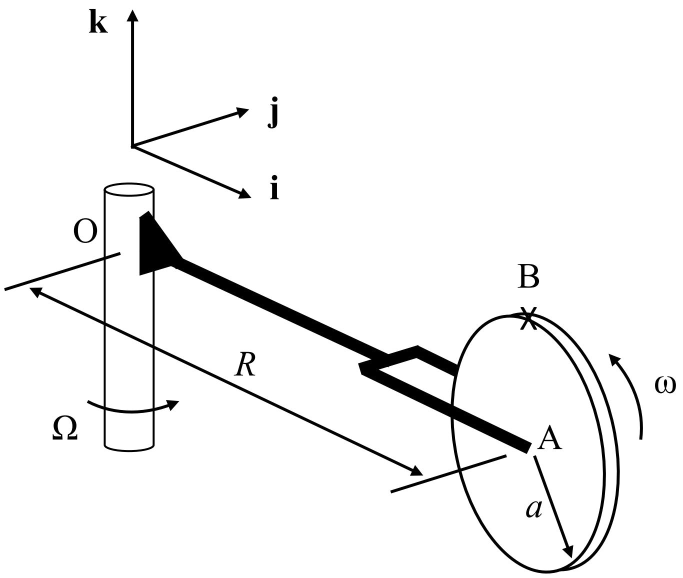

Fig. 3

SECTION B

*Answer not more than **two** questions from this section*

4 The planar mechanism sketched in plan view in Fig. 4 is driven by applying a moment M at S such that the link RS rotates with constant velocity ω about S. Link PQ can be approximated as massless while links QR and RS are uniform and each has a mass m. All joints are frictionless and the length of each link is indicated in Fig. 4.

(a) What are the angular velocities of links QR and PQ at the instant shown in Fig. 4? [5]

(b) What are the components of the acceleration of the centre of mass of QR parallel and perpendicular to QR? [8]

(c) What is the angular acceleration of QR? [4]

(d) Determine the applied moment M at S for the instant shown in Fig. 4. [8]

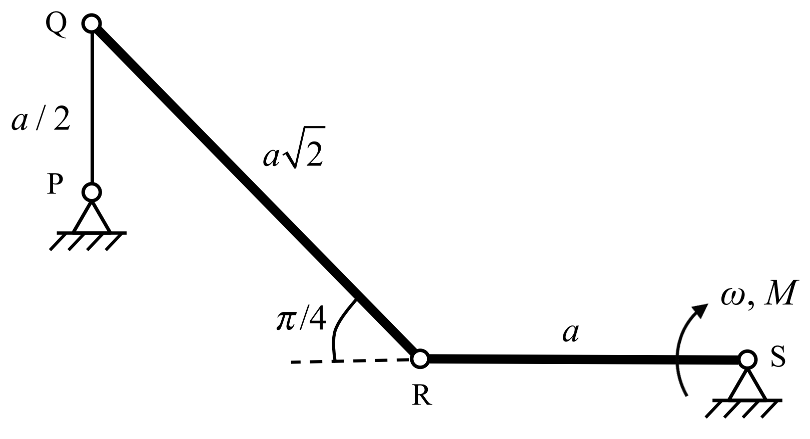

Fig. 4

5 A stationary ball of mass M and radius R lying on a horizontal surface is subjected to a horizontal impulse, P, as shown in Fig. 5. The impulse is applied at a height a above the centre of the ball, with $a < R$. The coefficient of friction between the ball and the surface is μ.

(a) For $a = 0$, calculate the velocities and accelerations (linear and angular) immediately after the application of the impulse. [5]

(b) Determine an expression for a_0, the value of a for which the ball rolls without slip immediately after the application of the impulse. [9]

(c) Describe the motion of the ball after application of the impulse when $a > a_0$ and determine the time after application of the impulse when the ball starts pure rolling. [11]

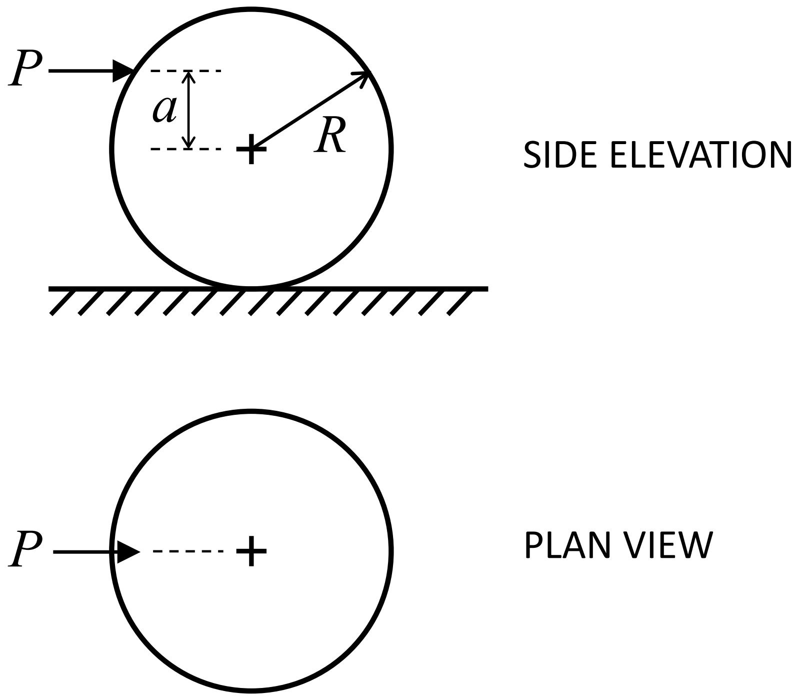

Fig. 5

6 (a) State the circumstances under which the mechanical energy of a system is conserved. [4]

(b) A long solid homogeneous cylinder of mass M and radius R is placed at the edge of a table such that the longitudinal axis of the cylinder and the edge of the table are in the same vertical plane, as shown in Fig. 6. There is sufficient friction present between the edge of the table and cylinder so that a small perturbation of the cylinder causes the cylinder to roll off the table without slipping.

(i) Calculate the angle through which the cylinder rotates about the edge of the table before losing contact with the edge. [8]

(ii) Calculate the angular velocity of the cylinder at the instant it loses contact with the edge of the table. [4]

(iii) Determine the ratio of the translational to rotational kinetic energies of the cylinder when the centre of mass of the cylinder is in the same horizontal plane as the surface of the table. [9]

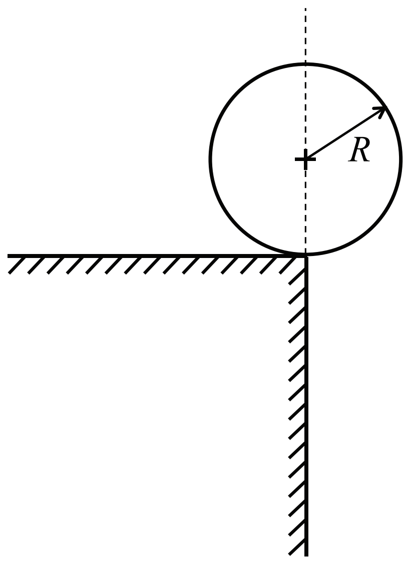

Fig. 6

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Answers

1.
(a) $-\dfrac{mu_1}{4}$

(b) $\dfrac{2\pi mga}{3u_1}$

(c) 787 Ns

2.
(b) $mg(a-b)(1-\cos\theta)$
(c) $\dfrac{3}{4}m\dot{\theta}^2(a-b)^2$

(d) ${\omega_n}^2 = \dfrac{2g}{3(a-b)}$

3.
(a) $\mathbf{v}_A = \Omega R\mathbf{j} \quad \mathbf{a}_A = -\Omega^2 R\mathbf{i}$

(b) $\mathbf{F}_O = -m\Omega^2 R\mathbf{i} + mg\mathbf{k}$
$\mathbf{M}_O = \dfrac{m\omega\Omega a^2}{2}\mathbf{i} - mgR\mathbf{j}$

(c) $\mathbf{v}_B = -\omega a\mathbf{i} + \Omega R\mathbf{j}$
$\mathbf{a}_B = -\Omega^2 R\mathbf{i} - 2\Omega\omega a\mathbf{j} - \omega^2 a\mathbf{k}$

(d) $\mathbf{F}_O = -\Omega^2 Rm\mathbf{i} - 2\Omega\omega am\mathbf{j} + m(g-\omega^2 a)\mathbf{k}$
$\mathbf{M}_O = 2\Omega\omega a^2 m\mathbf{i} + m(\omega^2 aR - \Omega^2 Ra - gR)\mathbf{j} - 2\Omega\omega amR\mathbf{k}$

4.
(a) $\omega_{QR} = \omega$ anticlockwise; $\omega_{PQ} = 2\omega$ anticlockwise

(b) $\sqrt{2}a\omega^2$ parallel; 0 perpendicular

(c) ω^2 anticlockwise

(d) $\dfrac{5ma^2\omega^2}{6}$ anticlockwise

5.
(a) $v = \dfrac{P}{M} \quad \omega = 0 \quad \dot{v} = -\mu g \quad \dot{\omega} = \dfrac{5\mu g}{2R}$

(b) $\dfrac{2}{5}R$

(c) $\dfrac{2}{7}\dfrac{P}{\mu Mg}\left(\dfrac{5a}{2R} - 1\right)$

6.

(b)

(i) $\cos^{-1}\left(\frac{4}{7}\right)$

(ii) $\sqrt{\frac{4g}{7R}}$

(iii) 6