

EGT1  
ENGINEERING TRIPOS PART IB

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Tuesday 6 June 2017 2 to 4

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**Paper 4**

**THERMOFLUID MECHANICS**

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

*All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**

Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

Engineering Data Book

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**SECTION A**

Answer not more than **two** questions from this section.

1 A humidification system is shown in Fig. 1.

(a) Water is supplied at 25 °C and 1 bar. First, it enters an adiabatic, reversible pump, which raises its pressure to 100 bar. Next, it is heated at constant pressure until its dryness fraction is 50%, and then passed through an adiabatic throttle to drop the pressure back to 1 bar. Determine (per kg of water flowing):

- (i) the work input to the pump; [2]
- (ii) the heat supplied by the water heater; [4]
- (iii) the temperature, state, and entropy of the fluid leaving the throttle. [6]

(b) The fluid leaving the throttle enters a humidifier, where it is mixed with dry air at 50 °C and 1 bar. The resulting wet air leaves the humidifier at 50 °C and 1 bar, with relative humidity 50%.

- (i) What is the ratio between the water and dry-air mass flow rates? [4]
- (ii) What is the dew point temperature of the wet air? [3]

- (c) (i) The water vapour in the wet air may be treated as a perfect gas. Explain how this property may be used to deduce its enthalpy from tabulated saturation values. [2]
- (ii) Evaluate  $q$ , the required humidifier heat transfer (per kg of water flowing). Is heat added or removed? [4]

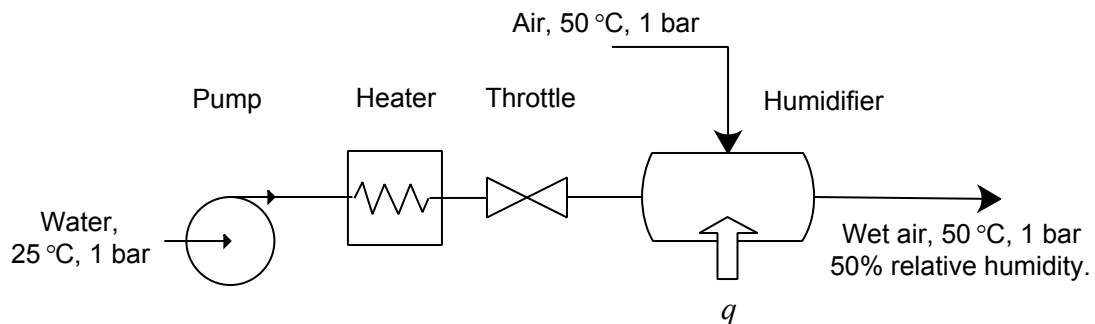


Fig. 1

2 The indirectly heated gas turbine system shown in Fig. 2 uses one turbine to drive its compressor and a second to generate power. The compressor has an isentropic efficiency of 0.8, and a pressure ratio of 10. The turbines both have an isentropic efficiency of 0.9. There is no pressure drop in the heater. A  $1 \text{ kg s}^{-1}$  air flow enters the compressor at the environment condition:  $25 \text{ }^\circ\text{C}$  and 1 bar. The first turbine's inlet temperature is 1200 K, and the second turbine exhausts to the environment. You can assume that the air behaves as a perfect gas and that all turbomachinery is adiabatic.

- (a) Calculate:
- (i) the temperature at the exit of the compressor; [2]
  - (ii) the pressure ratio across the first turbine; [5]
  - (iii) the power output from the second turbine. [4]
- (b) (i) How much power potential has been gained by the air in the heater? [3]
- (ii) What is the total loss of power potential due to irreversibilities in the system? [5]
- (iii) Explain why the difference between your answers to parts (i) and (ii) is not equal to the work output from the second turbine. [2]
- (c) The heat for the system comes from a constant-pressure,  $1 \text{ kg s}^{-1}$  flow of hot combustion gases leaving a burner at 1500 K, with  $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$ . How much power potential is lost in transferring this heat from the combustion gases to the air in the gas turbine cycle? [4]

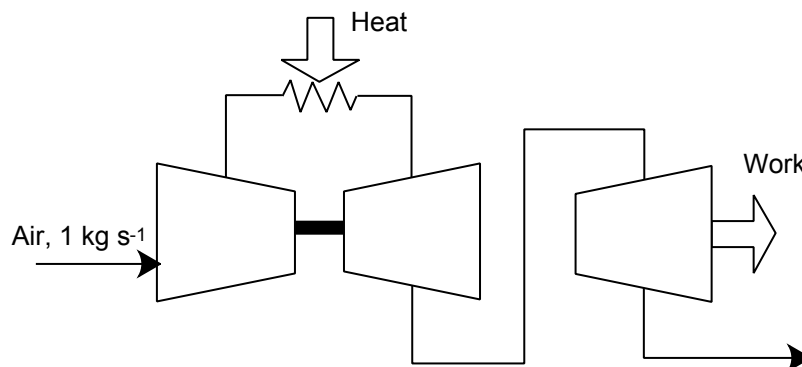


Fig. 2

3 A fluid with a Prandtl number of unity is passing through a heat-exchanger tube. The fluid has a velocity of  $100 \text{ m s}^{-1}$ , density  $1 \text{ kg m}^{-3}$ , heat capacity  $1 \text{ kJ kg}^{-1}$  and thermal conductivity  $0.03 \text{ W m}^{-1} \text{ K}^{-1}$ . The flow is fully developed and end effects can be neglected. The inner radius of the tube is 10 mm and the tube wall consists of two materials. The inner material has thermal conductivity  $10 \text{ W m}^{-1} \text{ K}^{-1}$  and thickness 5 mm. The outer material has thermal conductivity  $5 \text{ W m}^{-1} \text{ K}^{-1}$  and thickness 3 mm. Physical properties can be assumed to be independent of temperature.

(a) By considering a suitable control surface or differential volume, show that the thermal resistance for radial heat transfer through unit length of an annulus with thermal conductivity  $\lambda$ , inner radius  $r_1$  and outer radius  $r_2$  is

$$\frac{1}{2\pi\lambda} \ln\left(\frac{r_2}{r_1}\right)$$

and hence determine the total thermal resistance of 1 m of the tube wall. [8]

(b) (i) Briefly describe what is meant by Reynolds analogy in the context of heat transfer, and suggest why it applies equally as well inside a pipe as along a flat plate. [4]

(ii) At a particular point along the length of the tube the temperatures of the inner and outer walls are  $140 \text{ }^\circ\text{C}$  and  $150 \text{ }^\circ\text{C}$  respectively. The temperature difference between the inner wall and the bulk fluid is  $52 \text{ }^\circ\text{C}$ . What is the pressure drop in the fluid per unit length of tube? [8]

(c) The Nusselt number in the tube is proportional to  $\text{Re}^{0.8}$ , where Re is the flow Reynolds number. If the velocity of the fluid is reduced to  $20 \text{ m s}^{-1}$ , what is the new pressure drop per unit length of tube? [5]

## SECTION B

Answer not more than **two** questions from this section.

4 A viscous liquid, with density  $\rho$  and dynamic viscosity  $\mu$ , flows steadily down a plane inclined at angle  $\theta$  to the horizontal (Fig. 3). The depth of the liquid,  $h$ , and its velocity parallel to the plane,  $V(y)$ , are independent of the stream-wise coordinate,  $x$ .

(a) (i) How does the gauge pressure in the liquid vary with position? Justify your answer. [4]

(ii) Find the velocity profile  $V(y)$ . [7]

(b) A thin plate of mass-per-unit-area  $M$  is placed on the liquid surface at  $y = h$  and released. What is the new velocity profile, once steady conditions have been re-established? [5]

(c) The plate of part (b) is now dragged upwards in the direction parallel to the plane, with speed  $U$ . What value of  $U$  results in zero net volumetric flow rate down the plane? [9]

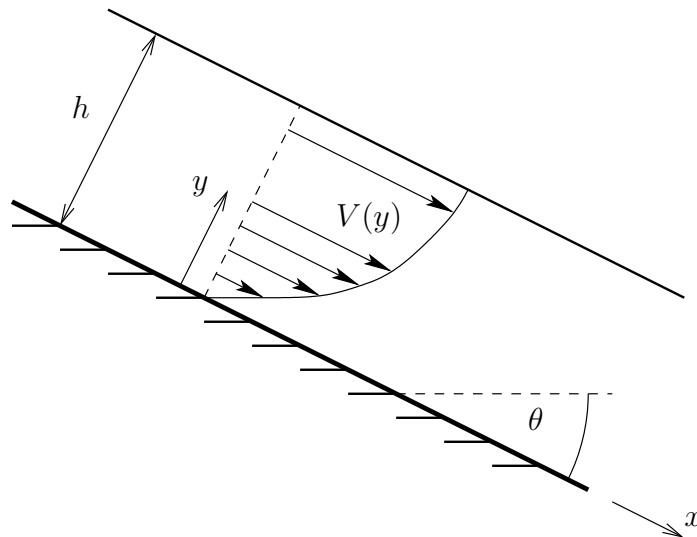


Fig. 3

5 (a) An irrigation system consists of a set of nozzles supplied with a volumetric flow rate  $Q$  of water from a reservoir. The nozzle exit flows have parallel streamlines, and the combined nozzle exit area is  $A_n$ . The pipe connecting the reservoir to the nozzles is of length  $L$ , with diameter  $d$  and skin-friction coefficient  $c_f$ . Losses at the pipe entry and in the nozzle flows are negligible. The water in the reservoir has density  $\rho$  and surface height  $H$  (relative to the nozzles). Find an expression for  $H$ . [8]

(b) Figure 4 (not to scale) shows an irrigation facility with two reservoirs. The main reservoir has variable surface height  $H_1$ , and is connected to the nozzles by a 1 km supply pipe of diameter 1 m. The second reservoir acts as a reserve. Its supply pipe is 10 km long, its surface is at constant height  $H_2 = 1000$  m, and it can provide a maximum flow of  $0.1 \text{ m}^3 \text{ s}^{-1}$ . The skin-friction coefficient of both supply pipes is 0.005 and they meet at the junction J. Losses at, and downstream of, J are negligible.

(i) The system is to be designed such that flow from the main reservoir ceases when the height  $H_1$  is 10 m, and the reserve flow is then at its limit. On this basis, specify the total nozzle area and the diameter of the reserve-reservoir supply pipe. [6]

(ii) At what value of  $H_1$  are the reserve-reservoir and main-reservoir flow rates equal? [5]

(iii) For the conditions of part (ii), compare the total head at the junction J with that in the main reservoir, and comment. [2]

(iv) When the main reservoir is full,  $H_1$  is 100 m. Perform an approximate calculation to estimate the total irrigation flow rate in this case. (Your answer to part (iii) may help.) [4]

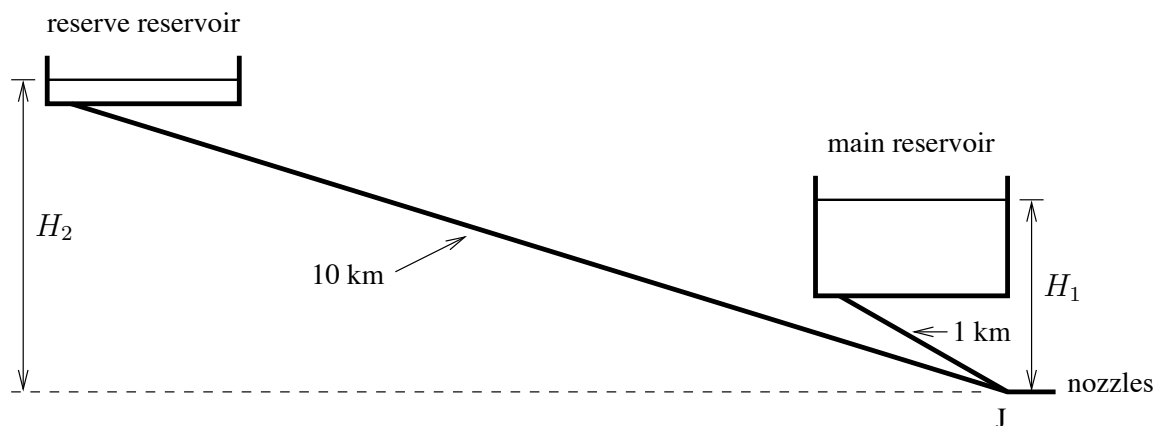


Fig. 4

6 Figure 5 shows a chemical mixing tank. Fluid of density  $\rho$  and dynamic viscosity  $\mu$  fills a cylindrical vessel (height  $h$ , diameter  $D$ ), and is stirred by an impeller rotating with angular velocity  $\Omega$ .

(a) The torque,  $Q$ , required to drive the impeller depends only on  $\rho$ ,  $\mu$ ,  $h$ ,  $D$  and  $\Omega$ . Derive the general form of this dependence in terms of dimensionless groups. [8]

(b) A quarter-scale model of the tank is constructed for testing. The fluid properties are unchanged. How should the angular velocity be scaled in order to ensure dynamical similarity between the model and its full-scale counterpart? [4]

(c) When dynamical similarity is achieved, what is the ratio between the torques required to stir the model and the full-scale tank? [4]

(d) The contents of the tank have to be maintained at constant temperature. In the model test, the angular velocity is  $8 \text{ rad s}^{-1}$ , the torque is  $1000 \text{ N m}$ , and  $7.8 \text{ kW}$  of internal cooling must be provided.

(i) Assuming that all the mechanical power supplied to the impeller is dissipated as heat, and no chemical reactions occur in the fluid, what is the rate of heat loss from the model surface to the surroundings? [3]

(ii) The manufacturers wish to avoid the cost and complexity of a cooling system on the full-size tank. Discuss, with supporting calculations, whether this is likely to be possible. [6]

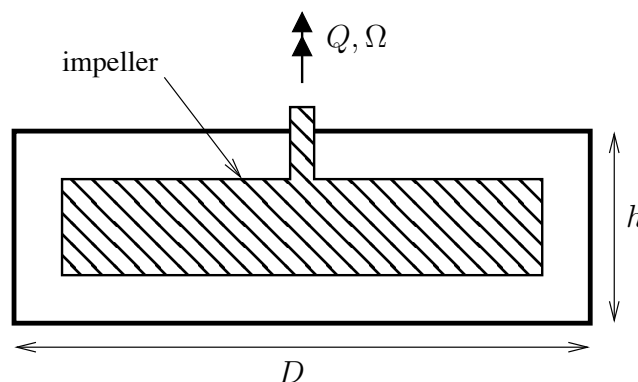


Fig. 5

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ENGINEERING TRIPOS PART IB

**Paper 4: THERMOFLUID MECHANICS**

**ANSWERS**

1(a) (i) 9.93kJ/kg (ii) 1950kJ/kg  
(iii) 99.6°C; two-phase with dryness fraction 0.731; 5.73kJ/kgK

(b) (i) 0.0409 (ii) 36.6°C

(c) (ii) 525kJ/kg added

2(a) (i) 645K (ii) 3.88 (iii) 183kW

(b) (i) 372kW (ii) 57.1kW

(c) 47.6kW

3(a) 0.0123K/W

(b) (ii) 4990Pa/m

(c) 275Pa/m

4(a) (i)  $p = \rho g(h - y)\cos\theta$  (ii)  $V(y) = \frac{\rho g \sin\theta}{\mu} y \left( h - \frac{y}{2} \right)$

(b)  $V(y) = \frac{g \sin\theta}{\mu} y \left[ M + \rho \left( h - \frac{y}{2} \right) \right]$

(c)  $U = \frac{\rho g h^2}{6\mu} \sin\theta$

5(a)  $H = \frac{Q^2}{g} \left[ \frac{1}{2A_n^2} + \frac{32c_f L}{\pi^2 d^5} \right]$

(b) (i)  $7.14 \times 10^{-3} \text{m}^2$ ; 0.176m (ii) 38.9m  
(iii) 0.016m difference (iv)  $0.316 \text{m}^3/\text{s}$

6(a)  $\frac{Q}{\rho\Omega^2 D^5} = f\left(\frac{\mu}{\rho\Omega D^2}, \frac{h}{D}\right)$ , or equivalent

(b) Model rotates 16 times faster

(c) Model torque is 1/4 full-scale value

(d) (i) 200W