EGT1
ENGINEERING TRIPOS PART IB

Thursday 8 June 2017   2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper, graph paper, semilog graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

Answer not more than ***two*** *questions from this section.*

1 The *RLC* circuit in Fig. 1 has input voltage, u, and output voltage, y. The transfer function for the circuit between $\bar{u}$ and $\bar{y}$ is given by

$$G(s) = \frac{1}{LCs^2 + (L/R)s + 1}$$

For simplicity, in what follows take $L = 1\,\mathrm{H}$, $C = 1\,\mathrm{F}$ and assume $R > 0\ \Omega$.

(a) (i) Compute the poles of the transfer function as a function of the resistance, R. Is $G(s)$ asymptotically stable for any value of R? [4]

(ii) Compute the impulse response for $R > 0.5\ \Omega$. What is the effect of a smaller resistance, R, on the transients? Sketch the impulse response. [5]

(b) Compute the steady-state output, y, to a step input voltage $u = V$. [4]

(c) Take $R = 1\ \Omega$.

(i) Sketch the Bode and Nyquist diagrams of $G(s)$ and estimate the gain margin. [6]

(ii) Using the Nyquist criterion, find the gains, k, that guarantee stability of the closed-loop system made by the interconnection of $G(s)$ with the proportional controller $u = -ky$. Consider both positive and negative values of k. [6]

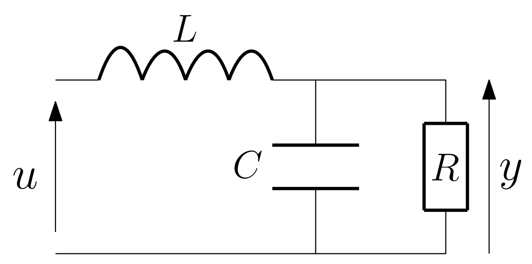

Fig. 1

2 The block diagram in Fig. 2 represents an industrial control plant. The controller, $K_1(s)$, within the (fast) inner control loop stabilises the industrial process, $G_1(s)$. Regulation is achieved by the controller, $K_2(s)$, within the (slow) outer control loop. Transport delays are modeled by $G_2(s)$.

(a) In terms of $G_1(s)$, $G_2(s)$, $K_1(s)$ and $K_2(s)$:

(i) derive the transfer functions $G_{u_1,y}(s)$ and $G_{u_2,y}(s)$ respectively from $\bar{u}_1$ to $\bar{y}$ and from $\bar{u}_2$ to $\bar{y}$; [3]

(ii) derive the sensitivity and complementary sensitivity functions $S(s)$ and $T(s)$ respectively from $\bar{r}$ to $\bar{e}$ and from $\bar{r}$ to $\bar{y}$. [4]

(b) Take $G_1(s) = 1/s$ (first order integrator), $G_2(s) = e^{-Ds}$ (simple delay) and $K_1(s) = k$ (proportional controller).

(i) Show that $G_{u_1,y}(s)$ and $G_{u_2,y}(s)$ are asymptotically stable for any $k > 0$. [4]

(ii) Compute the steady-state response of $G_{u_2,y}(s)$ to the step input $u_2(t) = 1$, for $t \geq 0$. Sketch the complete response. How does the response change for larger values of k and D? [6]

(c) Design $K_2(s)$ to achieve a zero steady-state error to constant references.

(i) Select between a proportional controller, $K_2(s) = 1$, and an integral controller $K_2(s) = 1/s$. Motivate your answer by computing the steady-state error, e, to constant reference, r, assuming that the closed loop is stable. [5]

(ii) What is the steady-state output, y, to a constant reference, r? [3]

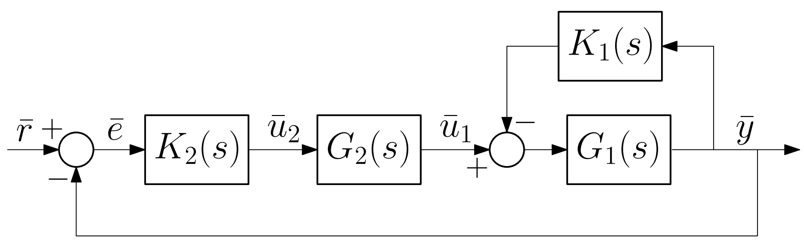

Fig. 2

3 A loudspeaker converts electrical audio signals into sounds. The electrical signal drives a current through a voice coil generating a magnetic field. The *voice coil actuator* force, F_{vca}, generated by the current, I, through the coil is regulated by the equation (Lorentz force)

$$F_{vca} = k_F I$$

where the force sensitivity, k_F, is approximated by a constant. The voice coil electrical behaviour is captured by the equivalent circuit

$$E = RI + k_B \frac{dx}{dt} + L\frac{dI}{dt}$$

where E is the external voltage, R and L are the resistance and the inductance of the coil, and $k_B dx/dt$ is the back-electromotive force induced by the displacement of the speaker. Finally, the speaker mechanics are regulated by the equation

$$F_{vca} - F_{ext} = m\frac{d^2x}{dt^2}$$

balancing the Lorentz force and the external force, F_{ext} (mechanical load). The mass of the speaker is m.

(a) Consider the input $u = E$, the output $y = x$ and set $F_{ext} = 0$. Derive the transfer function $G(s)$ from $\bar{u}$ to $\bar{y}$. [5]

(b) A *good* loudspeaker must have a flat response (constant gain at each frequency). Take $k_F = 10\,\mathrm{N\,A^{-1}}$, $k_B = 10\,\mathrm{V\,s\,m^{-1}}$, $R = 1\,\Omega$, $L = 5\times10^{-3}\,\mathrm{H}$ and $m = 1\,\mathrm{kg}$.

(i) Sketch the Bode diagram of $G(s)$. [5]

(ii) Estimate the magnitude of the steady-state response to the sinusoidal input $u = \cos(\omega_i t)$ for frequencies $\omega_1 = 10\sqrt{2}\,\mathrm{rad\,s^{-1}}$ and $\omega_2 = 10^3\sqrt{2}\,\mathrm{rad\,s^{-1}}$. Is the speaker *good*? [5]

(c) The performance of the loudspeaker can be improved by control design. Consider a proportional controller $u = k_C(-y + r)$, where $k_C \geq 0$ is the control gain (Fig. 3).

(i) From the Bode diagram of $G(s)$ sketched in part (b)(i), find the gain margin of the system. What is the largest gain, k_C^*, that guarantees closed-loop stability for any $k_C < k_C^*$? [4]

(ii) The magnitude of the complementary sensitivity function $T(s) = \frac{k_C G(s)}{1+k_C G(s)}$ is reported in Fig. 4 for two different control gains (A and B). Which one corresponds to the larger control gain k_C? Discuss the effect of large values of k_C on the closed-loop performance. [6]

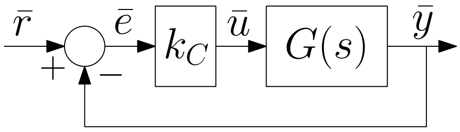
$\bar{r}$
$\bar{e}$
k_C
$\bar{u}$
$G(s)$
$\bar{y}$
+
−

Fig. 3

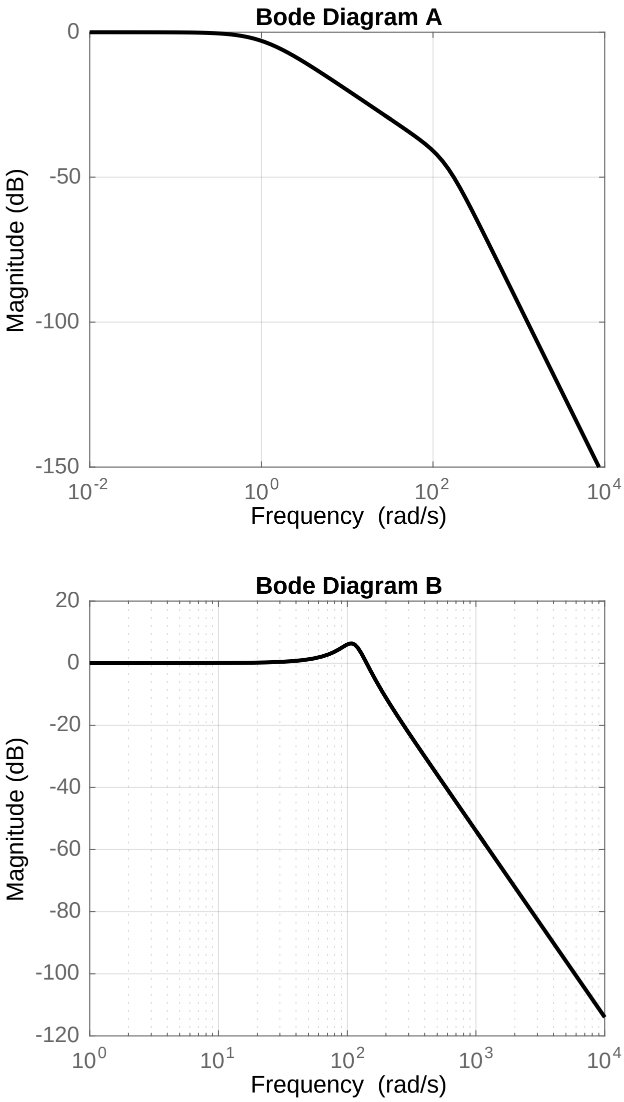
Bode Diagram A
Magnitude (dB)
0
-50
-100
-150
10^{-2}
10^{0}
10^{2}
10^{4}
Frequency (rad/s)
Bode Diagram B
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
-120
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)

Fig. 4

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) For $f(t) = \mathrm{e}^{-|t|}$, sketch $f(t)$ and show by direct integration that its Fourier transform, $F(\omega)$, is given by

$$F(\omega) = \frac{2}{1+\omega^2}$$

[4]

(b) Using *duality* find the Fourier transform of $g(t)$, where

$$g(t) = \frac{1}{1+t^2}$$

[3]

(c) If two functions $f_1(t)$ and $f_2(t)$ have Fourier transforms of $F_1(\omega)$ and $F_2(\omega)$ respectively, derive the following result:

$$\int_{-\infty}^{+\infty} f_1(t) f_2^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_1(\omega) F_2^*(\omega)\, d\omega$$

[6]

(d) Using the results of parts (b) and (c), show that the following integral

$$\int_{-\infty}^{+\infty} \frac{1}{1+t^2}\, \mathrm{sinc}(t)\, dt$$

takes the form $\alpha(1-\mathrm{e}^n)$, and find α and n, where n is an integer. [7]

(e) By considering the convolution of the Fourier transforms of $\frac{1}{1+t^2}$ and $\mathrm{sinc}(t)$, outline an alternative method for evaluating the integral in part (d). [5]

5 (a) $x(t)$ (t in seconds) is a signal which takes the form

$$x(t) = \cos(0.2\pi t) + \cos(6\pi t) + \cos(16.4\pi t)$$

and is sampled every 0.125 s (i.e. sampling period $T = 0.125$ s).

(i) Sketch the spectrum of the sampled signal in the range -10 Hz to $+10$ Hz, explaining carefully how each peak arises. [5]

(ii) Suppose we now take a set of N samples (at nT for $n = 0, 1, .., N-1$) from our signal, $\{x_0, x_1, ..., x_{N-1}\}$, and from these form the discrete Fourier Transform (DFT), $\{X_0, X_1, ..., X_{N-1}\}$. How large should N be to ensure that all peaks in the spectrum can be resolved? (You do not need to take windowing into account.) [5]

(iii) In order to avoid aliasing, at what frequency should the signal, $x(t)$, be sampled? [2]

(b) To transmit a signal over a channel we first sample and then quantise the signal.

(i) If $Q(x)$ is the quantised value of a signal sample, z, using a step size of D, the quantisation noise, $e_Q(z)$, is given by

$$e_Q(z) = z - Q(z)$$

Explain how e_Q can be modelled as a random variable. [2]

(ii) Assuming that the signal to be quantised is a triangular wave taking values between $-V$ and $+V$ volts, find the average power SNR (signal to noise ratio) in terms of V and D. [5]

(iii) How many levels, L, would be required to achieve a power SNR of at least 15 dB? [3]

(iv) If we have an n-bit uniform quantiser, what is the least number of bits required to achieve this SNR of at least 15 dB? [3]

6 (a) Channel coding consists of encoding and decoding processes which exploit the redundancy introduced by adding additional bits to our source bits in a controlled manner.

(i) For a BSC (binary symmetric channel), explain the encoding and decoding for an $(n,1)$ (n odd) *repetition code*. [3]

(ii) For a $(7,1)$ repetition code, find the probability of a decoding error occuring over a BSC(p) channel, with the crossover probability, p, taking the value 0.1. [4]

(iii) Comment on the performance of repetition codes in terms of *rate* vs. *probability of detection error*, as the repetition length, n, is increased. [2]

(iv) For a $(7,4)$ Hamming code, 3 parity-check bits are appended to each block of 4 data bits. If the data bits are s_1, s_2, s_3, s_4, the 7-bit Hamming codeword is given by

$$[c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7] = [s_1,\ s_2,\ s_3,\ s_4,\ s_1 \oplus s_2 \oplus s_3,\ s_2 \oplus s_3 \oplus s_4,\ s_1 \oplus s_3 \oplus s_4]$$

where $\oplus$ denotes modulo-two addition.
Using this example of a $(7,4)$ Hamming code, describe how such *block codes* work and compare them to repetition codes. [4]

(b) If many users need to communicate using the same channel bandwidth, *multiple access* schemes are used.

(i) Describe the following three multiple access schemes: FDMA, TDMA and CDMA. The main differences between these schemes should be outlined. [4]

(ii) For CDMA (code division multiple access) and K users, K unique and orthogonal signature functions, $c_i(t)$ for $i = 1, 2, ..., K$, are needed. For $K = 4$, describe a possible set of such functions, checking their orthogonality. [4]

(iii) Using the orthogonal signature functions described in part (b)(ii), describe how one might create a 5th orthogonal function, $c_5(t)$, if an extra user were to be added. [4]

END OF PAPER

Numerical Answers

1a(i) Poles are $s = \frac{1}{2}\left(\frac{1}{R} \pm \sqrt{\frac{1}{R^2} - 4}\right)$.

1(a)(ii) Impulse response is $e^{\sigma t}\sin(\omega t)$.

1(b)(i) $\lim_{s\to 0} sG(s)\frac{V}{s} = G(0)V = V$.

2(a)(i)

$$G_{u_1,y}(s) = \frac{G_1(s)}{1+G_1(s)K_1(s)}$$

$$G_{u_2,y}(s) = \frac{G_1(s)G_2(s)}{1+G_1(s)K_1(s)}$$

2(a)(ii)

$$S(s) = \frac{1+G_1(s)K_1(s)}{1+G_1(s)K_1(s)+K_2(s)G_1(s)G_2(s)}$$

$$T(s) = \frac{K_2(s)G_1(s)G_2(s)}{1+G_1(s)K_1(s)+K_2(s)G_1(s)G_2(s)}$$

2(b)(ii) Steady state response:

$$\lim_{s\to 0} sG_{u_2,y}(s)\frac{1}{s} = G_{u_2,y}(0) = \frac{1}{k}$$

3(a) Transfer function:

$$G(s) = \frac{k_F}{s\left(mLs^2+mRs+k_Bk_F\right)}$$

3(b)(ii) $10^{\frac{-44}{20}} \simeq 6.3\cdot 10^{-3}$ and $10^{\frac{-124}{20}} \simeq 6.3\cdot 10^{-7}$

4(b)

$$G(\omega) = \pi \mathrm{e}^{-|\omega|}$$

4(d)

$$\pi\left(1-\frac{1}{e}\right)$$

5(a)(ii) $N \geq 80$

5(a)(iii) 16.4 Hz

5(b)(ii)

$$\mathrm{SNR} = \frac{V^2/3}{D^2/12} = \frac{4V^2}{D^2}$$

5(b)(iii) at least 6 levels are needed

5(b)(iv) Need 3 bits

6(a)(ii) 2.73×10^{-3}.