

EGT1
ENGINEERING TRIPOS PART IB

Friday 9 June 2017 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer **one** question from Section A. In addition:

*If you are not taking the Foreign Language option, answer **four** questions, taken from only two of sections B–H. Not more than **two** questions from each section may be answered.*

*If you are taking the Foreign Language option, answer **two** questions from one of Sections B–H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

Answers to questions in each section should be tied together and handed in separately.

Section A: Introductory Business Economics	2
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STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachments: Data Sheet for Section B (6 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A: *Introductory Business Economics*

*Answer not more than **one** question from this section.*

- 1 (a) Consider the market for butter. Use supply and demand curves to represent and explain changes in the market equilibrium following:
- (i) a decrease in the price of margarine;
 - (ii) a decrease in the price of milk;
 - (iii) an increase in consumer income;
 - (iv) the introduction of a new and efficient technology in butter production. [5]
- (b) With reference to market structure:
- (i) define a monopolist firm and its market characteristics;
 - (ii) describe the different types of entry barriers;
 - (iii) show and analyse diagrammatically the market equilibrium. [10]
- (c) What is the Circular Flow of Income in macroeconomic analysis? Explain and analyse the impact of leakages and injections on the flow. [10]
- 2 (a) Considering a market structure where there is an oligopoly:
- (i) explain the different models of oligopoly;
 - (ii) show diagrammatically and explain the profit maximisation output. [5]
- (b) Discuss the importance of the concept of Game Theory in making economic decisions by firms and explain the concept of the Prisoner's Dilemma. [10]
- (c) Explain, with the aid of diagrams and equations, the fundamental principles of Keynesian Consumption Theory. [10]

SECTION B: Civil and Structural Engineering

Answer not more than **two** questions from this section.

Note Data Sheet at end of the paper.

3 A loaded beam supports part of the opening to an underground facility. It has two simple supports as shown in Fig. 1. The loading increases linearly from one end to the other as shown; these figures include an allowance for the beam's own self weight. The concrete has a cube strength f_{cu} of 30 N/mm² and a steel yield stress f_y of 460 N/mm².

- (a) Calculate the bending moment distribution for the beam. [4]
- (b) If the beam width is fixed at 500 mm, and the beam is to be *just* adequate in resisting sagging bending, calculate:
 - (i) the required effective depth of the beam; and [4]
 - (ii) the amount of steel required and a suitable combination of bars. [4]
- (c) Calculate the reinforcement required in the beam at the location of the maximum hogging bending moment if the beam has a constant effective depth. [5]
- (d) Calculate the amount of shear reinforcement required at the point where the shear force is largest. [4]
- (e) Sketch the reinforcement layout in the complete beam. [4]

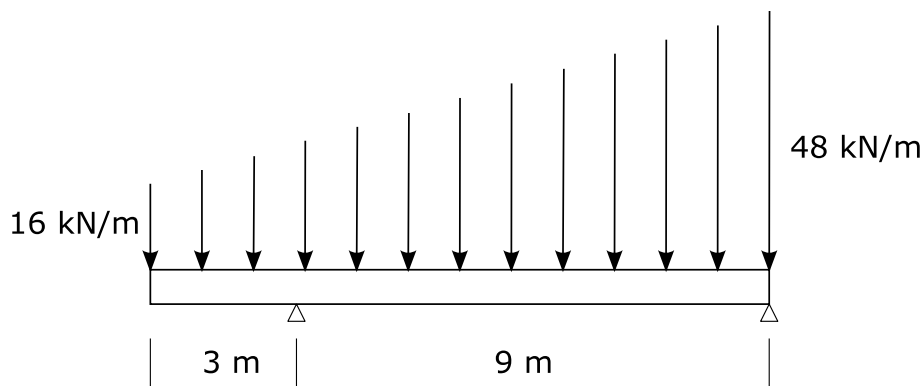


Fig. 1

4 In order to support a 9 m deep temporary excavation an impermeable reinforced concrete diaphragm wall is installed in the ground to a depth of 12 m, as shown in Fig. 2. The ground profile is a loose sand down to a depth of 9 m, underlain by a stiff clay of low permeability. The ground water table is 1 m below ground level. The critical state angle of friction of the sand is 30 degrees, and the unit weight above and below the water table is 17 kN/m³ and 19 kN/m³ respectively. The unit weight of the stiff clay is 20 kN/m³ and its undrained shear strength c_u is 75 kN/m².

Soon after excavation commences, a prop is installed at a depth of 1 m below original ground level. The excavation then continues to a depth of 9 m. A surcharge of 20 kN/m² is applied at the ground surface behind the wall.

(a) Assuming that the wall moves sufficiently for active pressures to be generated behind the wall over its entire length and passive pressures in front of the wall, calculate the short term total horizontal pressures acting on both sides of the wall, and sketch the pressure distribution. [15]

(b) The diaphragm wall is constructed with concrete with a cube strength f_{cu} of 30 N/mm². Assuming that the maximum design bending moment is 2000 kNm/m length of wall, determine a suitable thickness for the wall. [5]

(c) Design a suitable layout for the reinforcement, assuming that the steel has a yield strength of 460 N/mm². [5]

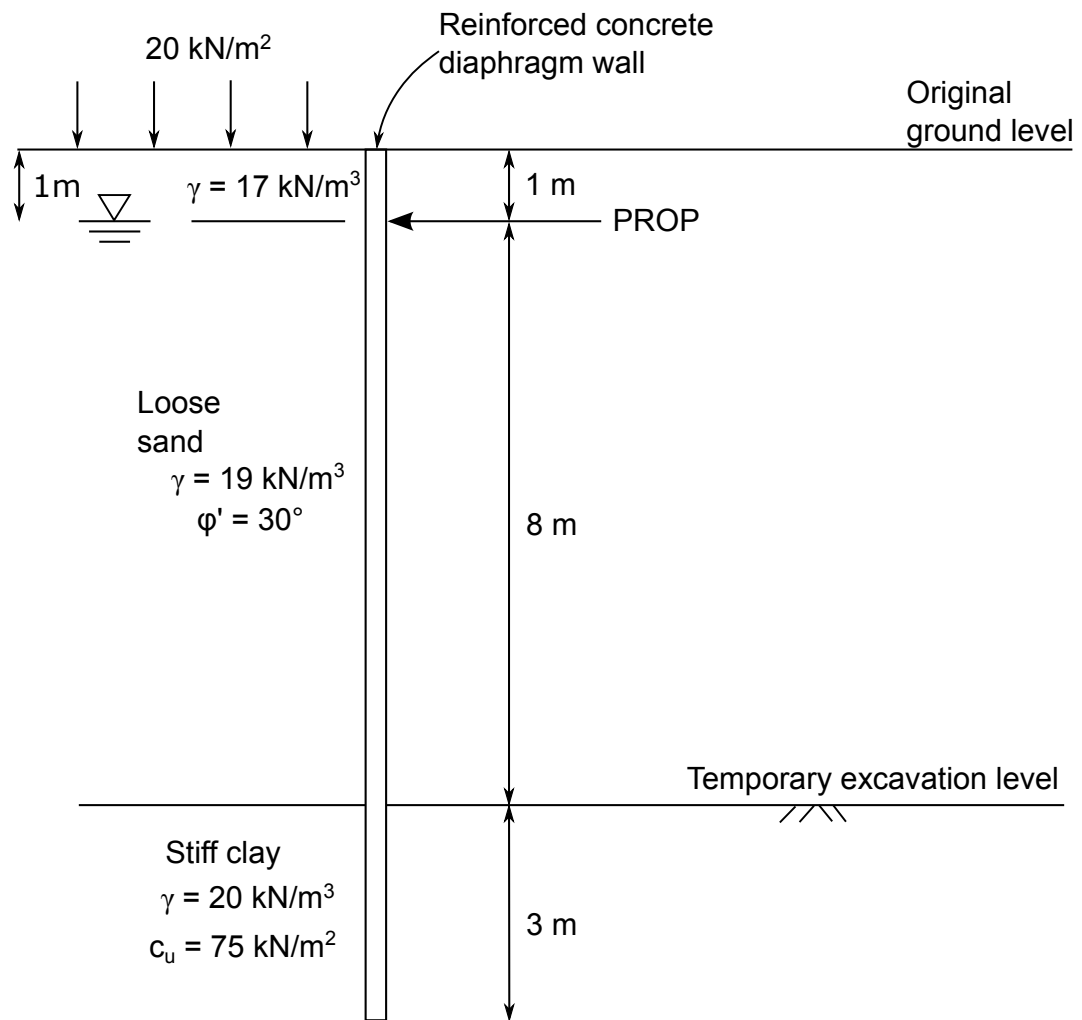


Fig. 2

- 5 (a) Describe the significance of soil permeability in the context of tunnelling below the water table, and give two examples of techniques that can be used to overcome potential problems. [5]
- (b) Explain why a tunnel constructed directly beneath a masonry building may be less likely to cause damage than when it is below and to one side of the building. [5]
- (c) Explain how compensation grouting can be used to control damage to buildings in an underground construction project and the role of instrumentation in the process. What should be the major aim of the process when it is used to protect a masonry building? [4]
- (d) Explain the difference between segmental linings and sprayed concrete for lining a tunnel, giving advantages and disadvantages of each. [5]
- (e) Explain why it is safe to construct an open face Crossrail station tunnel in London Clay but not in the soft marine clays of Bangkok or Singapore. Use the concept of stability ratio to illustrate your answer, and consider tunnels to be constructed at depths of 20 m. Take the density of both soils as 20 kN/m^3 . Assume the undrained shear strength of London Clay at 20 m is 200 kN/m^2 and soft marine clays of Bangkok or Singapore at 20 m is 50 kN/m^2 . [6]

SECTION C: Mechanics, Materials and Design

Answer not more than **two** questions from this section.

6 (a) Sketch the changes of the axial velocity and tangential velocity with position as wind passes through the rotor plane of a wind turbine. Annotate your sketches with expressions containing the axial induction factor a and the angular induction factor a' for the velocities before, during and after the wind has passed through the rotor plane. [5]

(b) Blade Element Momentum (BEM) theory is used to calculate the local forces on a wind-turbine blade and to alleviate some of the difficulties in calculating the induced velocities at the rotor. Explain why an iterative procedure must be used to calculate a and a' . Describe the starting assumptions and iterative steps of BEM theory. [5]

(c) A horizontal axis wind turbine with an upwind rotor has three blades, each with a twist angle of $\theta = 18^\circ$ and chord of 1.5 m at a radius $r = 5$ m. The lift and drag coefficients, C_L and C_D respectively, are approximately given as a function of angle of attack α by:

$$C_L = 2\pi\alpha, C_D = 0.01 \text{ for } 0 < \alpha < 0.3 \text{ rad.}$$

The wind turbine typically operates at an incident wind speed $V_0 = 8 \text{ m s}^{-1}$ with a fixed angular velocity ω of 30 rpm. Applying BEM theory at a radius $r = 5$ m and iterating until stable values are found gives $a = 0.114$, $a' = 0.025$.

- (i) Estimate new values for a and a' at $r = 5$ m, and the corresponding wind speed, for the condition where the blades are just approaching stall with $\alpha = 15^\circ$. [10]
- (ii) Estimate the contributions to the flap-wise and edge-wise bending moments on the blade from the section of a blade with $4.5 \text{ m} < r < 5.5 \text{ m}$ at the stall wind speed. Take the density of air as 1.23 kg m^{-3} . [5]

Note that

$$a = \left[\frac{4 \sin^2 \phi}{\sigma C_N} + 1 \right]^{-1}, \quad a' = \left[\frac{4 \sin \phi \cos \phi}{\sigma C_T} - 1 \right]^{-1}, \quad \tan \phi = \frac{(1 - a)V_0}{(1 + a')r\omega}$$

where C_N and C_T are the normal and tangential force coefficients and σ is the rotor solidity.

7 (a) Give three advantages of the doubly-fed induction generator (DFIG) for the generation of electricity by wind turbines. Draw a schematic diagram of a DFIG connected to the three-phase grid, and explain its principles of operation. [6]

(b) A wind turbine utilizing a DFIG is operated at its optimum tip-speed ratio of 7, at which its power coefficient C_p is 0.38, for wind speeds v between cut-in (2.5 m s^{-1}) and rated (10 m s^{-1}). Its slip-rings are short-circuited at the most probable wind speed, which is 6 m s^{-1} . Its stator winding is connected to the 3-phase, 50 Hz grid. The DFIG is mechanically coupled to the turbine via a gearbox of ratio 30. The blade diameter of the turbine is 80 m. Find:

(i) the number of poles that the generator should have in order to operate optimally at the most probable wind speed; [2]

(ii) the generator rating; [3]

(iii) the approximate slip of the generator at rated wind speed. [3]

The following may be quoted without proof: $P = \frac{1}{2}C_p\rho Av^3$, $\lambda = \omega R/v$. Take ρ to be 1.23 kg m^{-3} .

(c) Why are epicyclic gearboxes commonly used in wind turbine drive trains? Sketch a typical gearbox arrangement for a large commercial turbine. [3]

(d) An epicyclic gearbox is used as the first stage in a wind-turbine drive chain. The blade assembly is connected to the carrier of the epicyclic, the output is from the sun and the ring is held fixed. The radius of the sun wheel is 0.1 m and the radii of the three planet wheels are 0.15 m. The blades rotate at 30 rpm, delivering 200 kW of power. Neglecting power losses in the gearbox, find the output speed and torque of the epicyclic gear and the contact forces between each of the gears in the gearbox. [8]

8 (a) Show that an elastic beam of length L , second moment of area I and Young's modulus E loaded in bending can be modelled by two rigid rods of length $L/2$ connected by a rotational spring of stiffness $k = EI/L$, as illustrated in Fig. 3. [4]

(b) Propose a model containing two point masses and two rotational springs to represent the vibration in bending of a blade of length L , mass m and bending stiffness EI which is constant along the length of the blade. Assume that the blade is rigidly connected to a stationary hub and that the mass is uniformly distributed along the blade. Hence derive an expression for the lowest natural frequency of vibration of the blade in bending, in terms of E , I , m and L . [16]

(c) Outline how an accurate estimate of the resonant frequencies of the hub-and-blade assembly illustrated in Fig. 4 might be obtained. Sketch the expected mode shapes. [5]

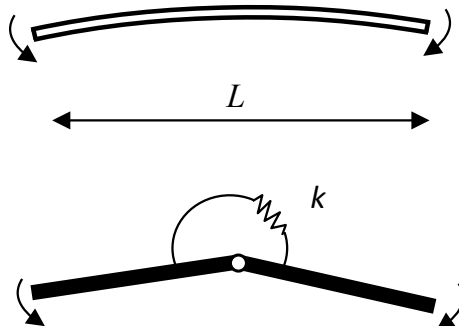


Fig. 3

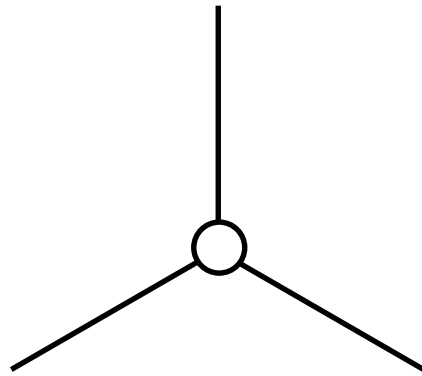


Fig. 4

SECTION D: Aerothermal Engineering

Answer not more than **two** questions from this section.

9 (a) A single shaft turbojet flies at an altitude of 10,000 m where the ambient conditions are $T = 222 \text{ K}$, $p = 26 \text{ kPa}$. The Mach number at the compressor inlet plane is 0.8. Calculate the stagnation temperature and the stagnation pressure at the compressor inlet plane. [3]

(b) The compressor of the turbojet has an isentropic efficiency of 90% and a pressure ratio of 20. Calculate the stagnation temperature at the exit of the compressor. [4]

(c) The compressor has 10 stages and all stages have a design stage work coefficient $\psi = \Delta h_0 / U^2 = 0.4$. The compressor has a rotational speed of 6000 rpm. All compressor stages are designed to have the same mean radius. Calculate the mean radius of the compressor. [6]

(d) The compressor has a mass flow rate of 50 kg s^{-1} . Calculate the height of the first blade row of the compressor. [6]

(e) The work coefficient is usually defined for an entire stage. It can also be defined at a fixed radius. The work coefficient at the hub radius, ψ_{hub} , is defined as

$$\psi_{hub} = \frac{\Delta h_{0hub}}{U_{hub}^2}$$

where U_{hub} is the blade speed at the hub radius at stage inlet and Δh_{0hub} is the stagnation enthalpy rise across the stage at the hub radius. The first stage of the compressor is designed to have the same increase in stagnation enthalpy, from inlet to exit, at all radial heights, i.e. the stagnation enthalpy rise at the hub, Δh_{0hub} , is equal to the stagnation enthalpy rise at mid-height, Δh_{0mid} . The work coefficient at the mid-height radius, ψ_{mid} , of the first stage of the compressor is 0.4. Calculate the work coefficient at the hub radius, ψ_{hub} , of the first stage of the compressor. Comment on this value and the implications for blade design. [6]

Assume that the working fluid is air throughout with $\gamma = 1.4$, $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$ and $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$.

10 (a) For a simple turbojet operating with a choked exit nozzle the non-dimensional relationship between the non-dimensional thrust \tilde{F} and the non-dimensional mass flow \tilde{m} can be written as $\tilde{F} = f(\tilde{m})$ where

$$\tilde{F} = \frac{F_G + p_a A_N}{D^2 p_{02}} \quad \text{and} \quad \tilde{m} = \frac{\dot{m} \sqrt{c_p T_{02}}}{p_{02} A_N}$$

and all the terms have their usual meaning. Explain carefully the significance of using the term $F_G + p_a A_N$ in the expression for \tilde{F} . Explain why the non-dimensional thrust \tilde{F} can be written as a function of the non-dimensional mass flow \tilde{m} alone. [5]

(b) The turbojet is tested in a stationary test at an ambient pressure of 101 kPa and an ambient temperature of 288 K. The turbojet is to be used to power an aircraft with a cruise Mach number of 0.8 at an altitude where the ambient pressure is 26 kPa and the ambient temperature is 222 K. During flight the turbojet operates at the same non-dimensional conditions as in the stationary test. Calculate the ratio of the mass flows at the stationary operating condition and the flight condition. [5]

(c) Explain how gross thrust and net thrust are used in these engine calculations. The nozzle area of the turbojet is 0.08 m^2 . During the stationary test the gross thrust and exit jet velocity are measured to be 30 kN and 500 m s^{-1} . Calculate the net thrust of the engine in flight. [9]

(d) State the equation for non-dimensional fuel flow rate and provide a physical explanation for its non-dimensional form. Calculate the ratio of the fuel mass flow rate in the stationary test to the fuel mass flow rate in flight. [6]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$.

11 (a) An aircraft is designed to operate on long-haul routes. The aircraft is considered for use on a medium-haul route. If the aircraft operator wanted to minimise fuel burn on the new route what advice would you give them about changing the cruise altitude? Explain your reasoning. [6]

(b) Derive Breguet's range equation for the distance, s , between the start and end of cruise:

$$s = \frac{V}{g} \frac{L/D}{\text{sfc}} \ln \left(\frac{W_{\text{start}}}{W_{\text{end}}} \right)$$

where g is the acceleration due to gravity, V is the flight speed, sfc is the thrust specific fuel consumption, L/D is the aircraft lift-to-drag ratio and W_{start} and W_{end} are the total aircraft weights at the start and end of cruise respectively. State any assumptions made in the derivation. [6]

(c) The aircraft operator from section (a) decides to re-engine the aircraft to reduce fuel burn on a route of fixed range. The propulsive efficiency and thermal efficiency of the new engines are 10% and 5% higher than the original engines ($\eta_{p \text{ new}} = 1.1 \times \eta_{p \text{ old}}$, $\eta_{th \text{ new}} = 1.05 \times \eta_{th \text{ old}}$). Before the aircraft is re-engined the ratio of the total aircraft weight at the start of cruise to the total aircraft weight at the end of cruise, $W_{\text{start}}/W_{\text{end}} = 1.5$. Calculate the ratio of the total aircraft weight at the start of cruise to the total aircraft weight at the end of cruise after the aircraft has been re-engined. [7]

(d) The aircraft is powered by turbofan engines. The manufacturer of the engines decides to re-design them in order to further increase the propulsive efficiency. What are the practical challenges? Sketch the layout of the new turbofan engine labelling key features. What determines the maximum possible propulsive efficiency which can be achieved in practice? [6]

SECTION E: *Electrical Engineering*

Answer not more than **two** questions from this section.

12 (a) The key element for a smartphone has always been the display. Indeed the display in the *iPhone7* is one of the most superior in terms of contrast ratio and screen reflectance for liquid crystal based technology. If the display screen of the *iPhone7* has a diagonal 4.7 inches with an aspect ratio 16:9, what is the display size in pixels if the resolution is 326 pixels per inch. [2]

(b) If the display screen yields a maximum contrast ratio of 1762 with a dark black luminance of 2.5 cd m^{-2} , what is the peak white luminance? Assume screen reflectance is negligible. [5]

(c) Name three basic thin film transistor (TFT) technologies that are of current interest in display screens and tabulate their relative performance in terms of carrier mobility and current applications. [8]

(d) A thin film transistor for eventual applications in OLED TVs is being fabricated using a new trial channel material, indium silicon oxide. Its electron mobility $\mu = 50 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. The transistor will have channel width $W = 10 \text{ }\mu\text{m}$, gate length $L = 3.6 \text{ }\mu\text{m}$, and channel thickness $t_{ch} = 20 \text{ nm}$. Assuming that the gate induces a uniform carrier density $N = 5 \times 10^{14} \text{ cm}^{-3}$ in the channel, what is the drift current I_d for a drain-source voltage of 2 V? Note that elementary charge $e = 1.6 \times 10^{-19} \text{ C}$. [10]

13 (a) Explain how Gauss's law can be used to obtain the electric field created by a cube of uniform electronic charge. Be clear with your arguments about the direction of the field. [5]

(b) Hence derive the expression for the gate threshold voltage to turn off a depletion mode n-type FET with a channel doping density of N electrons m^{-3} , a channel thickness h , and a relative dielectric constant ϵ_r . [6]

(c) From (b), calculate the gate threshold voltage V_t for this case where $h = 50 \text{ nm}$, $N = 2.5 \times 10^{23} \text{ m}^{-3}$ and $\epsilon_0 \epsilon_r = 10^{-10} \text{ F m}^{-1}$. Sketch the I_{ds} vs. V_{gs} characteristic for this case. [6]

(d) A floating gate of Si ($\epsilon_0 \epsilon_r = 10^{-10} \text{ F m}^{-1}$) in a Flash memory-like device has thickness $t = 30 \text{ nm}$, and a very thin second gate oxide layer of SiO_2 is added on top of the floating gate, as sketched in Fig. 5. The floating gate is charged up to a charge density of $2.5 \times 10^{24} \text{ C m}^{-3}$. Calculate the new value of the gate threshold voltage. [8]

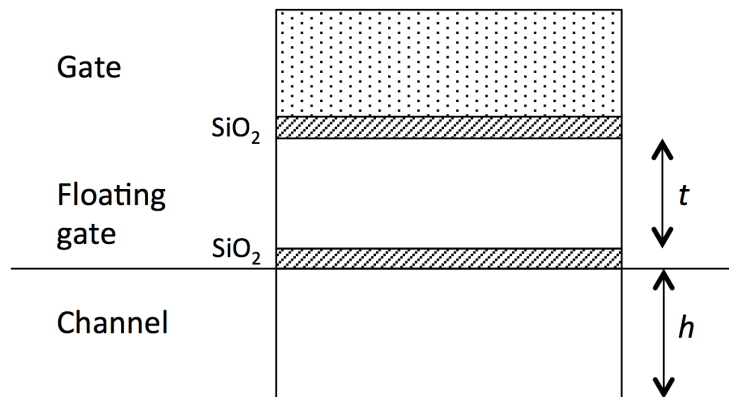


Fig. 5

- 14 (a) Derive an expression for the wavelength of an electron in terms of its kinetic energy T and effective mass m^* . [6]
- (b) Explain what is meant by quantum mechanical tunnelling, for example by sketching the wavefunction of an electron with total energy E travelling from left to right through a tunnel barrier of height V_0 and thickness d ; the potential is $V = 0$ outside the barrier. [6]
- (c) Derive an approximate expression for the tunnelling probability P in terms of the parameters E , m^* , V_0 and d . [5]
- (d) Calculate P for the case of a 1.5 nm thick SiO_2 layer assuming that the effective electron mass m^* is half of the free mass, $E = 0.5$ eV, $V_0 = 3.5$ eV, Planck's constant $= 6.626 \times 10^{-34}$ kg m² s⁻¹, and the free electron mass is $m = 0.9 \times 10^{-30}$ kg. [8]

SECTION F: *Information Engineering*

Answer not more than **two** questions from this section.

15 (a) We often need to rotate an image to correct for unintended rotation of the camera; for example, to make true verticals appear vertical in the image.

(i) Suppose we wish to rotate an $M \times N$ image anticlockwise by an angle θ . Sketch the rotated image with respect to the original image. If the rotated image has size $P \times Q$, show how we obtain P and Q in terms of M, N and θ . [3]

(ii) Write down the 2D rotation matrix R which rotates a vector $\mathbf{x} = [x_1, x_2]^T$ anticlockwise through an angle θ . [2]

(iii) If the rotation is about the centre of the image, show how we can use R to rotate original pixel coordinates, $\mathbf{u} = [u, v]^T$, to coordinates in our new rotated image, $\mathbf{p} = [p, q]^T$. [3]

(iv) The rotated coordinates, $\mathbf{p} = [p, q]^T$, will not necessarily be located at pixel locations in the rotated image. Explain how we map the continuous rotated coordinates to integer pixel coordinates. [3]

(b) 2D filters are used to perform a variety of operations on images.

(i) A 2D filter, $g(x, y)$, is applied to an intensity image, $I(x, y)$. Write down an expression for the output image, $I_c(x, y)$, and discuss how one deals with image boundaries. [3]

(ii) If the filter $g(x, y)$ is separable, i.e. $g(x, y) = g_r(x)g_c(y)$, explain how the 2D filter can be implemented as two 1D filters. [2]

(iii) Compare the number of operations (multiply and add) needed for a full 2D filter compared to two 1D filters. Take the 2D filter to be of size $N \times N$. [3]

(iv) If $G(\omega_1, \omega_2)$ is a lowpass filter, explain why $H(\omega_1, \omega_2)$ behaves as a highpass filter when H takes the following form:

$$H(\omega_1, \omega_2) = \alpha - \beta G(\omega_1, \omega_2)$$

where α and β are positive constants, taking appropriate values. [3]

(v) For various representative values of α and β , describe the effect of the highpass filter in (b)(iv) on edges. You may illustrate the effects in 1D. [3]

16 (a) A grey scale image, $I(x,y)$, is to be smoothed by convolving it with Gaussian filters of different scales, $G_{\sigma}(x,y)$, as part of the feature detection process.

(i) Why is smoothing required? Describe the effects of increasing the scale parameter, σ . [3]

(ii) Show how discrete samples from the *scale-space* representation of the image, $S(x,y,\sigma) = G_{\sigma}(x,y) * I(x,y)$, can be represented efficiently in an *image pyramid*. Your answer should include details of the implementation of smoothing and sub-sampling to avoid convolution with large filters. [4]

(iii) The images are filtered with a *band-pass* filter to localise features at different scales. What is meant by band-pass filtering? Show that the *Laplacian of a Gaussian* is a band-pass filter and describe why it can be implemented efficiently using the differences between smoothed images in the image pyramid. [4]

(iv) Show how image features such as *blob-like* shapes can be localised in an image and describe how to determine an appropriate scale and feature size. [3]

(b) Consider an algorithm to match image features in different images. The neighbourhood of each image feature is first normalised to a 16×16 patch of pixels by sampling pixels at an appropriate scale and orientation.

(i) Show how the similarity of the patch of pixels (after normalising for the image scale and orientation) in different images can be obtained by looking at the *cross-correlation* between the pixels in each patch. [4]

(ii) How should the cross-correlation score be modified so that it is less sensitive to photometric changes? What are the shortcomings of using cross-correlation? [4]

(iii) List two other descriptors that can be used for matching image features and show how they obtain better invariance to lighting and viewpoint changes. [3]

- 17 (a) A *single-layer perceptron* is to be used to classify a simple pattern in an image.
- (i) Sketch a single *artificial neuron*. Identify the D inputs, \mathbf{x} , and weights, \mathbf{w} , and the single output, y . Give an expression for computing the *activity* of the neuron and include details of the non-linearity used to compute the output. [6]
 - (ii) What is meant by *supervised learning*? Show how a training set of input and desired output pairs can be used to learn the weights, \mathbf{w} . Include details of a suitable objective function or cost that should be minimised during training. [6]
 - (iii) Comment on the classification boundaries that can be obtained with a single layer perceptron. [3]
- (b) A *convolutional neural network* is to be used to determine the categories of objects in small 32×32 RGB images by training using a labelled dataset of 60,000 images downloaded from the internet.
- (i) What are the advantages of using convolutions? [3]
 - (ii) Sketch the anatomy of a typical convolutional neural network and identify the role of each layer. Include details of the number of filters and their sizes. Explain the roles of *pooling* and the non-linearities used in the network. [7]

SECTION G: Bioengineering

Answer not more than **two** questions from this section.

18 (a) Describe the path of the illuminating and reflecting light from the fundus, when using both the Fundus Camera and the Scanning Laser Ophthalmoscope. Explain the reasons for each approach and contrast the resulting optical properties. [6]

(b) In Optical Coherence Tomography (OCT), the fundus is illuminated with a laser pulse E with n periods of a frequency ω_0 :

$$E = \begin{cases} e^{j\omega_0 t} & -\frac{n\pi}{\omega_0} < t < \frac{n\pi}{\omega_0} \\ 0 & \text{otherwise} \end{cases}$$

(i) What is the bandwidth b of this pulse, defined as the width of the main lobe of the frequency response, and how does this relate to the spatial depth resolution of OCT? [7]

(ii) Suggest approximate values for n and ω_0 typically used in OCT. [2]

(c) The images in Fig. 6 below show two types of visualisation from OCT data.

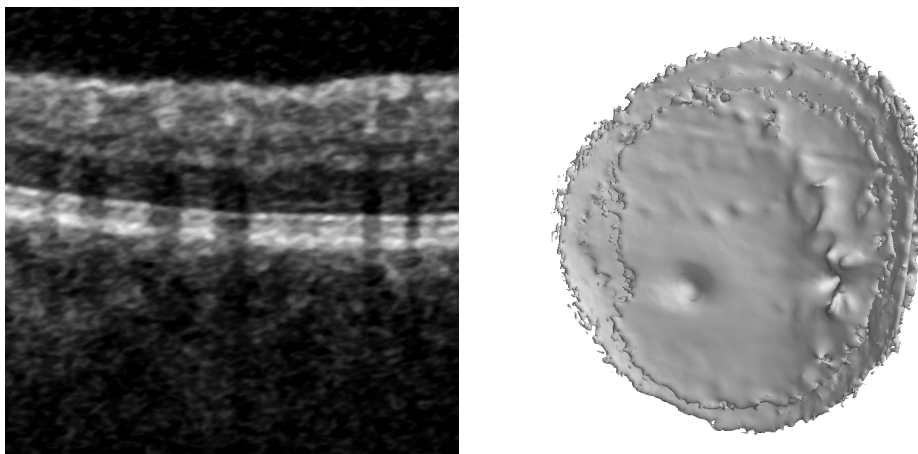


Fig. 6

(i) For each image, explain what type of visualisation is being used and what it can display and describe one feature of the eye which can be seen in that visualisation. [6]

(ii) For each image, describe one feature which is an artefact of the visualisation process and briefly explain how this artefact arises. [4]

- 19 (a) Sketch a section of the human eye in the vertical plane that divides it into left and right halves and annotate it by placing the following labels: cornea, lens, retina, fovea, optic nerve. [4]
- (b) Explain the mechanism underlying the physiological blind spot in vertebrates and give an example of an animal species that does not have such a blind spot. [4]
- (c) Describe briefly
- (i) simple cells in the primary visual cortex;
 - (ii) cortical minicolumns;
 - (iii) the transmission of visual information from photoreceptors to optic nerve fibres. [7]
- (d) Give two examples of convolution-based algorithms for the detection of luminance edges in an image. Sketch the associated spatial filters, and compare the advantages of the two algorithms. Which is closest to the processing of visual inputs performed by the retina? [5]
- (e) What is an “orientation map” in the primary visual cortex? Describe an experiment that could be performed to reveal such maps. Include details of recording techniques and visual stimuli. [5]

- 20 (a) Describe the tissue structure of the crystalline lens in the eye. [5]
- (b) Describe lens accommodation in young, normal, healthy eyes. Explain how the accommodation reflex differs from accommodation. [6]
- (c) Lenses are typically characterised by the quantity *diopeters*, D , measuring lens power.
- (i) How does diopter relate to focal length of a lens?
- (ii) Use a drawing to show the difference between $+D$ and $-D$ lenses.
- (iii) A lens whose focal length is 50 cm is placed in contact with a second lens whose focal length is 10 cm. What is the power of the combination lens? [6]
- (d) Explain why reading glasses are increasingly likely to be necessary as the eyes age. What type and strength of reading glasses are typically used to correct the problem? [8]

SECTION H: *Manufacturing and Management*

*Answer not more than **two** questions from this section.*

21 (a) Explain, using examples to illustrate your answer, the advantages and disadvantages of each of the following types of business model:

- (i) selling a product;
- (ii) selling a product plus consumables;
- (iii) selling a product plus a service. [6]

(b) Compare, using examples to illustrate your answer, the characteristics of each of the following types of production system:

- (i) project;
- (ii) job shop;
- (iii) batch production;
- (iv) mass production. [9]

(c) Discuss why there are conflicting objectives between supply and demand in a manufacturing firm, and describe strategies that can be used to manage these conflicting objectives. [10]

22 A team of students has developed a robust, low-cost, digital medical heart monitor for use in disaster zones where normal medical facilities are not available. For this product, describe the processes by which the students could:

- (a) assess the scale of the potential market; [8]
- (b) understand the requirements of stakeholders; [8]
- (c) create a detailed product specification. [9]

23 (a) Describe the differences between these four types of Intellectual Property:

- (i) copyright;
- (ii) trademark;
- (iii) design rights;
- (iv) patents. [8]

(b) For a patented technology, compare the advantages and disadvantages for a firm using a business model based upon:

- (i) licensing the patented technology to another firm;
- (ii) manufacturing and selling a product that incorporates the patented technology.

[8]

(c) (i) Describe the key elements of a business plan for commercialising an idea based around a patented technology using a partnership-based business model.

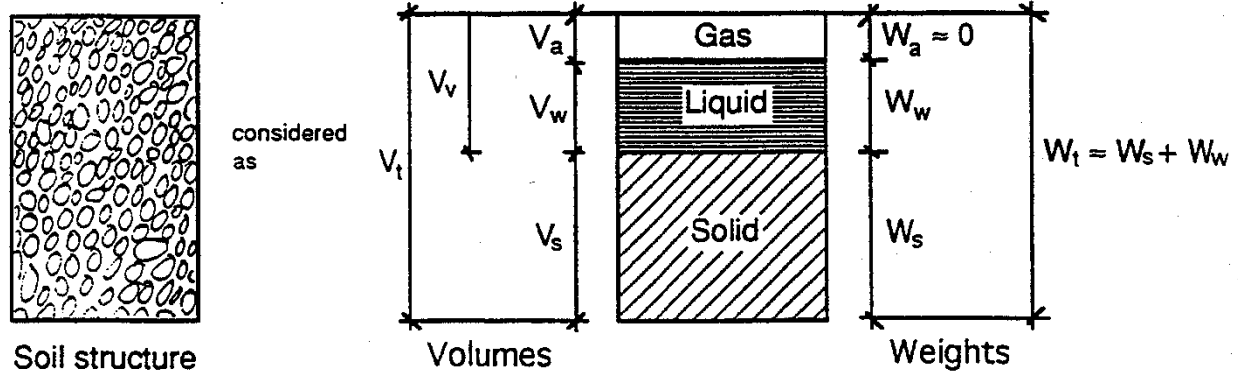
- (ii) For the business plan described in part (i), discuss the specific issues a venture capitalist would focus upon when examining this business plan. Explain why you think these issues are particularly important to a venture capitalist. [9]

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Data sheet – Soil Mechanics

General definitions



Specific gravity of solid

$$G_s$$

Voids ratio

$$e = V_v / V_s$$

Specific volume

$$v = V_t / V_s = 1 + e$$

Water content

$$w = (W_w / W_s)$$

Degree of saturation

$$S_r = V_w / V_v = (w G_s / e)$$

Unit weight of water

$$\gamma_w = 9.81 \text{ kN/m}^3 \text{ (although we assume } 10 \text{ kN/m}^3 \text{)}$$

Unit weight of soil

$$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$$

Buoyant (effective or submerged) unit weight

$$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w \quad (\text{soil saturated})$$

Unit weight of dry soil

$$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$$

Relative density

$$I_d = \frac{(e_{\max} - e)}{(e_{\max} - e_{\min})}$$

where e_{\max} is the maximum voids ratio achievable in the quick tilt test (for sands), and e_{\min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than	200 mm
Cobbles	between	200 mm and 60 mm
Gravel	between	60 mm and 2 mm
Sand	between	2 mm and 0.06 mm
Silt	between	0.06 mm and 0.002 mm
Clay	smaller than	0.002 mm (two microns)

D equivalent diameter of soil particle

D_{10} , D_{60} etc particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

$$\begin{aligned} \text{total stress } \sigma &= \text{effective stress } \sigma' + \text{pore water pressure } u \\ \tau &= \tau' + 0 \end{aligned}$$

and

$$\begin{aligned} \sigma_v &= \text{vertical stress} \\ \sigma_h &= \text{horizontal stresses} \\ \tau &= \text{shear stress} \end{aligned}$$

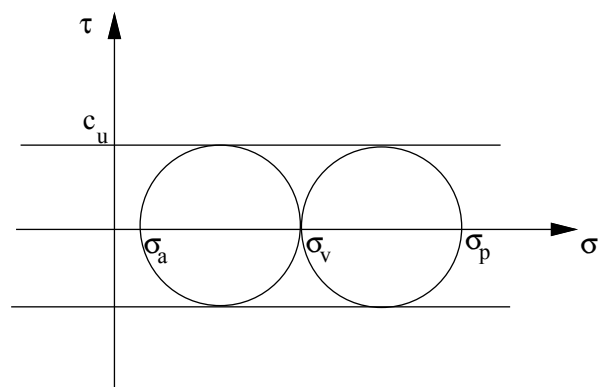
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2 c_u$$

$$\sigma_p = \sigma_v + 2 c_u$$

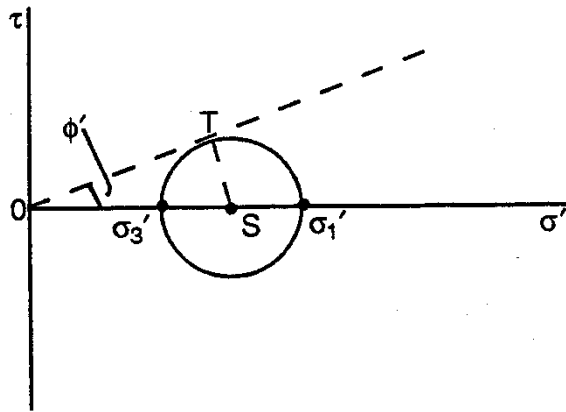
where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'

where $\tau = \sigma' \tan \phi'$



$$\sin \phi' = TS/OS$$

$$= \frac{(\sigma'_1 - \sigma'_3)/2}{(\sigma'_1 + \sigma'_3)/2}$$

$$\therefore \phi' = \sin^{-1} \left[\frac{\left(\frac{\sigma'_1}{\sigma'_3} \right) - 1}{\left(\frac{\sigma'_1}{\sigma'_3} \right) + 1} \right]$$

Earth pressure coefficient K:

$$\sigma'_h = K\sigma'_v$$

Active pressure:

$$\sigma'_v > \sigma'_h$$

$$\therefore \sigma'_1 = \sigma'_v$$

$$\sigma'_3 = \sigma'_h$$

$$K_a = (1 - \sin \phi') / (1 + \sin \phi')$$

Passive pressure:

$$\sigma'_h > \sigma'_v$$

$$\therefore \sigma'_1 = \sigma'_h$$

[We assume principal stresses

$$\sigma'_3 = \sigma'_v$$

are horizontal and vertical]

$$K_p = (1 + \sin \phi') / (1 - \sin \phi') = \frac{1}{K_a}$$

Angle of shearing resistance:

at peak strength ϕ'_{\max} at $\left(\frac{\sigma'_1}{\sigma'_3} \right)_{\max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{\max}$ and

$$\phi'_{\max} = \phi'_{\text{crit}} + \phi'_{\text{dilatancy}}$$

where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero ($\phi'_{\text{dilatancy}} \rightarrow 0$) as $\phi'_{\max} \rightarrow \phi'_{\text{crit}}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values ($\pm 2^\circ$):

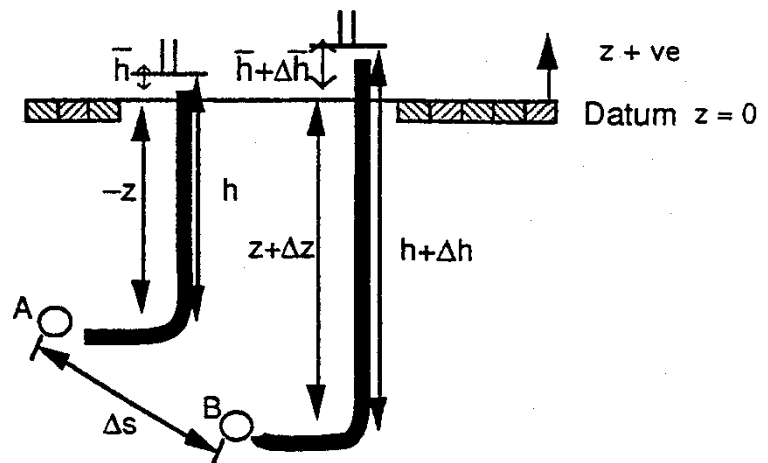
	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure

Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$



Total pore water pressure head at A: $u = \gamma_w h = \gamma_w (\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})]$

Hydraulic gradient A-B $i = -\frac{\Delta \bar{h}}{\Delta s} = -\frac{\Delta \bar{u}}{\gamma_w \Delta s}$

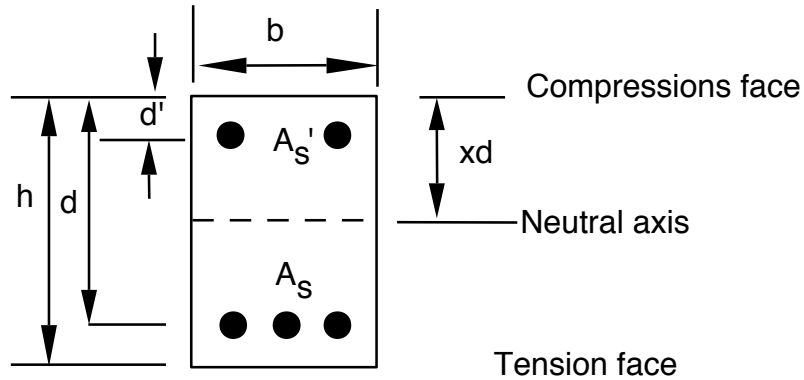
Darcy's law $v = ki$
 v = average or superficial seepage velocity
 k = coefficient of permeability

Typical permeabilities

$D_{10} > 10\text{mm}$:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$:	$k \cong 0.01(D_{10}\text{ in mm})^2 \text{ m/s}$
clays	:	$k \cong 10^{-9} \text{ to } 10^{-11} \text{ m/s}$

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu} . At failure in bending, stress in concrete = $0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y . At failure in bending, stress in bars in tension = $0.87f_y$, stress in bars in compression = $0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15 f_{cu} b d^2$$

$$M = 0.87 f_y A_s d (1 - x/2)$$

$$x = 2.175 (f_y / f_{cu}) (A_s / b d) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} b d^2 + 0.75 f_y A'_s (d - d')$$

$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} b d$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33} \cdot (400/d)^{0.25} \quad (\text{N/mm}^2)$

and $v_s = 0.87f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm ²)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460 \text{ N/mm}^2$

Plain mild steel $f_y = 250 \text{ N/mm}^2$

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: 24 kN/m^3

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										