Version GRH/3

EGT3
ENGINEERING TRIPOS PART IIB

Friday 28 April 2017 9.30 to 11

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A10 Flow Instability data sheet (two pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A fluid subject to a horizontal velocity $u(z)$ is initially stably stratified with a smoothly varying density profile $\rho(z)$ where z denotes the vertical coordinate. Assuming $du/dz > 0$, develop an energy-based argument to show that the flow is unstable for

$$\frac{-g}{\rho_0}\frac{d\rho}{dz} < \frac{1}{4}\left(\frac{du}{dz}\right)^2$$

where g is the acceleration due to gravity and ρ_0 is a reference density. [30%]

(b) A liquid fills the annular gap formed between two long concentric cylinders. The inner cylinder, of radius r_1, rotates about its vertical axis with angular velocity Ω_1. The outer cylinder, of radius r_2, rotates with angular velocity Ω_2 about the same axis. You may take $\Omega_1 > 0$, $\Omega_2 > 0$ and $r_2 > r_1$. Consider the stability of this Taylor-Couette problem and show that stability for an inviscid fluid requires

$$\frac{d}{dr}\left[(\Omega r^2)^2\right] \geq 0$$

where Ω denotes the angular velocity at a radial distance r from the vertical axis. [35%]

(c) Consider a slender incompressible cylindrical liquid jet of density ρ propagating horizontally in air. You may assume that the diameter d of the jet is sufficiently small that surface tension γ cannot be neglected. The jet is subject to a small amplitude perturbation.

(i) Use dimensional arguments to develop a scaling for the growth rate s of the perturbation.

(ii) By considering sections through the jet perpendicular to its longitudinal axis, discuss with clear physical reasoning the stability of the jet to both axisymmetric and non-axisymmetric disturbances. To support your discussion provide clearly labelled schematics that illustrate the jet for circumferential wavenumbers of $n = 2$ and $n = 4$. [35%]

2 (a) Explain the principles and approach behind a linear stability analysis. Include a description of why a normal mode analysis may be regarded as enabling the influence of all possible small amplitude disturbances to the flow to be assessed. [20%]

(b) Consider a thin layer of incompressible inviscid liquid (density ρ and surface tension γ) at rest between the free plane surfaces $z = \pm a$. Show that the stability of the layer is governed by

$$\nabla^2 p' = 0 \quad -\infty < (x,y) < \infty \quad -a < z < a$$

for a perturbation p' of the resting state, with boundary conditions

$$\rho \frac{\partial^2 p'}{\partial t^2} = \pm \gamma \frac{\partial^2}{\partial x^2}\left(\frac{\partial p'}{\partial z}\right) \quad \text{at} \quad z = \pm a$$

where t denotes time. Take normal modes of the form $p' \propto e^{ikx+st}$ (where k denotes the wavenumber) to show that for the upper surface

$$\frac{a^3\rho}{\gamma} s^2 = -\alpha^3 \tanh(\alpha) \quad \text{where} \quad \alpha = ka$$

Hence, explain why the layer is stable. [80%]

3 To a first approximation, the General Dynamics F16 backward-swept wing in Fig. 1a can be modelled by the 2D airfoil in Fig. 1b. The vertical forces (positive downwards) and moments (positive clockwise) on the wing are resolved through the elastic axis (E.A.). The structural elasticity forces are modelled by a linear spring with constant k_y and a torsional spring with constant k_θ. For the simplified analysis in this question, the aerodynamic forces are modelled as

$$F_y = -\lambda\theta \qquad F_\theta = -c_a F_y \qquad \text{where} \quad \lambda \equiv \frac{1}{2}\rho U^2 c \left.\frac{\partial C_L}{\partial \alpha}\right|_0 > 0$$

These forces act on the aerodynamic centre (A.C.) but are resolved through the E.A. by considering the E.A.'s motion relative to the centre of mass (C.M.). This gives the undamped translational (y–direction) and torsional (θ–direction) equations of motion

$$\begin{aligned} m\ddot{y} + S_x\ddot{\theta} + k_y y &= F_y \\ I_\theta\ddot{\theta} + S_x\ddot{y} + k_\theta\theta &= F_\theta \end{aligned}$$

where m is the mass of the wing, I_θ is the moment of inertia of the wing about the elastic axis, and S_x/m is the distance between the E.A. and the C.M.

(a) By considering displacements of the form $y = Y_0 e^{st}$ and rotations of the form $\theta = \theta_0 e^{st}$, show that there are two possible types of unstable motion. Describe these motions and derive the conditions required for them to be unstable. [60%]

(b) In 1976, the USA considered building an F16 with a forward-swept wing (Fig. 1c). From the point of view of stability, discuss any advantages and disadvantages of the forward-swept wing. [40%]

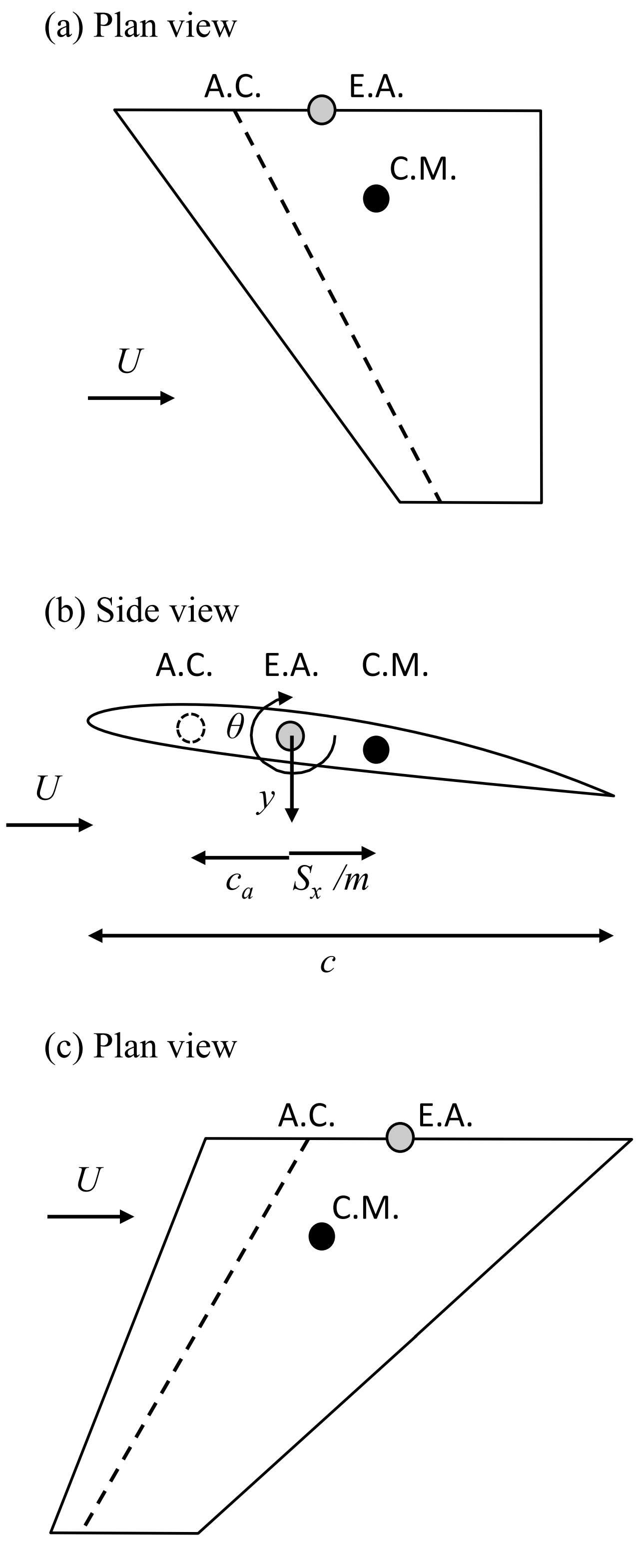

Fig. 1

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4 An inkjet print head is to be modelled as a channel of length $l = 1000\ \mu$m, height $h = 50\ \mu$m, width $w = 50\ \mu$m (Fig. 2). The ink feed, not shown, is through the top plate. Ink is expelled through an orifice of diameter $d = 10\ \mu$m at the centre of the bottom plate. The ink is expelled when the piezo-electric plates at each end ($x = \pm l/2$) move towards each other, forced by an electric signal. Each plate is modelled as a thin plate of mass m, held frictionlessly by a spring with constant k. The ink has density $\rho = 1000$ kg m^{-3} and sound speed $c = 1500$ m s^{-1}. The first acoustic mode is planar with $x-$velocity profile $u(x,t) = u_w(t)\sin(x\pi/l)$, where u_w is the velocity of the plate at $x = l/2$.

(a) Calculate the frequency of the first acoustic mode. [10%]

(b) Assuming that the drop diameter is the same as the orifice diameter, calculate the displacement of each piezo-electric plate required to eject a single drop. [10%]

(c) A customer would like to eject 100 droplets per second, which is much lower than the frequency calculated in (a). Calculate the value of k that best exploits the resonance of the mass/spring system to produce these droplets. Explain why the fluid can be considered to be incompressible and why, to a good approximation, the value of m is not required. [30%]

(d) For another customer, the head is designed to eject droplets at the frequency of the first acoustic mode. Calculate the value of k that best exploits resonance within the system to produce these droplets. [40%]

(e) What other effects should be included to calculate k more accurately. [10%]

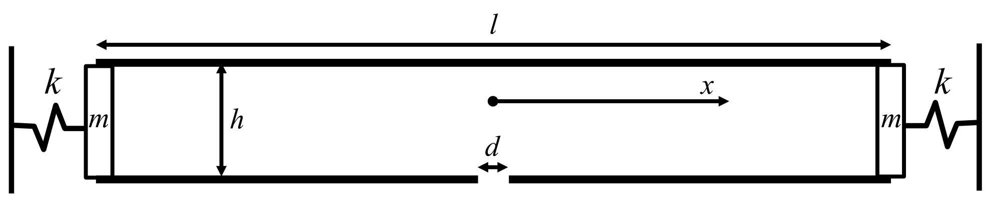

Fig. 2

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