Version AA/4

EGT3
ENGINEERING TRIPOS PART IIB

Monday 24 April 2017 9.30 to 11

Module 4A15

AEROACOUSTICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (6 pages).

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Show that the acoustic pressure field generated by a bubble of mean radius a pulsating with an angular frequency ω with a peak to peak change of radius amplitude of 2ε is given by

$$p'(r,t) = -\rho_0 \frac{\varepsilon}{r} \frac{(\omega a)^2}{1+ika} e^{-ik(r-a)+i\omega t}$$

where r is the distance from the centre of the bubble to the observer, t represents time, ρ_0 the mean density of the water surrounding the bubble and k the wavenumber. [50%]

(b) If the radius of the bubble is 1 mm, the frequency of oscillation is 5 kHz and $\varepsilon = 0.1$ mm, find the power radiated (in watts) by the bubble. Assume $\rho_0 = 1000\ \mathrm{kg/m^3}$ and the speed of sound $c_0 = 1450$ m/s. [50%]

2 The sound field radiated by a tuning fork can be modelled by the following differential equation

$$\frac{\partial^2 p'}{\partial t^2} - c_0^2 \nabla^2 p' = A \frac{d^2}{dx_1^2} \{\delta(x_1)\} \, \delta(x_2)\delta(x_3)e^{i\omega t}$$

where δ is the Dirac delta function, ω is the angular frequency, c_0 is the ambient speed of sound, A represents the source amplitude, p' represents the pressure perturbation, x_1, x_2 and x_3 represent the three spatial Cartesian coordinates with the origin at the tuning fork and t is the time.

(a) From this description of the source, what can you say about the direction of vibration of the prongs of the tuning fork? [5%]

(b) What kind of a source does the tuning fork represent? [5%]

(c) By forming a convolution of the Green's function for the wave equation with the source, show that the magnitude of the pressure field at a large distance x from the source and at an angle θ from the x_1 axis is given by

$$|p'| = \frac{A}{4\pi} \frac{\omega^2}{c_0^4} \frac{\cos^2 \theta}{x}$$

[80%]

(d) Describe qualitatively what would happen if, after the tuning fork is struck, one of the prongs of the turning fork is placed behind a rigid plate. [10%]

3 A patch of turbulent flow in a duct, that has rigid walls and a cross-sectional area S, produces a local wall pressure fluctuation. In a smooth walled duct the disturbances away from the turbulent region are evanescent. However when there is a small hole in the wall, sound is generated by the turbulent wall pressure fluctuation $p'_t(t)$ at the hole.

(a) Describe the resulting wave pattern. (Detailed calculations are not required) [15%]

(b) The hole has cross-sectional area A and the duct wall thickness gives the hole an effective length l. The pressure difference across the wall is $\rho_0 l \dot{u}(t)$, where ρ_0 is the mean density, $u(t)$ is the velocity through the hole and the dot denotes differentiation with respect to time. The hole connects the duct to a compact cavity of volume V. You may assume that the turbulent pressure fluctuation has a single frequency ω, $p'_t(t) = \hat{p}_t \exp(i\omega t)$, and that ω is sufficiently small so that only plane acoustic waves propagate in the duct.

(i) Determine the relationship between the pressure in the duct at the opening of the hole, $p'_1(t)$, and $u(t)$ in terms of ρ_0, A, l, V and c_0, where c_0 is the speed of sound. [30%]

(ii) Determine the plane sound waves generated in the duct if

$$p'_1(t) = p'_t(t) + P\exp(i\omega t)$$

where P is the amplitude of the plane wave. [40%]

(iii) What is the damping mechanism in the solution in part (ii)? [10%]

(iv) What additional damping would occur in practice? [5%]

4 (a) A plane wave of amplitude I travelling at an angle θ to the positive x axis, where $0 \leq \theta \leq \pi/2$, is incident on an infinite plane wall at $x = 0$. The complex impedance of the wall is Z.

(i) Show that the complex amplitude of the reflected wave, R, is given by

$$\frac{R}{I} = \frac{Z\cos\theta - \rho_0 c_0}{Z\cos\theta + \rho_0 c_0} \, ,$$

where ρ_0 and c_0 are the density and sound speed of the air for $x < 0$. [25%]

(ii) In the case when Z is a purely real and positive constant, determine the minimum and maximum possible values of $|R/I|$, being careful to distinguish between the case $Z > \rho_0 c_0$ and $Z < \rho_0 c_0$. [15%]

(iii) What happens to $|R/I|$ when Z is purely imaginary? [10%]

(b) A ray travels through $x \geq 0$ in a medium with spatially-varying sound speed

$$c(x) = c_0 \exp(\alpha x) \, ,$$

where c_0 and α are constants. The ray passes through the origin, where it makes an angle β to the positive x axis. Determine the equation of the ray path and sketch its shape in the cases $\alpha > 0$ and $\alpha < 0$. [50%]

You may assume without proof that

$$\int \frac{\exp(\alpha x)}{\sqrt{1 - \sin^2\beta \exp(2\alpha x)}} dx = \frac{\sin^{-1}(\exp(\alpha x)\sin\beta)}{\alpha \sin\beta}$$

END OF PAPER

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