EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 1 May 2018   2 to 3.40

Module 4A3

TURBOMACHINERY I

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: Excerpt from Compressible Flow Data Book (pages 1-16 of 38 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The diffusion factor, DF, for an axial compressor blade is defined as

$$DF = \frac{V_{\text{max}} - V_{\text{exit}}}{V_{\text{max}}}$$

where V_{max} is the maximum near surface velocity and V_{exit} is the exit velocity. Describe how the diffusion factor relates to the choice of pitch to chord ratio. Explain why this particular expression is difficult to use at the preliminary design stage of a turbomachine and discuss alternatives. [20%]

(b) Figure 1 shows an approximated blade surface pressure distribution for a typical axial compressor blade. The pressure distribution is plotted against axial distance x expressed as a fraction of the axial chord c_x. The relative inlet stagnation pressure is p_0, the blade exit static pressure is p_{exit} and p_{min} is the minimum pressure.

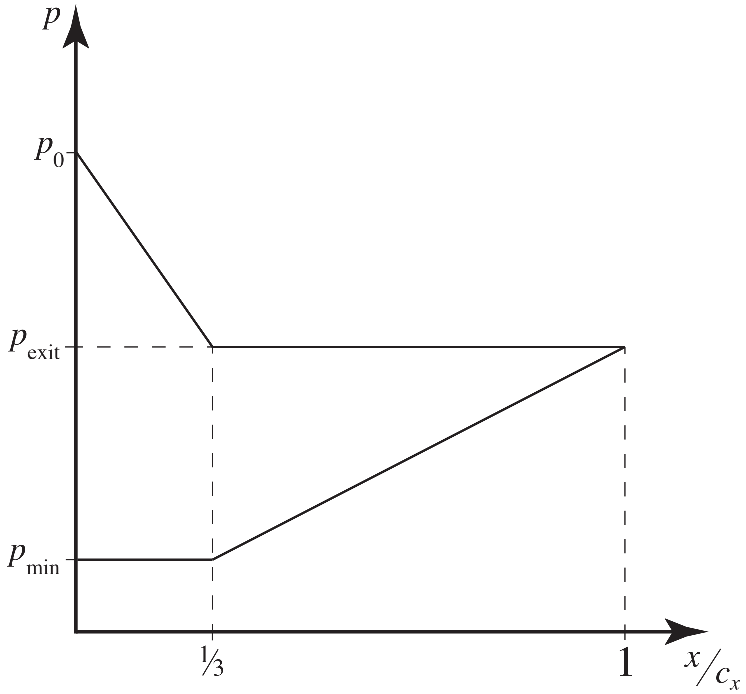
(i) Show that in loss-free incompressible flow with density ρ, the tangential force per unit span, F_θ, exerted by the blade on the flow is

$$F_\theta = \rho V_{\text{exit}}^2 \left(\frac{1}{3\,(1-DF)^2} - \frac{1}{4} \right) c_x$$

[35%]

(ii) By considering the change in tangential momentum of the flow across the blade row, obtain an expression for the blade pitch to axial chord ratio, s/c_x, as a function of the diffusion factor and the relative inlet and outlet angles. You should assume that the axial velocity is constant. [30%]

(c) Comment on how the diffusion factor for a given blade row would change with increasing Mach number. [15%]



Approximated blade surface
pressure distribution

Fig. 1

2 A single shaft research turbojet has a multistage axial compressor and a single stage axial turbine, as shown in Fig. 2. It is operated on a stationary test-bed at sea-level with inlet stagnation pressure $p_{01} = 101\,\text{kPa}$ and an inlet stagnation temperature $T_{01} = 288\,\text{K}$. At the design operating point the compressor stagnation pressure ratio $p_{02}/p_{01} = 6.0$; the inlet stagnation temperature at turbine inlet $T_{03} = 1200\,\text{K}$; the polytropic efficiency of the compressor is $\eta_{\text{pc}} = 0.85$ and that of the turbine is $\eta_{\text{pt}} = 0.9$. The turbine nozzle guide vanes and the propelling nozzle remain choked throughout the range of operation to be investigated.

It may be assumed that $c_p = 1005\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$ and $\gamma = 1.4$ for air flowing through the compressor and $c_{pe} = 1244\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$ and $\gamma_{\text{e}} = 1.3$ for the exhaust gas flowing through the turbine and the propelling nozzle.

(a) If the compressor and turbine polytropic efficiencies are assumed to remain constant as the speed of the turboshaft is varied, show that the compressor pressure ratio is given by

$$\frac{p_{02}}{p_{01}} = \left(1 + k\frac{c_{pe}}{c_p}\frac{T_{03}}{T_{01}}\right)^{\eta_{\text{pc}}\frac{\gamma}{\gamma-1}}$$

where k is a constant. Explain carefully any other assumptions that are made and find the value of k. [25%]

(b) Find the ratio of the turbine stator throat area to that of the propelling nozzle A_3/A_5. [25%]

(c) A test to examine the effect of volcanic ash deposition on the turbine nozzle guide vanes is performed by reducing the turbine stator throat area by 5% of its design value. Find the compressor pressure ratio at the design turbine inlet temperature $T_{03} = 1200\,\text{K}$ and the percentage change in engine mass flow and fuel flow rate. [25%]

(d) Explain what is meant by *compressor surge margin (stability margin)* and the factors that determine why it is necessary. Comment on the assumption of constant compressor polytropic efficiency in the analysis above. Is the reduced turbine stator throat area case likely to have more or less surge margin? Justify your answer. [25%]

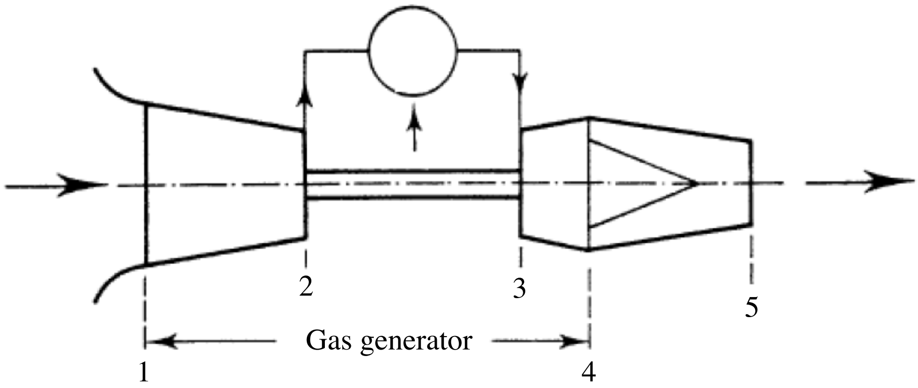


Fig. 2

(TURN OVER

3 (a) Show that the change in stagnation enthalpy across the rotor of a centrifugal pump or compressor is given by

$$h_{02} - h_{01} = \frac{1}{2}\left(U_2^2 - U_1^2\right) - \frac{1}{2}\left(V_{2,\mathrm{rel}}^2 - V_{1,\mathrm{rel}}^2\right) + \frac{1}{2}\left(V_2^2 - V_1^2\right)$$

where U is the blade speed, V_{rel} is the relative velocity and V is the absolute velocity of the flow. The subscripts 1 and 2 refer to rotor inlet and exit, respectively. Discuss the significance of the terms and explain why centrifugal machines can generally achieve higher stage pressure ratios than axial machines. [30%]

(b) A centrifugal compressor rotor has uniform axial inlet flow and has velocity components purely in the radial-tangential plane at outlet.

(i) Sketch and label typical velocity triangles for a rotor with backsweep χ and slip factor σ. [15%]

(ii) An application requires a mass flow rate of $7.84\,\mathrm{kg\,s^{-1}}$ of air and a rotor exit total pressure of 3 bar when the inlet stagnation conditions are 300 K and 1 bar. Consider a rotor with $45°$ of backsweep, an estimated slip factor of 0.9, a rotor total-to-total isentropic efficiency of 90% and rotational speed of 20 000 rpm. Calculate the required rotor exit radius for a blade exit span of 10 mm and an absolute rotor exit Mach number of 1. [35%]

(iii) Indicate how, for a given rotor speed and mass flow rate, the inlet tip radius can be chosen to minimise the blade tip inlet relative velocity. You may assume that the density and temperature vary slowly with changes in radius. Comment on why this is desirable. [20%]

Assume air behaves as a perfect gas with ratio of specific heats $\gamma = 1.4$ and specific heat at constant pressure $c_p = 1005\,\mathrm{J\,kg^{-1}K^{-1}}$.

END OF PAPER

4A3 2018 Answers:

1) –

2) (a) $k = 0.16$
(b) 0.471
(c) $k_{ash} = 0.17$, $PR_{ash} = 6.51$, 3% increase in mass flow, 0.8% increase in fuel flow
(d) –

3) (a) –
(b) (i) –
(ii) 0.263 m
(ii) –