EGT3
ENGINEERING TRIPOS PART IIB

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Thursday 03 May 2018 9.30 to 11.10

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**Module 4B23**

**OPTICAL FIBRE COMMUNICATION**

*Answer not more than **two** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 For a step index fibre with core radius $a$ and normalised wavenumber $V$, the $LP_{01}$ mode can be approximated by a Gaussian profile such that:

$$E(r) = E_0 \exp\left(-\frac{r^2}{r_0^2}\right)$$

where

$$r_0 = \frac{a}{\sqrt{\ln V}}$$

(a) Explain the role of $V$ in the modal analysis of step index optical fibres. Calculate the appropriate core diameter if a fibre is designed to be single moded for all wavelengths greater than 1260 nm, with core of refractive index $n_{co}$=1.455 and a cladding of refractive index $n_{cl}$=1.450. [20%]

(b) Given that $n_{co} \approx n_{cl}$, write down an expression for the time averaged power density as a function of $r$ and hence determine the total power $P$ carried in an optical fibre as a function of $E_0$, the impedance of free space $\eta_0$ and the fibre parameters ($V, a$ and $n_{co}$). [25%]

(c) The scattering of light at the core-cladding interface is proportional to the magnitude of the electric field at the interface $E(a)$. For a fibre with a constant optical power $P$ and radius $a$ show that assuming single mode operation, the scattering is maximised when $V^2 = \exp(1)$. Explain briefly why there should be a maximum. [25%]

(d) For the fibre with parameters from part (a), determine the maximum optical power that can be transmitted at $\lambda = 1550$ nm before electrical breakdown occurs at an electric field strength of $10^7$ V/m. Hence estimate the maximum data capacity that can be achieved in the conventional EDFA transmission window spanning from 1530 nm to 1565 nm, carefully stating any assumptions made. [30%]

2 A data rate of 400 Gbit/s is transmitted as PDM-16QAM at a symbol rate of 60 GBd with pulse shaping applied to obtain a minimum support rectangular spectrum. The signal modulates a laser with a wavelength of $\lambda = 1550$ nm and then is transmitted over 2000 km of optical fibre whose effective refractive index, $n(\lambda)$, varies with wavelength such that

$$n(\lambda) = 1.456 + \left(\frac{57}{\lambda}\right)^2 - \left(\frac{\lambda}{17500}\right)^2$$

where $\lambda$ is the free space wavelength measured in nm. After transmission the data is recovered using a digital coherent receiver.

(a) Explain the principle of the algorithms used to implement efficiently chromatic dispersion compensation in a digital coherent receiver. [20%]

(b) In a dispersive medium the propagation constant $\beta$ is given by $\beta = k_0 n(\lambda)$ where $k_0 = 2\pi/\lambda = \omega/c$ and $n(\lambda)$ is the wavelength dependent refractive index. Starting from the definition of the group velocity $v_g$, show that the group refractive index $n_g = c/v_g$ is given by

$$n_g = n - \lambda \frac{dn}{d\lambda}$$

[15%]

(c) Given that the dispersion coefficient $D$ is defined as

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right)$$

calculate the minimum number of taps $N_{CD}$ required to compensate the chromatic dispersion in the 60 GBd signal transmitted over the 2000 km link if the digital coherent receiver uses an oversampling rate of 16/15. [30%]

(d) Estimate the minimum power consumption required to realise digital chromatic dispersion compensation in the receiver if the energy required to perform a complex multiply is 1 pJ. You may assume that an $N = 2^m$ point FFT requires $(N/2)\log_2(N)$ complex multipliers where $m$ is an integer and there are no technological restrictions regarding the FFT size. [ 35%]

3 Noise and nonlinearities ultimately limit the capacity of optical fibre communication systems.

(a) What are the major sources of noise and nonlinearities encountered in a typical long haul optical fibre communication system employing digital coherent transceivers? [20%]

(b) In an unamplified system, shot noise presents a fundamental limit. This may be modelled as a stochastic process of electrons with charge $q$ arriving at a time $t_n$ such that the current $i(t)$ is given by

$$i(t) = q \sum_{n=-\infty}^{\infty} \delta(t - t_n)$$

(i) If the electrons arrive with a mean rate $\mu$ such that

$$\langle i(t) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} i(t)dt = q\mu$$

show that the autocorrelation is given by

$$\langle i(t)i(t+\tau) \rangle = q^2\mu^2 + q^2\mu\delta(\tau)$$

where the first and second term in $\langle i(t)i(t+\tau) \rangle$ corresponds to the signal and noise term respectively [25%]

(ii) The data is transmitted at a symbol rate $R_s$ and pulse shaping is applied such that at the receiver the signal and noise are filtered by a matched filter with frequency response

$$H(f) = \begin{cases} \cos\left(\frac{\pi f}{2R_s}\right) & -R_s < f < R_s \\ 0 & \text{otherwise} \end{cases}$$

Calculate the resulting signal to noise ratio of the filtered signal in terms of $\mu$ and $R_s$ and hence give the relationship between the signal to noise ratio and the number of photons per symbol. [25%]

(c) An unamplified system at $\lambda = 1550$ nm consists of a 100 Gbit/s signal transmitted as 31.5 GBd PDM-QPSK, over an optical fibre of length $L$ km with a loss of 0.2 dB/km. This is followed by a coherent receiver which includes a forward error correction code with a 26% coding overhead that achieves the Shannon capacity. Estimate the maximum distance the unamplified system can operate over, assuming the maximum launch power is +10 dBm. [30%]

**END OF PAPER**

Numerical answers for 4B23 2017/18

1 a) 8 µm, d) 199 Tbit/s

2 c) 1002 taps d) 2.0 W

3 c) 290 km