Version AAS/4

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 27 April 2017 9.30 to 11

Module 4C15

MEMS DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C15 MEMS Design data sheet (3 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The net van der Waals force of attraction *F(h)* between a sphere of radius *R* and a plane surface can be written as

$$F(h) = \frac{8\pi Rw}{3}\left\{\left(\frac{h}{h_0}\right)^{-2} - \frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}\right\}$$

where h is the smallest gap between the sphere and the plane and w is the work of adhesion of the materials concerned. Sketch the form of this relation and explain the physical significance of the dimension h_0. [30%]

(b) A nano-indenter is shown diagramatically in Fig. 1. The ceramic indenter I has a spherical tip of radius R and is supported by a spring of stiffness k within the frame which can be considered rigid. The specimen S, which also has a very high modulus, is brought into contact with the indenter by increasing its height z above the base of the frame. The gap h between the indenter and specimen is initially large compared to h_0. Confirm that when the surfaces snap together, provided $h >> h_0$, a good estimate of dimension h is

$$\sqrt[3]{\left(16\pi Rwh_0^2/3k\right)}.$$ [20%]

(c) Once the indenter and the specimen have made contact the specimen is retracted so that dimension z reduces until contact is broken. Show that the magnitude of the force P required to bring about separation is given by the expression $P = 2\pi Rw$. [15%]

(d) The spring stiffness is 20 N m^{-1} and the indenter radius $R = 65\times10^{-8}$ m. If S has moved down by 12×10^{-9} m at the instant when I and S separate, estimate the value of the work of adhesion for the materials of the contact. [20%]

(e) If both indenter and specimen were of the same material how could the measurement of P be used to estimate its surface energy? [15%]

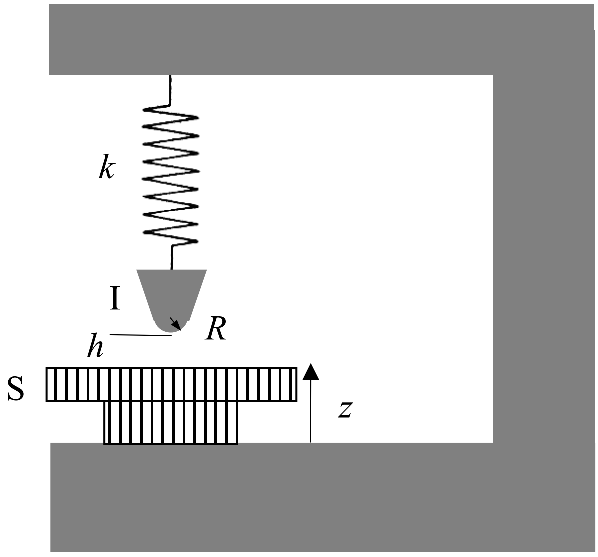

Fig. 1

2 A comb drive resonator is designed in a polysilicon surface micromachining process as shown in Fig. 2. The structural thickness of polysilicon is 6 μm and the mechanical parameters of the resonator are mass $m = 10^{-10}$ kg, stiffness $k = 10$ N m^{-1} and a Quality Factor of 100 for motion in air. The nominal overlap length L of the comb electrodes is 10 μm and the gap g between adjacent electrodes is 1 μm. One set of electrodes is used to actuate the resonator while the other set of electrodes is used to transduce the motional response. The number of electrode gaps N for each stationary electrode is as shown in Fig. 2.

(a) Obtain an expression for the force generated using a comb drive actuator as a function of g, N and other device parameters for the drive configuration shown in Fig. 2. [15%]

(b) Obtain an expression for the motional current as a function of the displacement x of the mass assuming that the potential difference across the electrodes is V_P. [15%]

(c) Estimate the amplitude of the motional current at resonance for applied voltages v_{ac} = 100 mV and V_P = 20 V. [20%]

(d) Derive expressions for the electromechanical motional parameters for this device and calculate specific values of these parameters for the drive conditions given in (c). [30%]

(e) Discuss process and design scaling approaches to reduce the motional resistance. [20%]

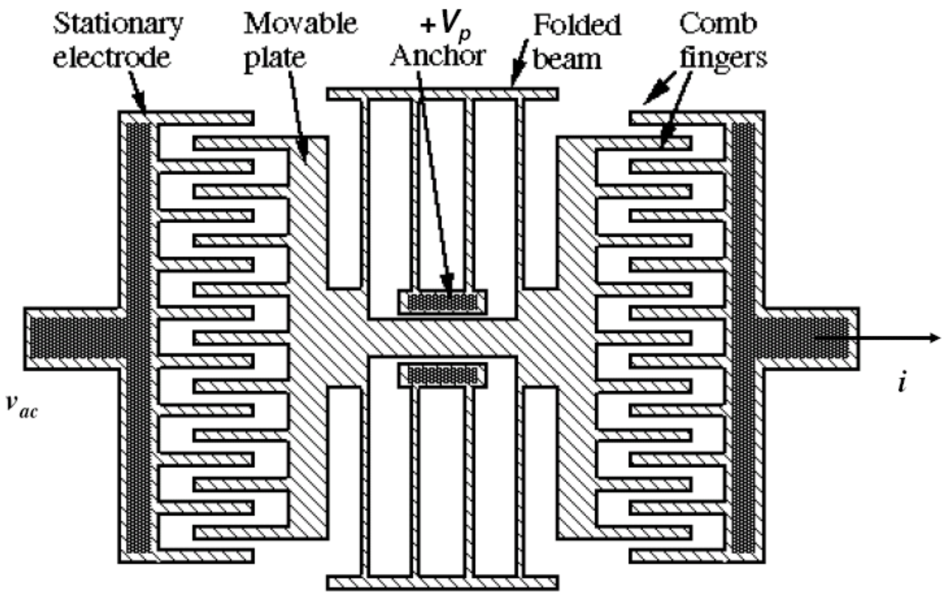

Fig. 2

3 A schematic of a capacitive accelerometer is shown in Fig. 3. The mechanical assembly consists of a suspended spring-supported mass. The response of the mass is transduced using a differential parallel-plate electrode arrangement. The mechanical parameters include the mass $m = 10^{-8}$ kg, stiffness $k = 10$ N m^{-1} and the damping constant, $c = 10^{-4}$ N m^{-1} s. The geometrical parameters for the device include a structural thickness $t = 20$ μm and a nominal electrode gap $g = 1$ μm. The electrode overlap length is $L = 100$ μm and the number of capacitive unit cells is $N = 100$. Equal and opposite values of dc voltage are applied to the two sets of fixed electrodes.

(a) Estimate the pull-in voltage if dc voltages of equal and opposite polarity are applied to the two fixed electrodes and the mass is grounded. [30%]

(b) Calculate the fractional change in capacitance in response to an external acceleration of 10 m s^{-2} along the sensitive axis. [30%]

(c) Estimate the thermo-mechanical noise equivalent acceleration response at 300 K. [20%]

(d) The natural frequency f_n of the device is sensitive to temperature and varies as $\partial f_n/\partial T = -30\times10^{-6} f_n$ K^{-1}. Estimate the change in the accelerometer response in (b) for a 1 K shift in ambient temperature. [20%]

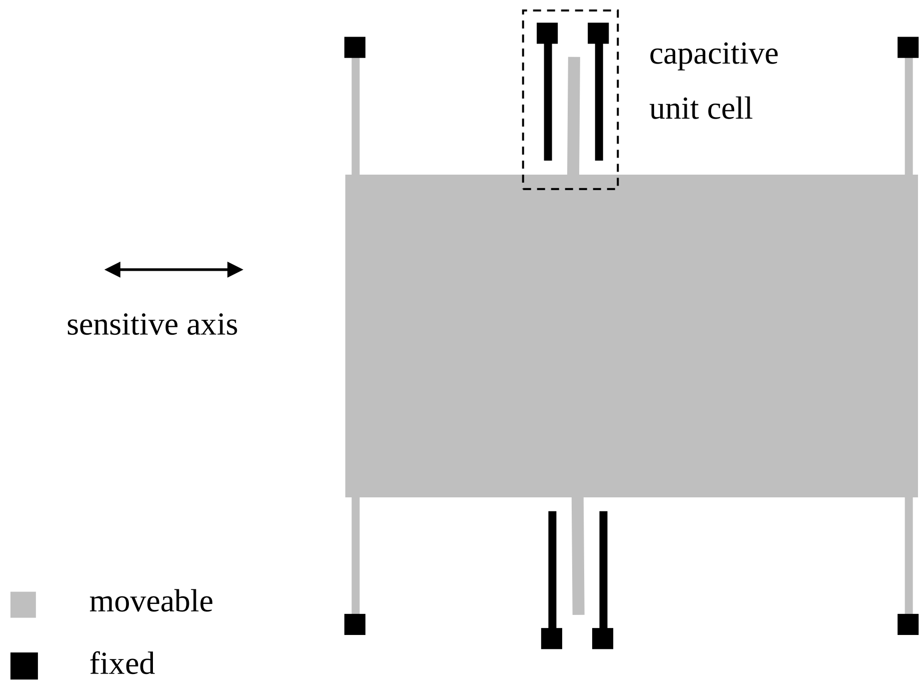

Fig. 3

4 A *z*-axis silicon micromachined vibrating rate gyroscope is shown in Fig. 4. The device consists of a 10 μm thick suspended mass in the form of a square plate of side dimension 1 mm. The mass is designed to be compliant along the two in-plane directions with a drive mode natural frequency of 10 kHz and a sense mode natural frequency of 11 kHz. The proof mass is driven in the drive mode using a comb drive configuration with electrode spacing of 1 μm and an initial electrode overlap length of 10 μm. The gap between the suspended mass and the fixed substrate is 2 μm. The device is packaged in an inert ambient at 1 atm and operated at room temperature, T = 300 K. The dynamic viscosity of the fluid surrounding the mechanical structure is $\eta = 1.8\times10^{-5}$ kg m^{-1} s^{-1}. The dissipation in the drive mode is dominated by viscous drag. The reduced Navier-Stokes equation to model viscous drag due to Couette flow is given by

$$\frac{\partial^2 U_x}{\partial y^2} = 0 ,$$

where U_x is the velocity associated with the fluid flow and y is the distance from the fixed substrate.

(a) Solve the above equation to obtain an expression for the viscous drag force on the proof mass and show that the damping constant c for motion in the drive direction can be expressed as $c = \eta A/h$ where A is the nominal area of overlap and h is the nominal constant gap between the plates. [20%]

(b) Hence, estimate the Quality factor for resonant motion in the drive mode. [10%]

(c) A sinusoidal ac voltage of magnitude 100 mV and a dc voltage of 100 V is applied between the comb electrodes and the mass with the ac signal represented by a sine wave at the resonant frequency of the device. Estimate the displacement of the device when driven under these conditions. The forces generated by the comb electrodes on either side of the mass act in phase. [20%]

(d) Estimate the response of the mass in the sense mode for an applied rotation rate Ω_z=1 rad s^{-1} about the sensitive axis assuming that the Quality factor for the drive mode and sense mode are identical. Comment on the validity of this assumption with reference to the device layout provided in Fig. 4. [20%]

(e) Estimate the thermo-mechanical noise limited resolution of the device. [20%]

(f) How does quadrature error affect the device response? [10%]

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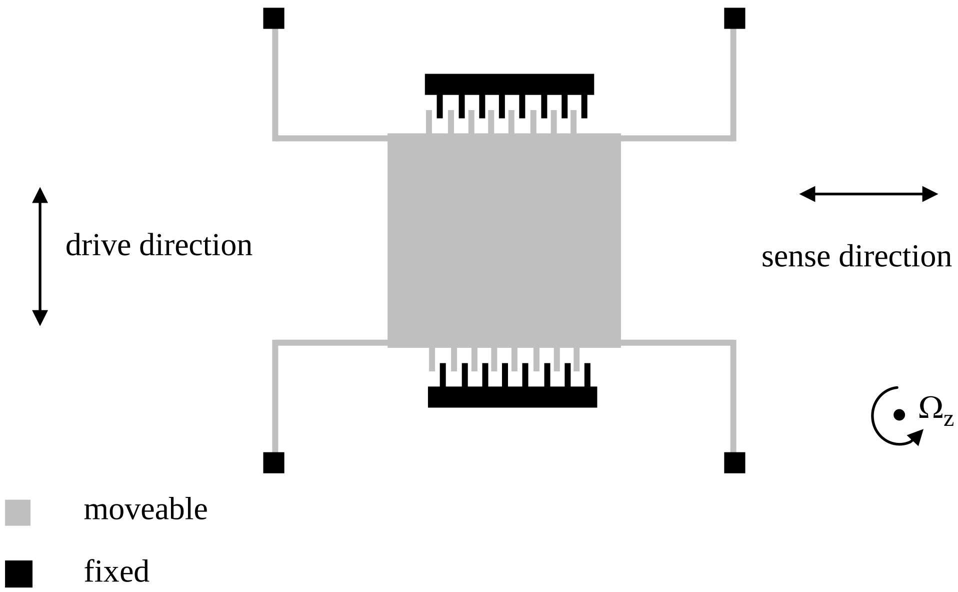

Fig. 4

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