Version HEMH/3

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 2 May 2017 9.30 to 11

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (9 pages).
Supplementary page: one extra copy of Figs. 1 and 2 (Question 1)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Vibration measurements are to be made for the purposes of modal analysis.

(i) Describe the advantages and disadvantages of an instrumented hammer versus an electromagnetic shaker as an input device. [20%]

(ii) Describe the advantages and disadvantages of an accelerometer versus a laser-Doppler vibrometer for response measurement. [20%]

(iii) For *each* of the combinations hammer/accelerometer and hammer/vibrometer, give an example of a measurement for which it would be an appropriate choice, and explain why. [20%]

(b) A measurement has been made of a vibration transfer function $H(\omega)$ of a structure, with velocity as the output variable. The response in the frequency range of the first three resonances is shown in Fig. 1 in the form of a decibel-magnitude plot, and in Fig. 2 as a Nyquist plot. The frequency resolution of the data is 0.5 Hz.

(i) Identify which peak in the first plot corresponds to which circle in the second, explaining your reasoning. [10%]

(ii) Use the information in the two plots to estimate the frequency and Q-factor of each mode, and also the amplitude factor (equal to the product of normalised modal amplitudes at the driving and observing points). [30%]

An additional copy of Figs. 1 and 2 is attached to the back of this paper. It may be detached, annotated to highlight significant features and handed in with your answers.

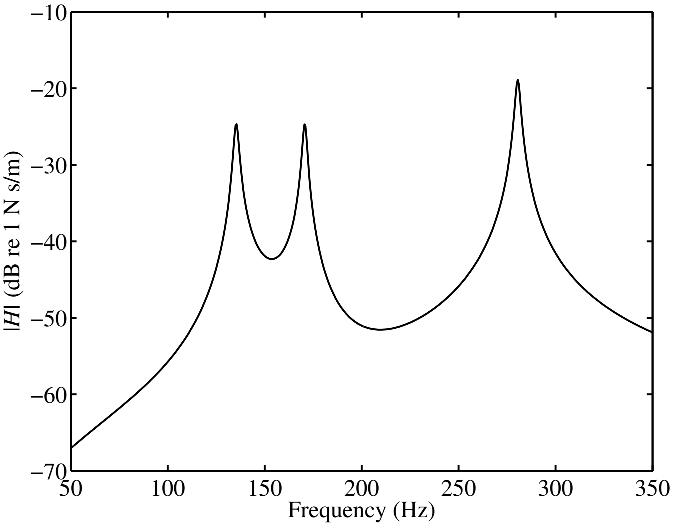

Fig. 1

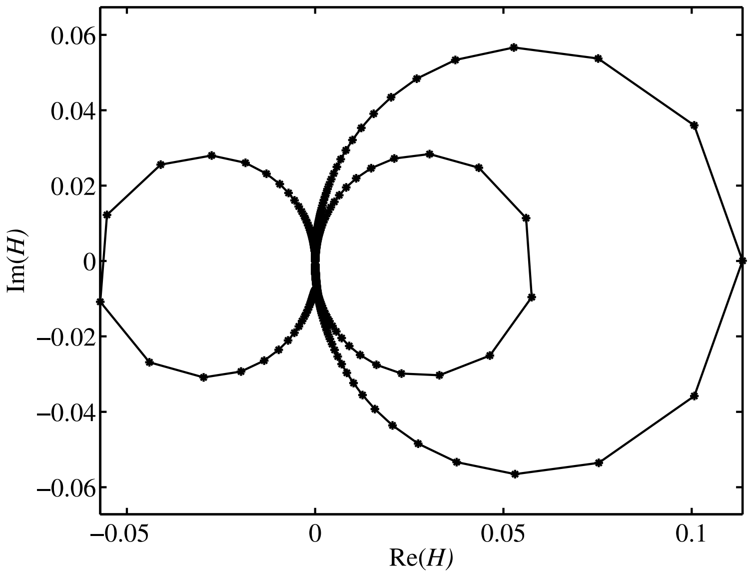

Fig. 2

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2 It is desired to add damping to a particular resonance of a structure by means of a tuned absorber. The system is represented in schematic form in Fig. 3. The structural mode has equivalent mass m and stiffness k, while the attached tuned absorber has mass λm and stiffness λk.

(a) Consider first the system without damping. Assuming that the mass ratio λ is very small, show that the two natural frequencies are given approximately by

$$\omega^2 \approx \frac{k}{m}(1 \pm \sqrt{\lambda}) \ .$$

Find corresponding approximate expressions for the associated mode shapes. [30%]

(b) Damping is now added by replacing the coupling spring by one with complex stiffness $\lambda k(1+i\eta)$, where η can be assumed to be small. Explain how Rayleigh's principle can be used to estimate the modal damping in a problem of this kind, then apply the method to estimate the modal Q-factors of the two modes of the system with the tuned absorber. [40%]

(c) Show that an approximate expression for the *modal overlap factor* (defined as the ratio of the average half-power bandwidth to the frequency spacing between the modes) is $\eta/(4\sqrt{\lambda})$. Sketch the expected form of a driving-point frequency response function measured on the mass m for the two cases $4\sqrt{\lambda} \gg \eta$ and $4\sqrt{\lambda} \ll \eta$. [20%]

(d) What considerations might influence the design of a practical tuned absorber? [10%]

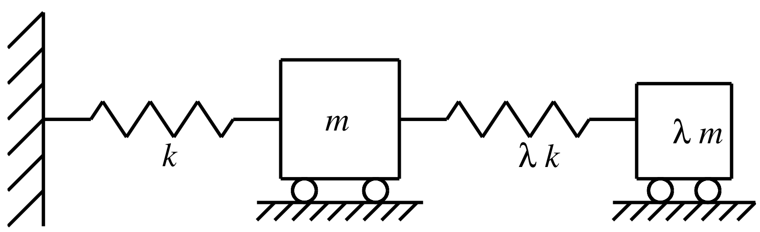

Fig. 3

3 (a) A circular drum has a membrane with tension T and mass per unit area m, with a fixed boundary of radius a. Using information from the data sheet, give expressions for the first ten natural frequencies and sketch the corresponding mode shapes. [30%]

(b) It is desired to make two drums in which the natural frequencies of corresponding modes are one octave apart on the musical scale, in other words with frequency ratios of 2. If only the radius is varied, how should it be changed? If instead the drums are all required to have the same radius, how could the tuning be achieved? What practical issues might put limits on the ability to achieve this effect? [15%]

(c) The drum from part (a) is now modified by constraining the motion of one point. Describe carefully what can be said about the frequencies and mode shapes of the ten modes described in (a) for the two cases:

(i) the constrained point is at the centre of the drum; [20%]

(ii) the constrained point is at a radius of $a/2$. [20%]

(d) How would the conclusions of part (c) be altered if the point constraint was replaced by a small concentrated mass? [15%]

4 (a) The damping properties of three materials (tool steel, cork and a ceramic honeycomb fabricated for use in a catalytic converter) are to be investigated. Three techniques are proposed:

(i) transient decay of vibration;

(ii) behaviour near peaks of vibration transfer functions in the frequency domain;

(iii) response to forced vibration.

Outline the advantages and disadvantages of each technique. With reference to information given in the Data Sheet, explain which method would you choose to measure the damping of the three candidate materials, commenting on any difficulties likely to be encountered in the measurement. [50%]

(b) Explain briefly how a Helmholtz resonance differs from a standing-wave resonance as in an organ pipe. [20%]

(i) A bottle has the form of a circular cylinder with cross-sectional area A and length x. The bottle is closed at one end, and has a circular cylindrical neck at the other end with length L and cross-sectional area S. Compare the Helmholtz resonance frequency of the bottle with the lowest standing-wave resonance frequency of the same bottle if the neck were removed so that the cylinder was simply open at the top. (You may assume that this standing-wave mode has a quarter-wavelength in the length of the cylinder.) [15%]

(ii) The neck section of the bottle described in (b)(ii) is gradually widened until it has the same radius as the cylinder. Describe what happens to the Helmholtz resonance mode during this process. [15%]

END OF PAPER

Version HEMH/3 Candidate Number:

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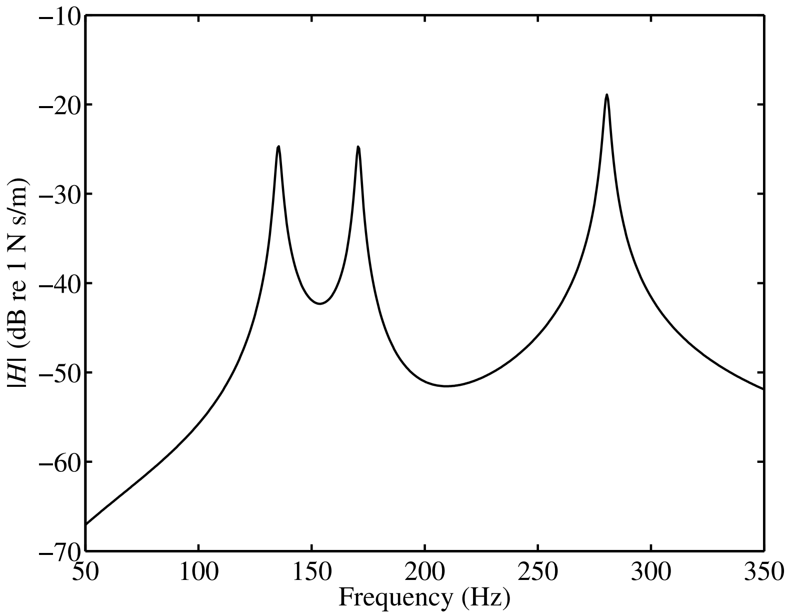
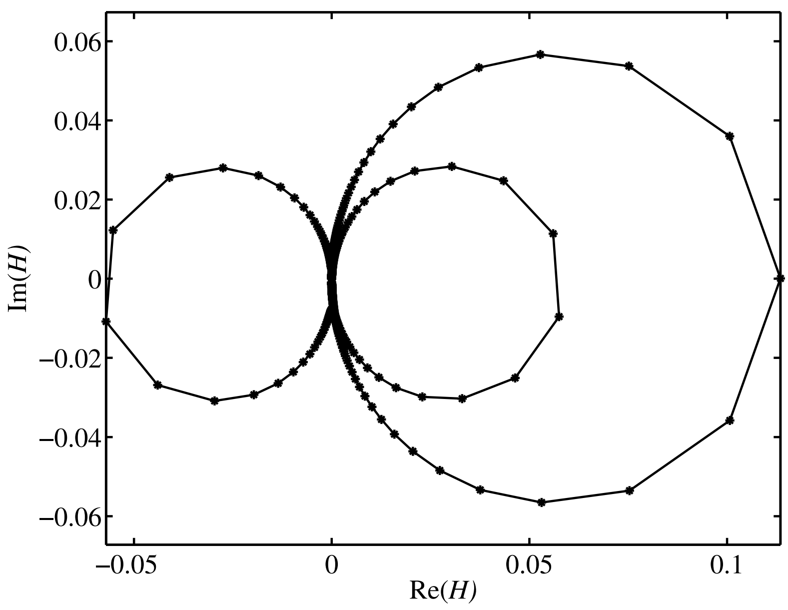
ENGINEERING TRIPOS PART IIB

Tuesday 2 May 2017, Module 4C6, Question 1.





Extra copy of Figs. 1 and 2 for Question 1.