EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 1 May 2018 2 to 3.40

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (9 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) An instrumented hammer is used for modal testing.

(i) Sketch the hammer, identifying key components. [10%]

(ii) The head of such a hammer has a total mass m. Determine the tip stiffness k required to achieve an impulse of duration b. [10%]

(iii) Sketch the frequency spectrum of the impulsive force delivered by the hammer and use your sketch to explain the rule of thumb that the hammer excites up to a frequency of $1/b$ Hz. [15%]

(b) A laser vibrometer is used to measure the vibration velocity of a steel panel struck by the hammer. The sampling rate of data acquisition is 2 kHz and the impulse data is sampled for a duration of 1 second. The driving-point velocity/force transfer function is computed from the FFT of velocity and force. At a particular location A on the plate a mode is identified at a frequency of 200 Hz, with a Q-factor of 20.

(i) Sketch the magnitude of the driving point transfer function in the vicinity of the peak for this mode, showing the spacing of the computed data points. [15%]

(ii) Sketch the corresponding modal circle, again showing the distribution of data points. [20%]

(c) The laser remains at A while the impulse is delivered at various other locations on the plate. Sketch modal circles for a variety of different impulse locations. Explain why the diameter of the modal circle changes with impulse position and identify a situation where the modal circle will have zero diameter and where the circle changes sign. [20%]

(d) The laser is moved to a different point B and in addition to the 200 Hz mode another mode is observed at 190 Hz with Q around 15.

(i) Explain why a new mode might have appeared and sketch the magnitude of the driving point transfer function for point B. [5%]

(ii) On a sketch show how the Nyquist plot might appear for a frequency range including the two modes. [5%]

2 Two rods with uniform cross-section of area A are joined with a butt joint, to form a single rod of length L. The join is at a distance a from the left-hand end. The left-hand section of rod has density ρ and Young's modulus E_1, while the right-hand section has the same density ρ but a different Young's modulus E_2. The corresponding wave speeds are $c_j = \sqrt{E_j/\rho}$ where $j = 1,2$. Distance x along the rod is measured from the left-hand end. The rod undergoes axial vibration with small displacement $w(x,t)$, and the ends are fixed so that $w(0,t) = w(L,t) = 0$.

(a) Explain why one boundary condition at the join requires

$$E_1 \left[\frac{\partial w}{\partial x}\right]_{x=a-} = E_2 \left[\frac{\partial w}{\partial x}\right]_{x=a+}$$

and write down the second boundary condition at this point. Hence, using the equation of motion from the data sheet, show that the natural frequencies ω of the coupled beam system satisfy

$$\frac{c_1}{E_1}\tan\frac{\omega a}{c_1} = -\frac{c_2}{E_2}\tan\frac{\omega(L-a)}{c_2}.$$

[30%]

(b) Use a graphical method to show the pattern of the roots of the equation for the natural frequencies from part (a). Hence show that these natural frequencies interlace with those obtained if the motion of the rod is constrained at the point $x = a$ so that $w(a,t) = 0$. [30%]

(c) Describe what happens in the special case for which $a/c_1 = (L-a)/c_2$. [15%]

(d) The effect of damping in the coupled rods is now considered. The two Young's moduli are replaced by complex values $E_j(1+i\eta_j)$ for $j = 1,2$, where $\eta_j \ll 1$. Use Rayleigh's principle together with expressions for potential and kinetic energy from the Data Sheet to show that the Q-factor of the nth mode, Q_n, satisfies

$$\frac{1}{Q_n} \approx J_1\eta_1 + J_2\eta_2$$

where J_1, J_2 are expressions involving integrals over the nth mode shape $u_n(x)$, satisfying $J_1 + J_2 = 1$. [25%]

3 (a) A possible design for a MEMS gas sensor is sketched in Fig. 1. A rectangular plate of mass *m* is able to vibrate in its own plane in the direction shown in the diagram, restrained by flexures (marked 'F') at the four corners. Electrodes near the plate allow excitation and sensing of the motion. One surface of the plate is coated with a material which can adsorb the gas to be detected. The adsorbed molecules increase the mass of the plate and shift the resonance frequency. This shift is detected and forms the basis of estimating the concentration of the gas.

(i) Discuss what determines the sensitivity of the sensor, and explain why low-damping design is desirable. [15%]

(ii) What factors should be taken into account when designing for low damping? Considerations of material, fabrication, geometry and operating conditions should be mentioned. [30%]

(iii) Suggest alternative designs that might allow lower damping and higher sensitivity to be achieved. [25%]

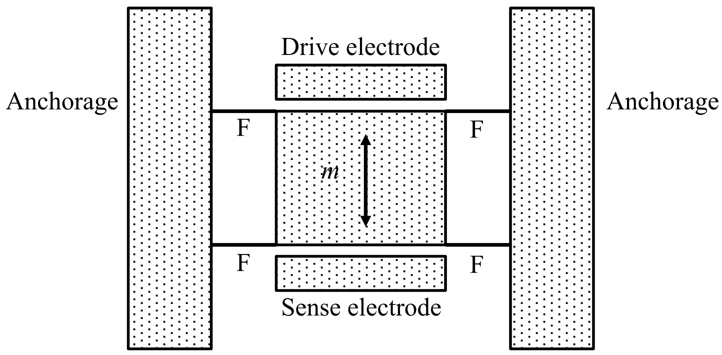

Fig. 1

(b) The body of an acoustic guitar is a thin-walled box with a circular soundhole of diameter 82 mm in the top plate.

(i) If the guitar has a Helmholtz resonance at 104 Hz and the speed of sound is $c = 340$ m/s, calculate the effective volume of the box. [10%]

(ii) Guitar makers sometimes use a device known as a "tornavoz" to lower the Helmholtz resonance frequency. This consists of a thin-walled cylindrical collar of the same diameter as the soundhole, glued to the underside of the top plate of the guitar around the soundhole and extending inside the box. Explain how this device works, and calculate the required height of the tornavoz cylinder in order to reduce the Helmholtz resonance frequency by one semitone to 98 Hz. [20%]

4 (a) A rectangular room of dimensions $L_x \times L_y \times L_z$ has hard walls, floor and ceiling. Sound pressure $p(x,y,z,t)$ inside the room satisfies the equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

where c is the speed of sound. The boundary condition on a hard surface requires that the gradient of pressure normal to the surface is zero. Use the method of separation of variables to find expressions for the mode shapes and corresponding natural frequencies of sound pressure in the room. [40%]

(b) Sinusoidal variation in space can be characterised by *wavenumber*: for example if variation with x is proportional to $\cos k_x x$, then k_x is the x component of wavenumber. Describe the geometrical distribution of the mode shapes from part (a) in the wavenumber space (k_x, k_y, k_z). Hence show that the approximate number of natural frequencies of the room below a frequency ω is given by

$$N(\omega) \approx \frac{L_x L_y L_z \omega^3}{6\pi^2 c^3}$$

provided ω is sufficiently high. [40%]

(c) A small rectangular concert hall has dimensions $L_x = 12$ m, $L_y = 15$ m, $L_z = 5$ m. If the speed of sound is $c = 340$ m/s and all the modes of the room have a Q factor 50, estimate the modal overlap factor at frequency 500 Hz. (Modal overlap factor is the number of natural frequencies lying within the half-power bandwidth of a mode at the given frequency.) [20%]

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Answers

1(b) 10 sample points within the half-power bandwidth

2(a) Second boundary condition $w|_{x=a-} = w|_{x=a+}$

(c) Natural frequencies where $\sin \omega a/c_1 = 0$ or $\cos \omega a/c_1 = 0$

(d) $J_1 = \dfrac{E_1 \int_0^a u_n'^2 dx}{E_1 \int_0^a u_n'^2 dx + E_2 \int_a^L u_n'^2 dx}$, $J_2 = \dfrac{E_2 \int_a^L u_n'^2 dx}{E_1 \int_0^a u_n'^2 dx + E_2 \int_a^L u_n'^2 dx}$

3(b)(i) 0.0205 m^3; (ii) height approximately 19 mm

4(a) Mode shapes $\cos \dfrac{m\pi}{L_x} \cos \dfrac{n\pi}{L_y} \cos \dfrac{q\pi}{L_z}$ where m, n and q take any values 0,1,2,3...

Corresponding natural frequency satisfies $\dfrac{\omega^2}{c^2} = \left[\dfrac{m\pi}{L_x}\right]^2 + \left[\dfrac{n\pi}{L_y}\right]^2 + \left[\dfrac{q\pi}{L_z}\right]^2$

(c) Modal overlap factor around 720