EGT3
ENGINEERING TRIPOS PART IIB

Friday 5 May $2017 \quad 9.30$ to 11

## Module 4C8

## VEHICLE DYNAMICS

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.

STATIONERY REQUIREMENTS
Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Attachment: Module 4C8 data sheet (3 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper.
You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Version DJC/4

1 (a) Describe with the aid of diagrams the main components and operating principles of a monotube damper and a twin-tube damper. State the significance of the differences in the two designs.
(b) Figure 1 shows a linear model with two degrees of freedom for predicting the vertical vibration response of a vehicle travelling along a road surface. It has sprung mass $m_{s}$, unsprung mass $m_{u}$, tyre stiffness $k t$, suspension stiffness $k$ and suspension damping $c$. The transfer function between vertical road displacement input $z_{r}$ and suspension working space output $z_{S}-z_{u}$ is given by

$$
\begin{aligned}
& \frac{z_{s}(j \omega)-z_{u}(j \omega)}{z_{r}(j \omega)} \\
& =\frac{\omega^{2} m_{s} k_{t}}{\omega^{4} m_{s} m_{u}-j \omega^{3}\left(m_{s}+m_{u}\right) c-\omega^{2}\left(m_{s}\left(k+k_{t}\right)+k m_{u}\right)+j \omega c k_{t}+k k_{t}} .
\end{aligned}
$$

If the double-sided mean square spectral density of the vertical road velocity input is $S_{0}$, find an expression for the mean square working space.

You may quote expressions from the data sheet and the following result:

$$
\int_{-\infty}^{\infty}|H(j \omega)|^{2} \mathrm{~d} \omega=\frac{\begin{array}{l}
\pi\left\{A_{0} B_{3}^{2}\left(A_{0} A_{3}-A_{1} A_{2}\right)+A_{0} A_{1} A_{4}\left(2 B_{1} B_{3}-B_{2}^{2}\right) \cdots\right. \\
\left.-A_{0} A_{3} A_{4}\left(B_{1}^{2}-2 B_{0} B_{2}\right)+A_{4} B_{0}^{2}\left(A_{1} A_{4}-A_{2} A_{3}\right)\right\}
\end{array}}{A_{0} A_{4}\left(A_{0} A_{3}^{2}+A_{1}^{2} A_{4}-A_{1} A_{2} A_{3}\right)}
$$

where

$$
H(j \omega)=\frac{B_{0}+(j \omega) B_{1}+(j \omega)^{2} B_{2}+(j \omega)^{3} B_{3}}{A_{0}+(j \omega) A_{1}+(j \omega)^{2} A_{2}+(j \omega)^{3} A_{3}+(j \omega)^{4} A_{4}}
$$



Fig. 1

## Version DJC/4

2 The vertical profiles of the left and right hand wheel tracks, $z_{\mathrm{L}}$ and $z_{\mathrm{R}}$, distance $2 T$ apart on a uniformly rough road surface can be defined in terms of an average vertical displacement $z_{\mathrm{V}}$ and a roll displacement $z_{\phi}$ as shown in Fig. 2(a). The mean square spectral densities of $z_{\mathrm{V}}$ and $z_{\phi}$ can be related using $S_{z \phi}(n)=|G(n)|^{2} S_{z_{\mathrm{V}}}(n)$, where $|G(n)|^{2}=n^{2}\left(n_{c}^{2}+n^{2}\right)^{-1}$ and $n_{\mathrm{c}}$ is a 'cut-off' wavenumber. The ratio of the mean square spectral densities of the average vertical displacement and the displacement of left or right profile is shown in Fig. 2(b). Also shown is the ratio of the mean square spectral densities of the roll displacement and the displacement of left or right profile.


Fig. 2(a)
(a) Explain the variation of the two ratios with wavenumber as shown in Fig. 2(b).
(b) Figure 2(c) shows a lateral-roll plane model of a vehicle, including the parameters and their values. Sketch approximate mode shapes and estimate natural frequencies of the vehicle at low speed. It is not necessary to derive equations of motion or calculate eigenvalues or eigenvectors, and it is not necessary to use all the parameter values.
(c) The vehicle in Fig. 2(c) travels along the uniformly rough road surface. The seat is at lateral offset $p$. The vertical acceleration of the seat is $\ddot{z}_{\mathrm{p}}$. The roll acceleration of the sprung mass is $\ddot{\theta}_{s}$. The ratio of the mean square of the seat acceleration due to roll, $p \ddot{\theta}_{s}$, to the mean square of the total seat acceleration, $\ddot{z}_{\mathrm{p}}$, is shown in Fig. 2(d). With reference to your answers to (a) and (b), explain why the ratio reduces as vehicle speed increases.


Fig. 2(c)


Fig. 2(d)

## Version DJC/4

3 (a) The equations of motion of a bicycle model for car handling (in the usual notation) are

$$
\begin{aligned}
& m(\dot{v}+u \Omega)+\left(C_{f}+C_{r}\right) \frac{v}{u}+\left(a C_{f}-b C_{r}\right) \frac{\Omega}{u}=C_{f} \delta+Y \\
& I \dot{\Omega}+\left(a C_{f}-b C_{r}\right) \frac{v}{u}+\left(a^{2} C_{f}+b^{2} C_{r}\right) \frac{\Omega}{u}=a C_{f} \delta+N .
\end{aligned}
$$

(i) State the assumptions needed to derive these equations of motion.
(ii) Use these equations to determine an expression for the critical speed of the vehicle and explain its implications for vehicle stability.
(b) A vehicle travels along a straight path at constant speed $u$, towing a trailer as shown in Fig. 3. The trailer has no suspension, is in horizontal plane motion and is free to rotate in the yaw plane about a vertical axis through point P . The rotation angle is $\theta$. The centre of mass of the trailer is located at point G, distance $a$ from P and distance $b$ from the axle of the trailer. The wheel track of the trailer is $2 d$ and is considered small in comparison to $a+b$. The combined cornering stiffness of the trailer tyres is $C$. The mass of the trailer is $m$ and the yaw moment of inertia about G is $I$. Consider the perturbed motion of the trailer when $\theta$ is small.
(i) Derive expressions for the slip angles of the trailer tyres and the lateral tyre force, considering both forward $(u>0)$ and reverse $(u<0)$ motion of the towing vehicle. State any assumptions.
(ii) Derive the equation of motion of the trailer.
(iii) Discuss the yaw stability of the trailer when the vehicle combination is in reverse motion $(u<0)$.


Fig. 3

## Version DJC/4

4 (a) Show that for motion along a straight track at steady speed $u$, the creep forces on a railway wheelset result in a total lateral force $Y$ of

$$
Y=2 C\left(\frac{\dot{y}}{u}+\theta\right)
$$

and a total yawing moment of

$$
N=2 d C\left(\frac{\varepsilon y}{r}-\frac{d \dot{\theta}}{u}\right)
$$

where $y$ is the lateral tracking error of the wheelset, $\theta$ is the yaw angle of the wheelset, $2 d$ is the track gauge, $r$ is the rolling radius of each wheel when $y=\theta=0, \varepsilon$ is the effective conicity of the wheelset and $C$ is the creep coefficient relating the lateral and longitudinal creep velocities to the corresponding creep forces. List your assumptions.
(b) A rigid railway bogie consists of two wheelsets connected by a rigid frame, with a spacing of $2 a$, as shown in Fig. 4. The bogie supports one end of a wagon of mass $M$ which causes a vertical load $M g / 2$ to be applied at the centre of the frame G. It is travelling on a straight track at speed $u$. The track runs along the side of a hill and so has a small, constant camber angle $\phi$. Determine the steady-state lateral tracking error $y$ at the centre of the frame $G$ and the steady-state yaw angle $\theta$. Comment on the implications for track design.


Fig. 4

## END OF PAPER

## Answers

1. (b)
$E\left[\left(z_{s}-z_{u}\right)^{2}\right]=\frac{S_{0} \pi\left(m_{s}+m_{u}\right)}{C}$
2. (b)

Approximate modes and frequencies:
sprung mass vertical 1.4 Hz
unsprung mass verrtical 11 Hz
sprung mass lateral/roll 1.2 Hz
unsprung mass lateral/roll 11 Hz
3. (a) (ii)
$U_{c}=\sqrt{\frac{C_{f} C_{r} l^{2}}{m\left(a C_{f}-b C_{r}\right)}}$ for $a C_{f}>b C_{r}$
(b) (i)
$\alpha=\frac{u \theta+(a+b) \dot{\theta}}{u}$
$Y=-C\left[\frac{u \theta+(a+b) \dot{\theta}}{|u|}\right]$
(ii)
$\left(I+m a^{2}\right) \ddot{\theta}+\frac{(a+b)^{2} C}{|u|} \dot{\theta}+(a+b) C \operatorname{sgn}(u) \theta=0$
4. (b)
$\theta=\frac{M g \sin \phi}{8 C}$
$y=0$

