Version DC/2

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 24 April 2018 2 to 3.40

Module 4C8

VEHICLE DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C8 datasheet, 2017 (3 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The car moves at steady forward speed u on a horizontal surface. It has mass m, yaw moment of inertia I, and lateral creep coefficients C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. A yawing moment N acts on the vehicle and a lateral force Y is applied at the centre of mass G. The equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v}+u\Omega)+\left(C_f+C_r\right)\frac{v}{u}+\left(aC_f-bC_r\right)\frac{\Omega}{u}=C_f\delta+Y$$

$$I\dot{\Omega}+\left(aC_f-bC_r\right)\frac{v}{u}+\left(a^2C_f+b^2C_r\right)\frac{\Omega}{u}=aC_f\delta+N$$

(a) State the assumptions needed to derive these equations of motion. [10%]

(b) Use the equations of motion to derive expressions for the steady state yaw rate and sideslip responses of the vehicle when a steady steer angle δ is applied and held, with no external forces or moments. The radius of the turn is R. [30%]

(c) Derive expressions for the steady state values of the sideslip angles of the front and rear tyres. [20%]

(d) For the case $u \to 0$, illustrate the configuration using a sketch of the vehicle showing the position of the turn centre, the steer angle δ, the various slip angles and other salient features. [20%]

(e) Determine an expression for the turn radius R and sketch its variation with speed for understeer, oversteer, neutral steer and real vehicles. [20%]

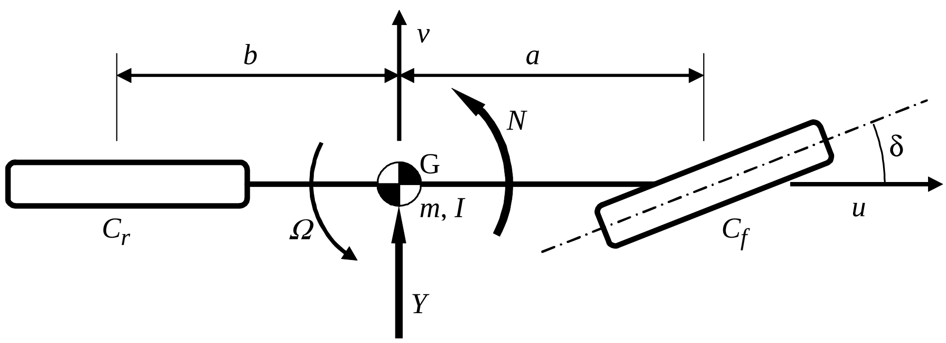

Fig. 1

2 (a) Explain why both lateral and longitudinal creep forces are important in curving of railway bogies at constant forward speed. Compare and contrast this situation with the tyre forces acting during constant-speed cornering of automobiles. [30%]

(b) The castered wheel of a supermarket trolley is idealised as shown in plan view in Fig. 2. A wheel of mass m with negligible diametral moment of inertia, is free to rotate about a horizontal axis through its mass centre. The bearings allow lateral displacement y of the wheel along the axle, restrained by springs with combined stiffness k. The wheel does not yaw with respect to the axle. The lateral creep coefficient of the tyre is C.

The axle is attached to a rigid yoke AB, of length a which is free to rotate about a vertical axis at A, with yaw angle θ. The yoke has moment of inertia I about A. Point A moves in a straight line at constant speed U, and the floor is smooth and level.

(i) Derive equations for small lateral oscillations of the system in the horizontal plane (neglecting gyroscopic effects.) [35%]

(ii) Use the Routh-Hurwitz criterion to determine the conditions for which the lateral motion of the system will be stable. [35%]

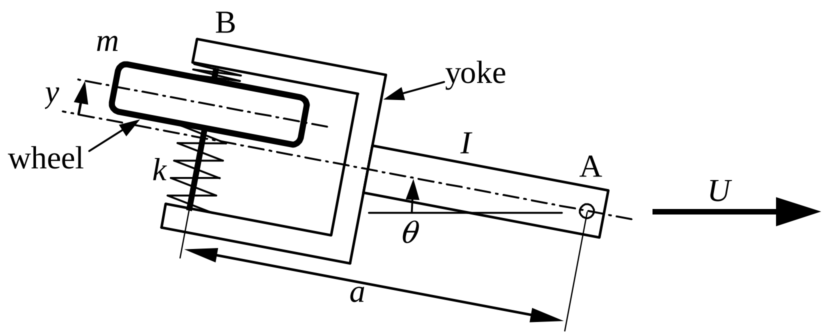

Fig. 2

3 Figure 3 shows a linear model with two degrees of freedom for predicting the vertical vibration of a vehicle travelling along a rough road surface. The model has sprung mass m_s, unsprung mass m_u, tyre stiffness k_t, suspension stiffness k and suspension damping c. Figure 3 also shows RMS values of sprung mass acceleration R and working space W due to vertical velocity input with spectral density $S_0 = 10^{-4}$ m^2 s^{-1} rad^{-1}, calculated using:

$$R = \sqrt{\mathrm{E}[\ddot{z}_s^2]} = \sqrt{\frac{\pi S_0[(m_s + m_u)k^2 + k_t c^2]}{m_s^2 c}}$$

$$W = \sqrt{\mathrm{E}[(z_s - z_u)^2]} = \sqrt{\frac{\pi S_0(m_s + m_u)}{c}}$$

The two sets of contour lines in Fig. 3 show constant values of RMS dynamic tyre force T and suspension stiffness k.

(a) When $k = 0$ find the relationship between R and W in the form $R = f\,(W, m_s, m_u, k_t, S_0)$. [15%]

(b) Find the locus of points, such as A, D and E in Fig. 3, at which R is a minimum for a given k. The locus should be expressed in the form $R = g\,(W, m_s, m_u, k_t, S_0)$. [50%]

(c) With reference to your answers to (a) and (b), and with reference to some or all of points A to E marked on the figure, explain the trade-offs involved in selecting values of suspension stiffness k and suspension damping c for a car. [35%]

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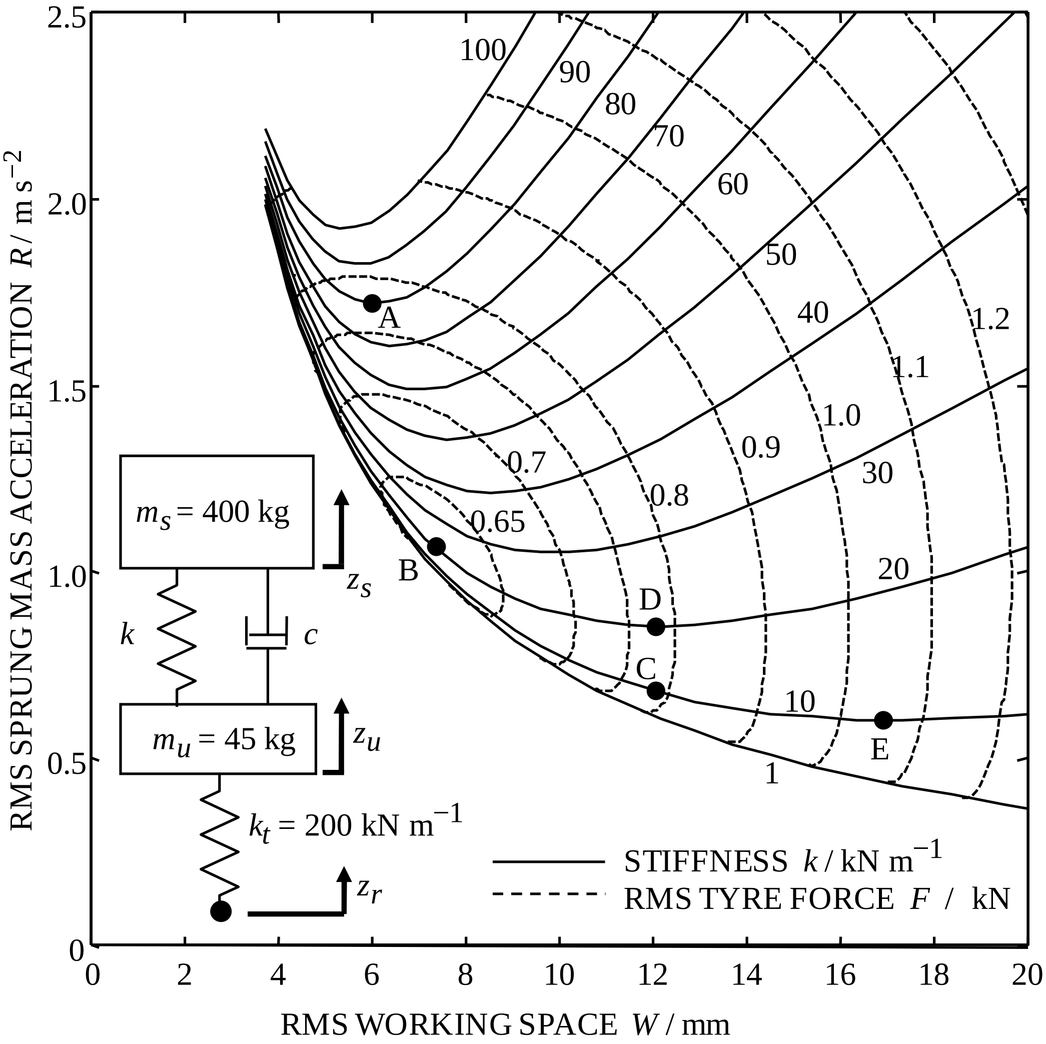

Fig. 3

(TURN OVER

4 (a) Describe the mechanism of ‘wheelbase filtering’. Show how vehicle speed U, wheelbase L and frequency f_{bounce} should be related to minimise excitation of the pure bounce mode of sprung mass vibration at its natural frequency f_{bounce}. [25%]

(b) Figure 4 shows a pitch-plane model of a car suspension. The sprung mass has mass m and pitch inertia J. The front and rear suspensions comprise a parallel spring k and damper c. The front and rear suspensions are linked by a massless rigid beam pivoted to the centre of the vehicle and connected to the front and rear axles by springs with stiffness s. The car travels at constant speed U. The vertical displacement inputs from the road surface are z_{r1} and z_{r2}.

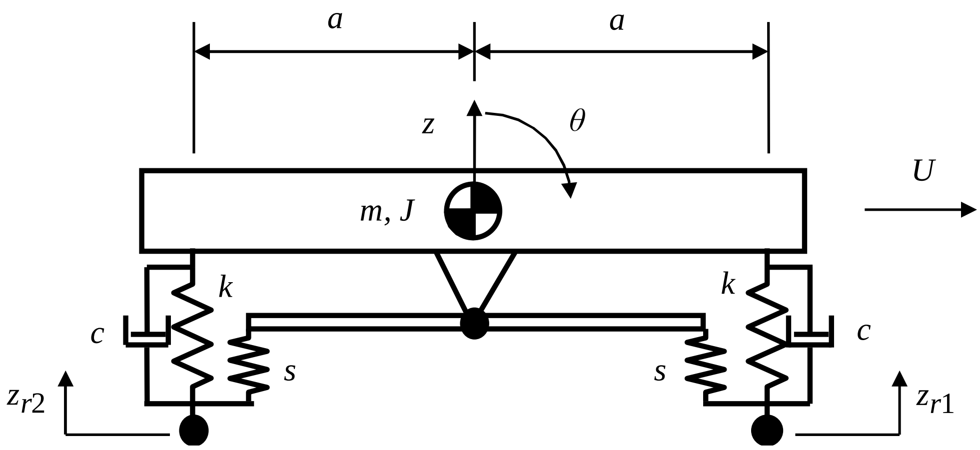

Fig. 4

(i) Derive expressions for the natural frequencies of the undamped vehicle and describe the corresponding mode shapes. [40%]

(ii) The stiffnesses k and s are to be chosen to minimise excitation of the sprung mass modes of vibration at a given speed U. If pitch inertia $J = ma^2$, derive expressions for suitable values of k and s. Comment on any other factors that might influence the choice of k and s. [35%]

END OF PAPER

Answers

1 (b) $\dfrac{\beta}{\delta}=\dfrac{\left(lbC_r-amu^2\right)C_f}{C_fC_rl^2-Csmu^2}$, $\dfrac{\Omega}{\delta}=\dfrac{C_fC_rlu}{C_fC_rl^2-Csmu^2}$

(c) $\dfrac{\alpha_f}{\delta}=\dfrac{-bC_rmu^2}{C_fC_rl^2-Csmu^2}$, $\dfrac{\alpha_r}{\delta}=\dfrac{-aC_fmu^2}{C_fC_rl^2-Csmu^2}$

(d) $\alpha_f=\alpha_r=0; \quad \beta=\dfrac{b}{l}\delta$ (e) $R=\dfrac{l}{\delta}\left(1-\dfrac{Csmu^2}{lC_fC_r}\right)$

2 (b) (i) $\begin{bmatrix} m & ma \\ 0 & I \end{bmatrix}\begin{pmatrix} \ddot{y} \\ \ddot{\theta} \end{pmatrix}+\begin{bmatrix} C/u & Ca/u \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \dot{y} \\ \dot{\theta} \end{pmatrix}+\begin{bmatrix} k & C \\ -ak & 0 \end{bmatrix}\begin{pmatrix} y \\ \theta \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (b)(ii) $k>\dfrac{C}{a}$

3 (a) $R=\dfrac{1}{W}\dfrac{\pi S_0}{m_s}\sqrt{k_t\left(m_s+m_u\right)}$ (b) $R_{\min}=\dfrac{\sqrt{2}}{W_{\min}}\dfrac{\pi S_0}{m_s}\sqrt{k_t\left(m_s+m_u\right)}$

4 (b) (i) $\omega_{bounce}=\sqrt{\dfrac{2(k+s)}{m}}; \quad \begin{pmatrix} z \\ \theta \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad \omega_{pitch}=\sqrt{\dfrac{2ka^2}{J}}; \quad \begin{pmatrix} z \\ \theta \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

(b) (ii) $k=0; \quad s=\dfrac{\pi^2u^2m}{8a^2}$