EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 2 May 2017 2 to 3.30

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C9 datasheet (3 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Using indicial notation, prove the identities relating vectors $\underline{a}$, $\underline{b}$, $\underline{c}$ and $\underline{d}$

(i) $(\underline{a}\otimes\underline{b}):(\underline{c}\otimes\underline{d}) = (\underline{a}\cdot\underline{c})(\underline{b}\cdot\underline{d})$ [10%]

(ii) $(\underline{a}\otimes\underline{b})\cdot\underline{c}+\underline{a}\cdot(\underline{b}\otimes\underline{c}) = \underline{a}(\underline{b}\cdot\underline{c})+\underline{c}(\underline{a}\cdot\underline{b})$ [10%]

(iii) $\underline{a}\times(\underline{b}\times\underline{c})+\underline{b}\times(\underline{c}\times\underline{a})+\underline{c}\times(\underline{a}\times\underline{b}) = 0.$ [10%]

(b) The yield function of a porous metallic solid is specified in terms of material constant α and the material strength Y as

$$\phi = \sqrt{\sigma_e^2 + \alpha^2\sigma_m^2} \leq Y$$

where the von-Mises stress is given specified in terms of the deviatoric stress s_{ij} as $\sigma_e^2 \equiv (3/2)s_{ij}s_{ij}$ and the hydrostatic stress $\sigma_m \equiv \sigma_{kk}/3$. Further, the plastic strain rate of the solid is given in terms of the hardening modulus h by the plastic normality relation

$$\dot{\varepsilon}_{ij}^p = \frac{1}{h}\frac{\partial\phi}{\partial\sigma_{ij}}\frac{\partial\phi}{\partial\sigma_{kl}}\dot{\sigma}_{kl}.$$

(i) Calculate the uniaxial compressive yield strength σ_Y in terms of Y and α. [15%]

(ii) Give expressions for the plastic strain rate $\dot{\varepsilon}_{ij}^p$ under an applied uniaxial stress rate $\dot{\Sigma}$ in the x_3 direction. Hence calculate the plastic Poisson's ratio defined as $\nu^p = -\dot{\varepsilon}_{11}^p/\dot{\varepsilon}_{33}^p$. [35%]

(iii) Comment on the pecularities of the response of the solid when $\alpha > 3/\sqrt{2}$. [20%]

2 (a) State Drucker's postulates and hence derive the upper bound theorem of plasticity. [35%]

(b) Figure 1 shows the section of a thick-walled cylindrical pressure vessel whose inner radius is a and outer radius is b. The vessel is internally pressurised to a gauge pressue p and made from a material with shear yield strength k. A proposed collapse mechanism comprises intersecting shear planes as indicated in Fig. 1. Each set of planes is symmetrical, make an angle 2β with respect to each other and tangentially intersect the inner surface of the vessel.

(i) For the special case of $2\beta = 90^{\mathrm{o}}$, estimate the upper bound collapse load p_{max} in terms of k and b/a. [30%]

(ii) Calculate a general expression for p_{max} in terms of β and hence comment on the effect of β on the collapse pressure. [35%]

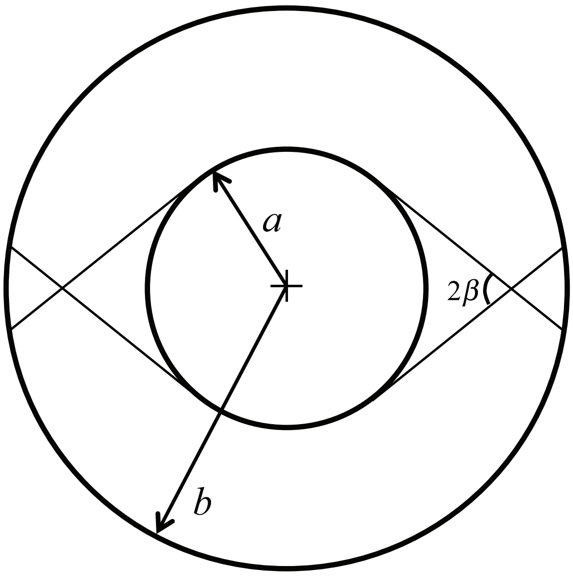

Fig. 1

3 A viscoelastic material is modelled using the constitutive relationship

$$\sigma_{ij}(t) = \int_0^t 2\mu_r(t-\tau)\frac{\partial \varepsilon_{ij}(\tau)}{\partial \tau}\,d\tau \;+\; 2\mu_r(t)\varepsilon_{ij}(0)$$

$$+\int_0^t \lambda_r(t-\tau)\frac{\partial \varepsilon_{kk}(\tau)}{\partial \tau}\delta_{ij}\,d\tau \;+\; \lambda_r(t)\varepsilon_{kk}(0)\delta_{ij} \;\;,$$

where $\sigma_{ij}(t)$ is the time-dependent stress, $\varepsilon_{ij}(t)$ is the strain, and $\mu_r(t)$ and $\lambda_r(t)$ are relaxation moduli. The Kronecker delta is denoted by δ_{ij}.

(a) Give a brief qualitative description of the physical interpretation of the four terms on the right hand side of this constitutive relationship. [15%]

(b) Characterisation experiments are performed on a particular viscoelastic material to determine the relaxation moduli $\mu_r(t)$ and $\lambda_r(t)$.

(i) A shear test imposes the deformation $\gamma_{12} = 2\varepsilon_{12} = \gamma_0 H(t)$, where $H(t)$ is a Heaviside step function. The measured response is

$$\sigma_{12}(t) = \gamma_0 c_1 e^{-c_2 t} \;\;,$$

where c_1 and c_2 are constants. Show that $\mu_r(t) = \sigma_{12}(t)/\gamma_0$. [15%]

(ii) A second test imposes a constant uniaxial strain rate $d\varepsilon_{11}/dt = \dot{\varepsilon}_0$ on an initially undeformed specimen, with all other strain components equal to zero. The measured response is

$$\sigma_{11}(t) = \dot{\varepsilon}_0 c_3 \left(1 - e^{-c_4 t}\right) \;\;,$$

where c_3 and c_4 are constants. Derive an expression for $\lambda_r(t)$. [40%]

(c) To simplify the model, it is assumed that $\lambda_r(t) = 2\mu_r(t)$, which gives a time-independent Poisson's ratio $\nu = 1/3$. Taking $\mu_r(t)$ to be the same as in part (b)(i), derive a differential equation relating $\sigma_{11}(t)$ and $\varepsilon_{11}(t)$ for the case of uniaxial stress in the x_1 direction (i.e. $\sigma_{22} = \sigma_{33} = 0$). [30%]

END OF PAPER

ANSWERS

1 (b)(i) $\dfrac{Y}{\left(1+\frac{\alpha^2}{9}\right)}$; (b)(ii) $\nu^p = \dfrac{\left(\frac{1}{2}-\frac{\alpha^2}{9}\right)}{1+\frac{\alpha^2}{9}}$

2 (b)(i) $p = k\sqrt{\left(\dfrac{b}{a}\right)^2 - 1}$; (b)(ii) $p = k\sqrt{\left(\dfrac{b}{a}\right)^2 - 1}$

3 (b)(ii) $\lambda_r(t) = c_3 c_4 \exp(-c_4 t) - 2c_1 \exp(-c_2 t)$

(c) $\dot{\sigma}_{11} + c_2\sigma_{11} = 4(1-\nu)c_1\dot{\varepsilon}_{11}$