Version VSD/3

EGT3
ENGINEERING TRIPOS PART IIB

Monday 30 April 2018 14.00 to 15.40

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C9 datasheet (3 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A slender beam, shown in Fig. 1, has length L and a square cross-section of side length B. The position of a material point within the plane of bending is $\underline{x} = x_1\underline{e}_1 + x_2\underline{e}_2$. The in-plane displacement is $\underline{u} = u_1\underline{e}_1 + u_2\underline{e}_2$. The beam is fully constrained at $x_1 = 0$, i.e. $u_1 = u_2 = 0$. Isotropic linear elastic material behaviour can be assumed, with Young's modulus E and Poisson ratio ν.

(a) The displacement field in the plane of bending is modelled by

$$\underline{u} = w(x_1)\,\underline{e}_2 - \frac{dw}{dx_1}\,x_2\,\underline{e}_1,$$

where w is the deflection of the neutral axis. Explain the physical basis of each term, including any kinematic assumptions. [15%]

(b) Show that the elastic strain energy density at a point in the beam is given by

$$U = \frac{1}{2}E\left(\frac{d^2w}{dx_1^2}\,x_2\right)^2.$$

State any assumptions made in the derivation. [15%]

(c) The beam is subjected to two applied loads: (1) a distributed bending moment per unit length of beam $M^e(x_1)$, acting over $0 \le x_1 \le L$, in the direction shown in Fig. 1; and (2) self weight, with the direction of gravity as shown in Fig. 1.

(i) Using the method of minimum potential energy, derive a differential equation governing $w(x_1)$ in the range $0 \le x_1 \le L$, and state the appropriate boundary conditions. [40%]

(ii) For the specific case $M^e(x_1) = M_0^e\,x_1/L$, where M_0^e is a constant, find an expression for the deflected shape $w(x_1)$ of the beam. [30%]

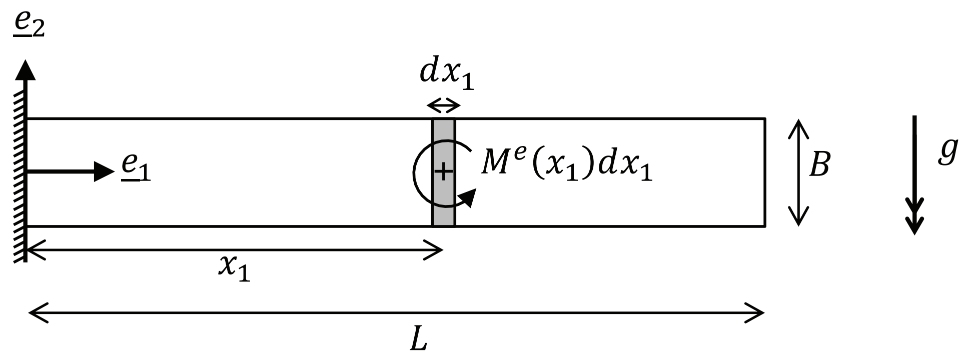

Fig. 1

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2 (a) Show that for a J_2 flow theory elastic perfectly-plastic solid

$$\sigma_{ij}\dot{\varepsilon}^p_{ij} = Y\dot{\varepsilon}^p_e .$$

Here, Y is the tensile yield strength while $\dot{\varepsilon}^p_{ij}$, $\dot{\varepsilon}^p_e$ and σ_{ij} are the plastic strain rate, the effective plastic strain rate and the stress, respectively. [25%]

(b) A metal matrix composite comprises a small volume fraction f_v of long circular ceramic fibres of radius a in a metal matrix of tensile yield strength Y. The matrix is a J_2 flow theory solid and, in comparison to the matrix, the fibres may be assumed to be rigid. The composite is subjected to a pure hydrostatic stress Σ_m as shown in Fig. 2 and a representative unit cell of the composite may be assumed to be a cylinder of radius b as indicated in Fig. 2.

(i) Show that $b \approx a/\sqrt{f_v}$ in the limit $b \gg a$. [10%]

(ii) Assume a radial velocity field within the matrix of the form $u = Ar + B/r$ where A and B are suitably chosen constants and r is the distance from the the centre of the unit cell. Derive an upper bound for the collapse load of the composite under hydrostatic loading. (The radial and tangential strain rates are related to u as $\dot{\varepsilon}_r = du/dr$ and $\dot{\varepsilon}_\theta = u/r$, respectively.) [45%]

(iii) Comment on the collapse load calculated above in the limits $f_v \to 0$ and $f_v \to 1$. [20%]

Note that $\int r\sqrt{1+a^4/r^4}dr = (a^2/2)[\sec\theta + \ln(\tan\theta/2)]$ where $\tan\theta = r^2/a^2$.

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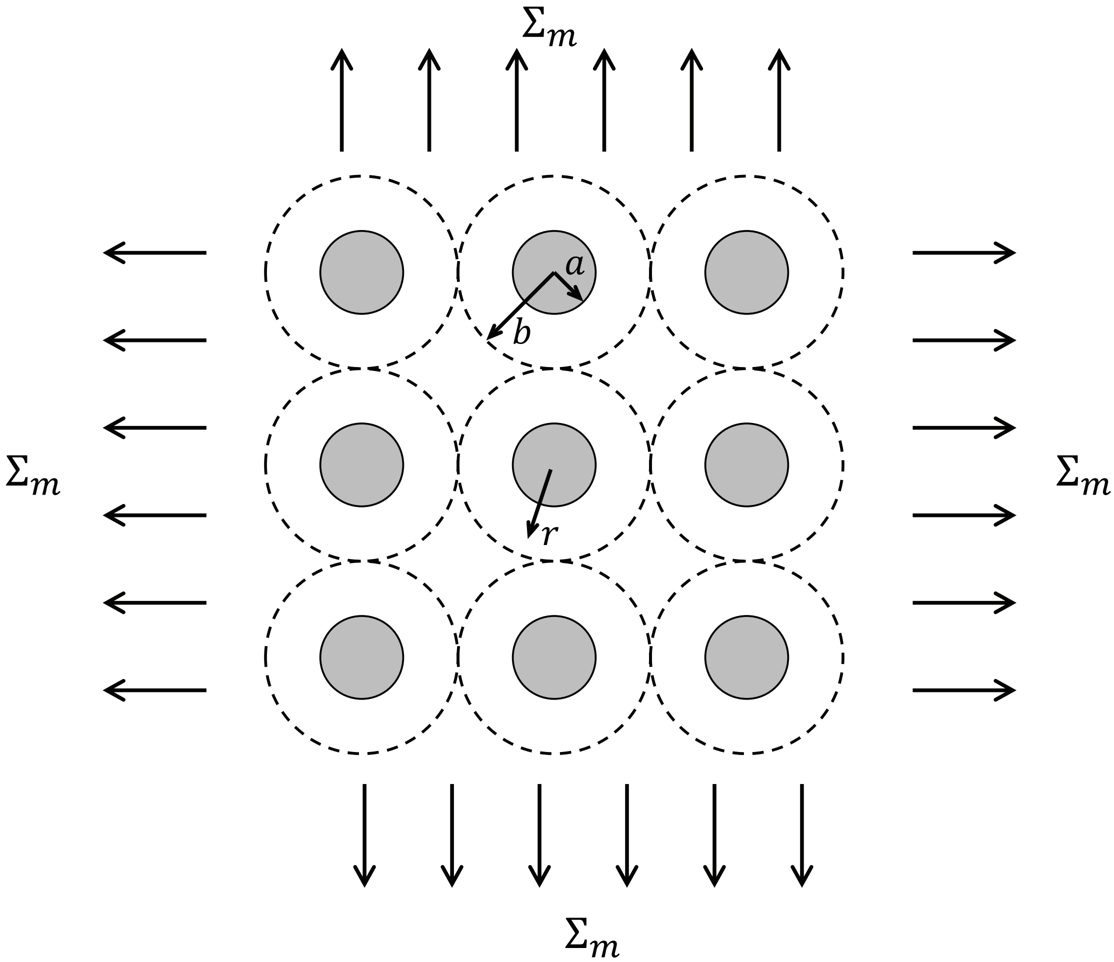

Fig. 2

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3 (a) Using index notation, prove the following:

(i) $e_{ijk}\, e_{kji} = -6$, where e_{ijk} is the permutation symbol. [10%]

(ii) If the strain tensor $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$, then $e_{ipk}\, e_{iql}\, \varepsilon_{pq,kl} = 0$. [15%]

(iii) If a body is subject to an external traction vector $\underline{t}^e$ acting over the surface S, then equilibrium dictates the external moment satisfies

$$\int_S \underline{x} \times \underline{t}^e \, dS = 0,$$

where, $\underline{x}$ is the position vector. [25%]

(b) For an elastic-perfectly plastic solid satisfying Drucker's postulates:

(i) define the limit stress state; [15%]

(ii) show that the rate of stress $\dot{\sigma}_{ij} = 0$ throughout the solid at the limit stress state. [35%]

END OF PAPER