EGT3
ENGINEERING TRIPOS PART IIB

Thursday 3 May 2018 9.30 to 11.10

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D6 Dynamics in Civil Engineering data sheets (5 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain the SRSS method of modal superposition for calculating the dynamic response of a structure. Comment on possible alternative approaches. [30%]

(b) Figure 1(a) shows the dynamic model used to assess the design of a new highway bridge. This comprises a uniform beam of total length $2L$, flexural rigidity EI and mass per unit length m, which is simply supported at both ends and the mid-span location.

Calculate the first two natural periods of the bridge in terms of the properties of the beam. [40%]

(c) The design requires consideration of a blast load case, in which the left-hand span is subjected to an incident pressure load. This is equivalent to a uniformly distributed upward force per unit length, which varies in time according to the triangular force pulse shown in Fig. 1(b).

Assuming that the bridge is initially at rest, and by considering the first two vibration modes, estimate the maximum deflection of the bridge due to the blast load when L = 15 m, $EI = 1.44 \times 10^{10}$ N m^2 and m = 28,000 kg m^{-1}. [30%]

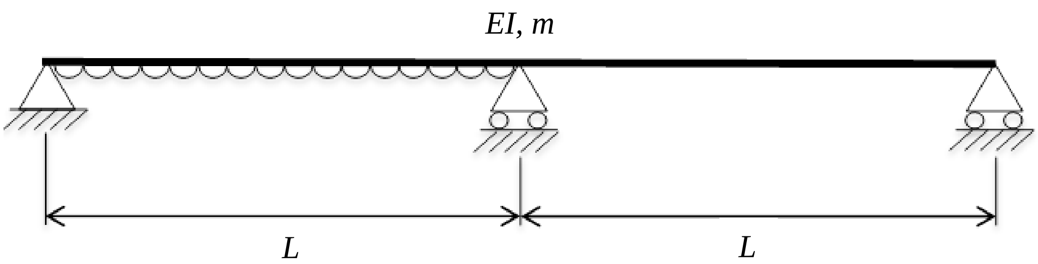

Fig. 1(a)

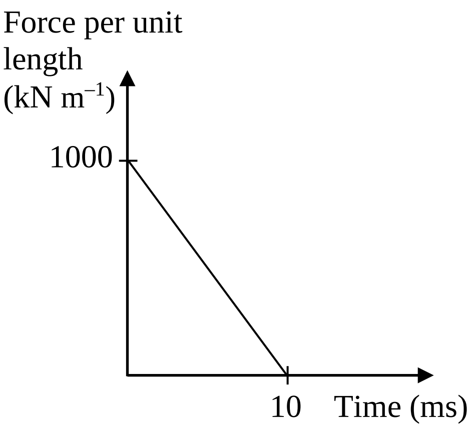

Fig.1(b)

2 (a) List four earthquake engineering design strategies and explain, using response spectra plots, how each design strategy works. Give one example of how each design strategy could be implemented (i.e. what devices or systems could be employed). [20%]

(b) List four types of building "irregularities" that prevent the designer from using the simple base shear design equation. For each building irregularity, state which of the assumptions used to derive the simple base shear design equation is violated. [20%]

(c) Figure 2(a) shows a two-storey sway-frame. The mass of the first storey is 2*m*, while the mass of the second storey is *m*. All columns have a flexural stiffness of *EI*.

(i) Determine the natural frequencies of the structure. [20%]

(ii) Assume *m* = 10,000 kg, the design PGA is 0.3g, and the design spectrum in Fig. 2(b) applies. Each column has an elastic shear capacity of 30 kN. Considering only the first mode response, determine the range of acceptable column flexural stiffness values (*EI*) required to ensure that the structure remains elastic for the design earthquake. [40%]

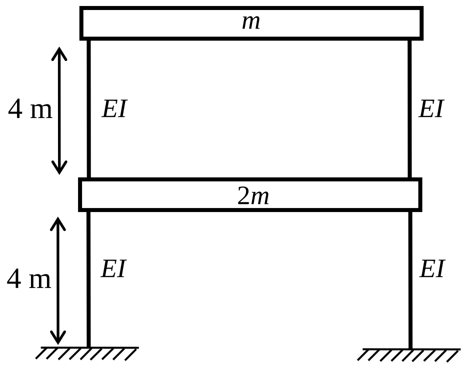

Fig. 2(a)

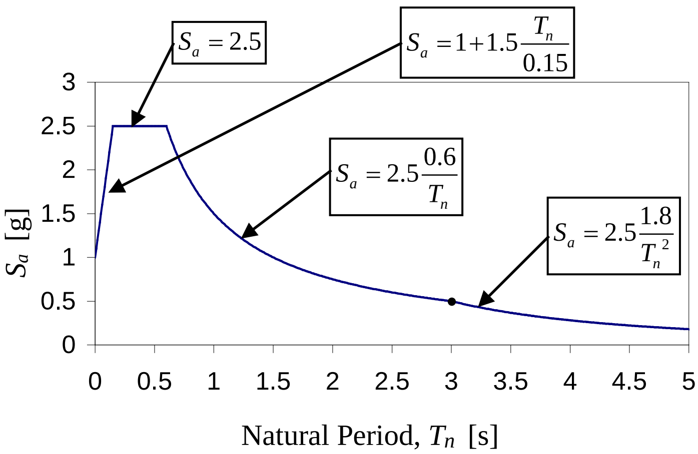

Fig. 2(b)

3 (a) Explain briefly the two ways in which the soil stiffness may degrade when a saturated soil body is subjected to earthquake loading. [10%]

(b) Explain how the finite element method can be used to solve a boundary value problem involving a two phase medium such as saturated soil. What special measures do you need to take into account to model the semi-infinite extent of the soil strata? [20%]

(c) A portal frame is supported on individual strip foundations as shown in Fig. 3. The sway stiffness of each wall is 2×10^6 N m^{-1} and they can be considered massless. The roof has a mass of 10,000 kg. Calculate the natural frequency of this portal frame assuming that the strip foundations provide complete fixity. [15%]

(d) The strip foundations are 0.5 m wide and are embedded to a depth of 0.25 m into a sandy soil layer as shown in Fig. 3. The unit weight of the sand is 17.5 kN m^{-3} and the void ratio is 0.65. The Poisson's ratio for sand is 0.3. Considering a reference plane 1 m below the ground level, calculate the horizontal and rotational stiffness of each of the strip foundations. The direction of earthquake shaking is shown in Fig. 3. Consider a 1 m length of the strip foundation into the plane of the paper. [20%]

(e) Propose a simple, discrete model for this structure to account for the soil-structure interaction. Using the model, calculate the natural frequencies in the horizontal and rotational modes. How do these compare to your answer in part (c) above? [25%]

(f) Speculate on any other ways in which this structure can suffer damage during a strong earthquake event. [10%]

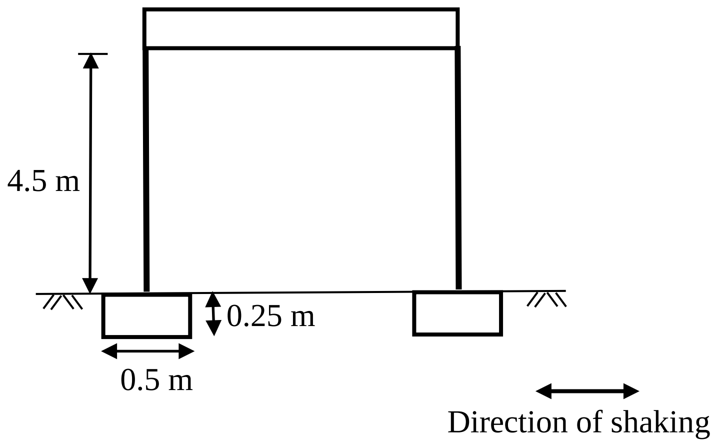

Fig. 3

4 (a) Explain what is meant by a flutter derivative, and how such might be measured in wind tunnel tests. [20%]

(b) Earthquake engineers make use of a technique called Response Spectrum Analysis. Explain why this would not be appropriate for determining the spectrum of the response of a structure to wind buffeting. [10%]

(c) Describe how the peak response of a large structure to wind buffeting may be estimated. [50%]

(d) Explain how the wind-induced vibrations of cables can be affected by rain. [10%]

(e) Describe two phenomena whereby the pressures on a building may be much greater than those calculated from graphs of the pressures in a free air blast. [10%]

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Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\;\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \quad \omega = 2\pi f$

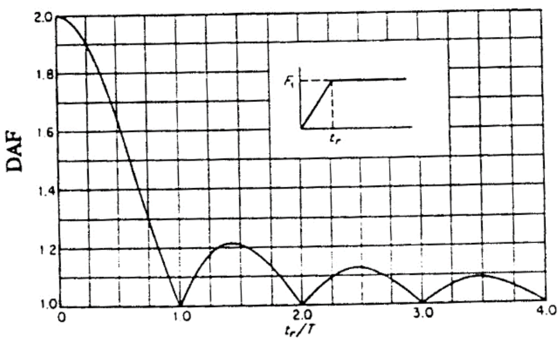
Modal analysis of simply-supported uniform beams

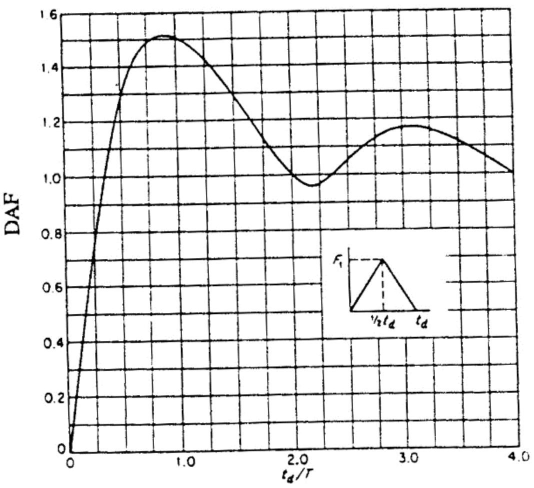
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

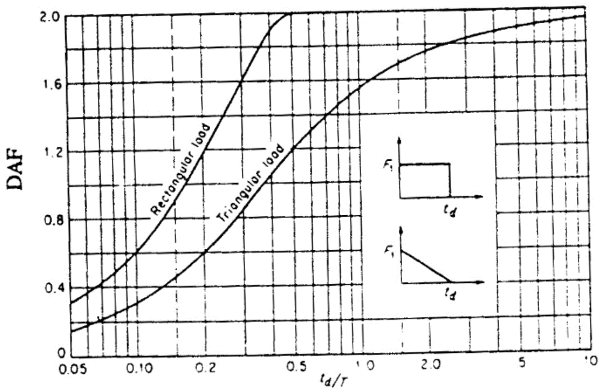
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

Dynamic amplification factors







Numerical Integration Schemes:

Central Difference Method:

Acceleration $\ddot{u}_t = \frac{1}{\Delta t^2}\left(u_{t-\Delta t} - 2\,u_t + u_{t+\Delta t}\right)$

Velocity $\dot{u}_t = \frac{1}{2\,\Delta t}\left(-u_{t-\Delta t} + u_{t+\Delta t}\right)$

Linear Acceleration Method:

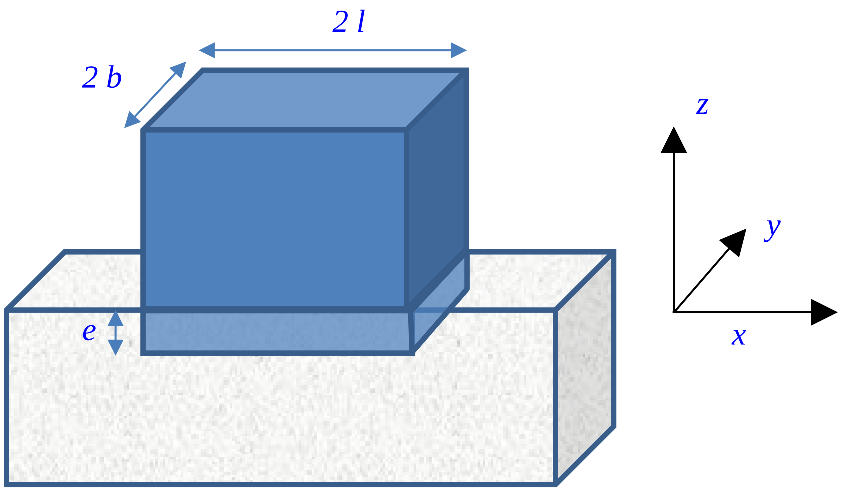
Acceleration $\ddot{u}(\tau) = \ddot{u}_n + (\ddot{u}_{n+1} - \ddot{u}_n)\,\frac{\tau}{\Delta t}$

Velocity $\dot{u}(\tau) = \dot{u}_n + \ddot{u}_n\tau + (\ddot{u}_{n+1} - \ddot{u}_n)\,\frac{\tau^2}{2\,\Delta t}$

Displacement $u(\tau) = u_n + \dot{u}_n\tau + \ddot{u}_n\frac{\tau^2}{2} + (\ddot{u}_{n+1} - \ddot{u}_n)\,\frac{\tau^3}{6\,\Delta t}$

Wolf Formulae:

Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_v = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \frac{l}{b}}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \left(\frac{l}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(\mathrm{G_s} + \mathrm{eS_r})\gamma_\mathrm{w}}{1+\mathrm{e}}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_\mathrm{d} = \frac{\mathrm{G_s}\gamma_\mathrm{w}}{1+\mathrm{e}}$$

Effective mean confining stress

$$\mathrm{p}' = \sigma_\mathrm{v}'\frac{\left(1+2\mathrm{K_o}\right)}{3}$$

where σ_v' is the effective vertical stress, $\mathrm{K_o}$ is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$\mathrm{K_o} = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{\max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p' is the effective mean confining pressure in **MPa,** e is the void ratio and G_{max} is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{\max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\, \sigma_v' \sin \phi'\right)^2 - \left(\frac{1 - K_o}{2}\, \sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2018

Q1) (b) $T_1 = \frac{2L^2}{\pi}\sqrt{\frac{m}{EI}}$ $T_2 = \frac{T_1}{4}$

(c) 3.4 mm

Q2) (c) (i) $\omega = \sqrt{\left(1 \pm \frac{\sqrt{2}}{2}\right)\frac{3EI}{8m}}$ (ii) $EI > 780$ kNm^2

Q3) (c) 3.18 Hz

(d) 296.1 MN/m 37.8 MNm/rad

(e) 38.73 Hz 2.65 Hz