EGT3
ENGINEERING TRIPOS PART IIB

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3 May 2017   2 to 3.30

Module 4F1

CONTROL SYSTEM DESIGN

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Supplementary pages: two extra copies of Fig. 2 (Question 3)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) State the definition of a conformal mapping, describe its properties and explain its usefulness in the analysis of feedback systems. [20%]

(b) An analogue implementation of a non-rational compensator is being considered, inspired by the passive circuit shown in Fig. 1, in which all capacitors have value 2 F and all resistors except the first have value 2 Ω. Let $Z(s) = \hat{v}(s)/\hat{i}(s)$ be the impedance of the circuit where v denotes the voltage across the terminals and i the terminal current.

(i) Let $X(s) = Z(s)+1$. The repetitive structure of the circuit for X can be used to show that $X = 2+(2s+X^{-1})^{-1}$. [You do not need to show this.] Deduce that

$$Z(s) = \sqrt{\frac{s+1}{s}}.$$

Use the high frequency behaviour of the circuit (or otherwise) to justify the positive square root. [15%]

(ii) Sketch the Bode diagram of $Z(s)$. [15%]

(iii) For the plant

$$G(s) = \frac{1000}{s(s+10)^2}$$

a controller with transfer function $K(s) = kZ(s)$ is selected in the standard negative feedback configuration. Sketch the complete Nyquist diagram of the return ratio of the system for $k = 1$. [Assume here and in the following question parts that $Z(s) \approx 1$ for values of s close to $10j$.] [15%]

(iv) Use the Nyquist stability criterion to assess the closed-loop stability of the feedback system for all k positive or negative. [15%]

(v) Without detailed calculation use the notion of conformal mapping to describe the behaviour of the feedback system for k in the vicinity of the value 2. [20%]

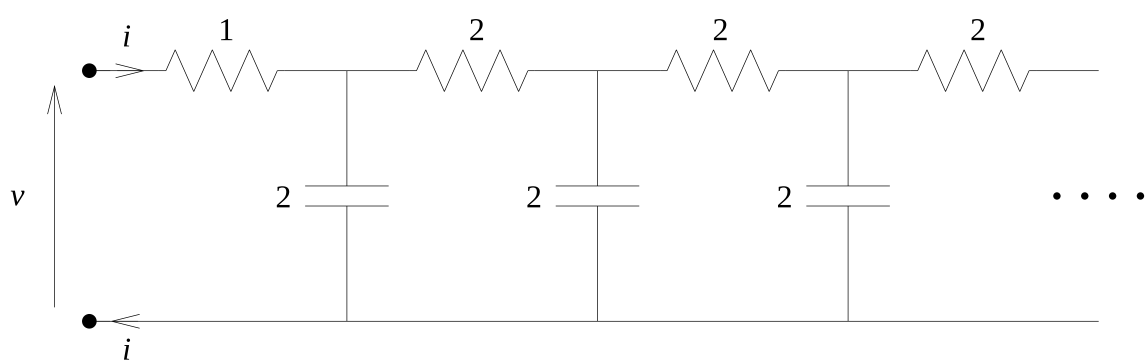

Fig. 1

2 (a) Let $F(s)$ be a stable real-rational transfer function and let $f(\omega)$ be a positive, continuous function.

(i) Consider a feedback system with $F(s)$ connected in feedback with a stable $\Delta(s)$ satisfying $|\Delta(j\omega)| \leq f(\omega)$ for all ω. Show that such a feedback system is stable if

$$|F(j\omega)|f(\omega) < 1$$

for all ω (including infinity). [15%]

(ii) Suppose that $|F(j\omega_0)|f(\omega_0) = 1$ for some $\omega_0 \neq 0$ and that $f(\omega)$ achieves its global minimum at ω_0. Show that there is a $\Delta(s)$ satisfying the conditions of (a)(i) in the form

$$c\frac{1-sT}{1+sT}$$

with c real and $T > 0$ for which the feedback system is not stable. [20%]

(b) Determine a necessary and sufficient condition for robust stability when a compensator $K(s)$ in negative feedback is used to stabilize a plant $G(s) = G_0(s)(1+\Delta(s))$ where $G_0(s)$ is a known transfer function and $\Delta(s)$ is assumed only to be stable and to satisfy a bound $|\Delta(j\omega)| < h(\omega)$ for all ω, where $h(\omega)$ is a positive, continuous function. [15%]

(c) The attitude stabilization system of a vertical takeoff (VTOL) aircraft is to be designed. At 40 knots the dynamics of the vehicle are approximately represented by the transfer function

$$\frac{1}{s^2+0.25}.$$

A rate gyro with transfer function $R(s) = s$ is placed in the feedback path, and the control actuator has transfer function $1+\Delta(s)$ where

$$\Delta(s) = \delta\frac{s+0.5}{s+8}$$

for some $\delta > 0$.

(i) Draw a block diagram of the system. [10%]

(ii) Find the largest δ_0 implied by the result in (b) for which the system is stable for $0 < \delta < \delta_0$. [25%]

(iii) Comment on whether this estimate is likely to be conservative. [15%]

3 Figure 2 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It is known that the system is closed-loop stable in the standard negative feedback configuration when $K(s) = 1$.

(a) (i) Sketch the Nyquist diagram of $G(s)$. [10%]

(ii) Use the Nyquist stability criterion to determine the number of poles of $G(s)$ in the right half plane. [10%]

(b) (i) Sketch on a copy of Fig. 2 the expected phase plot if $G(s)$ were stable and minimum phase and with the gain (magnitude) plot unchanged. [10%]

(ii) Use your plot and your answer from (a)(ii) to determine the number of right half plane zeros of $G(s)$. Estimate the location of any right half plane poles and zeros of $G(s)$ (if there are any). [10%]

(iii) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(c) A feedback compensator $K(s) = K_1(s)$ is selected where

$$K_1(s) = k\alpha \frac{s + \omega_c/\alpha}{s + \omega_c \alpha}$$

where $k = 0.72$, $\alpha = 1.8$ and $\omega_c = 20$.

(i) Sketch the Bode diagram of the return ratio $G(s)K_1(s)$ on a further copy of Fig. 2 and estimate the phase margin for the system. [15%]

(ii) Determine the gain margin of the system. [Hint: pay particular attention to both increases and decreases in gain.] [15%]

(iii) Design a further compensator $K_2(s)$ so that $G(s)K_1(s)K_2(s)$ has a phase margin of at least 45° and a gain margin of at least 6 dB. Sketch the Bode diagram of the return ratio $G(s)K_1(s)K_2(s)$ on the same copy of Fig. 2 as used in part (c)(i). [20%]

Two copies of Fig. 2 are provided on separate sheets. These should be handed in with your answers.

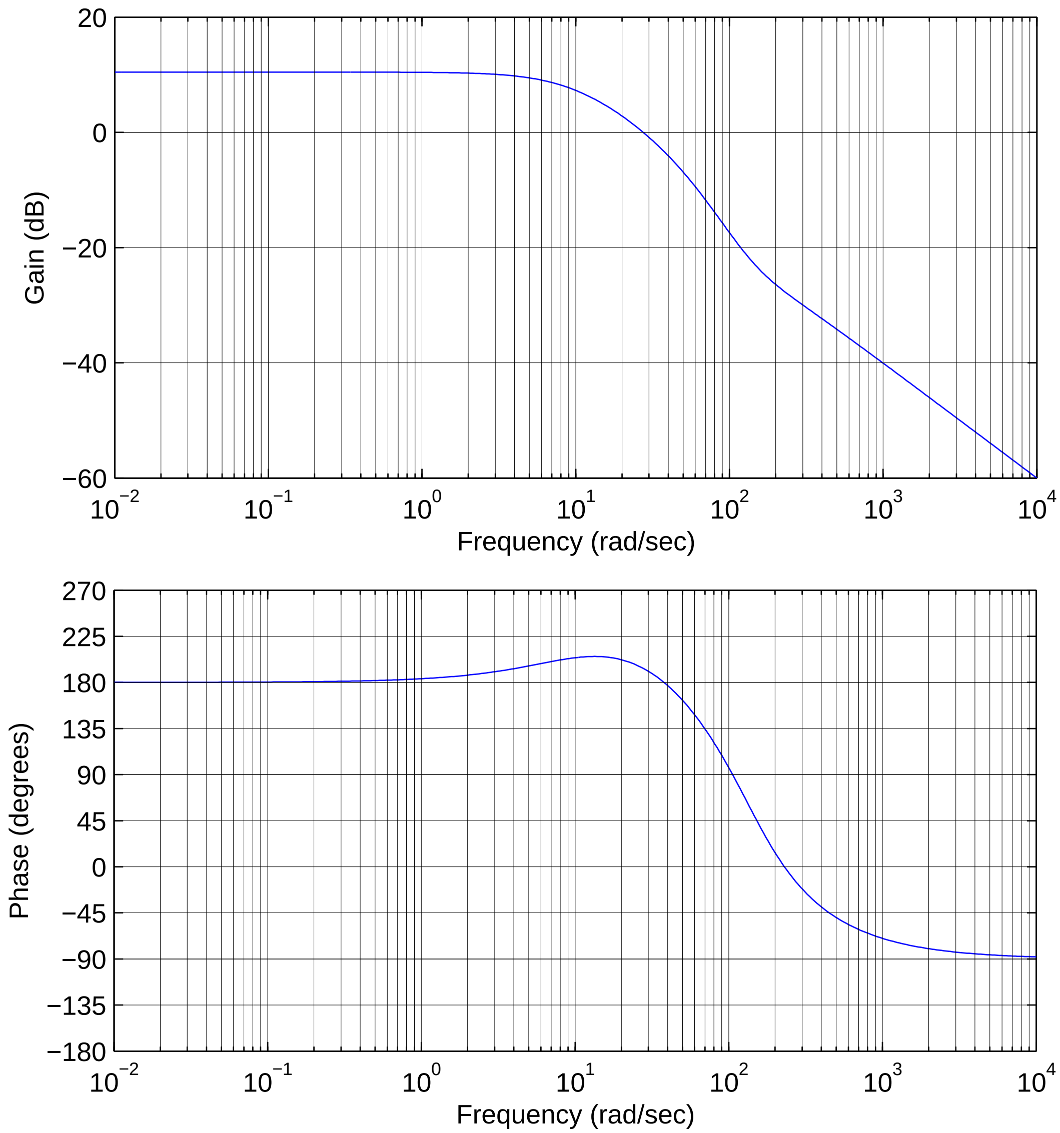

Fig. 2

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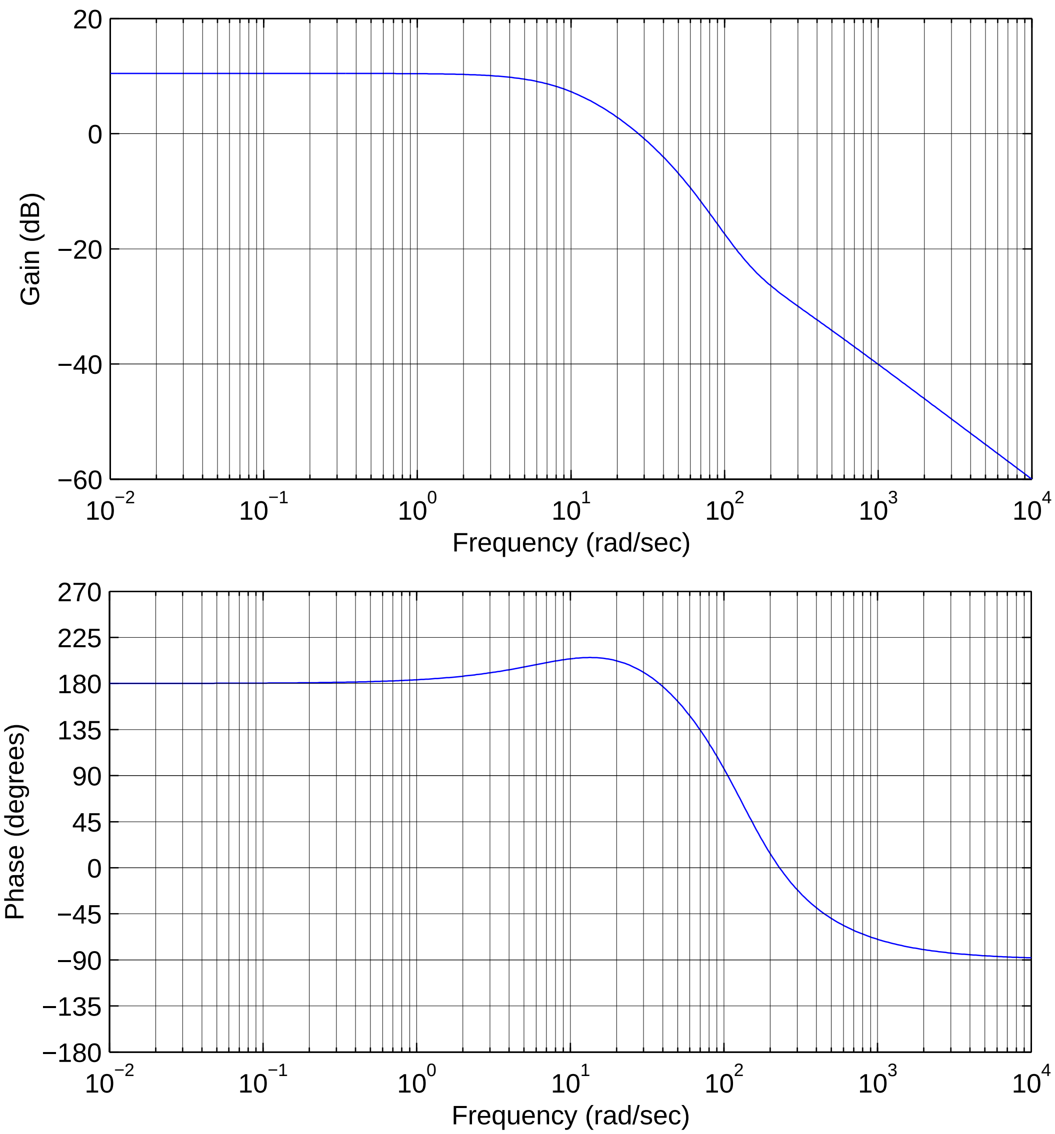
3 May 2017, Module 4F1, Question 3.



Extra copy of Fig. 2: Bode diagram for Question 3.

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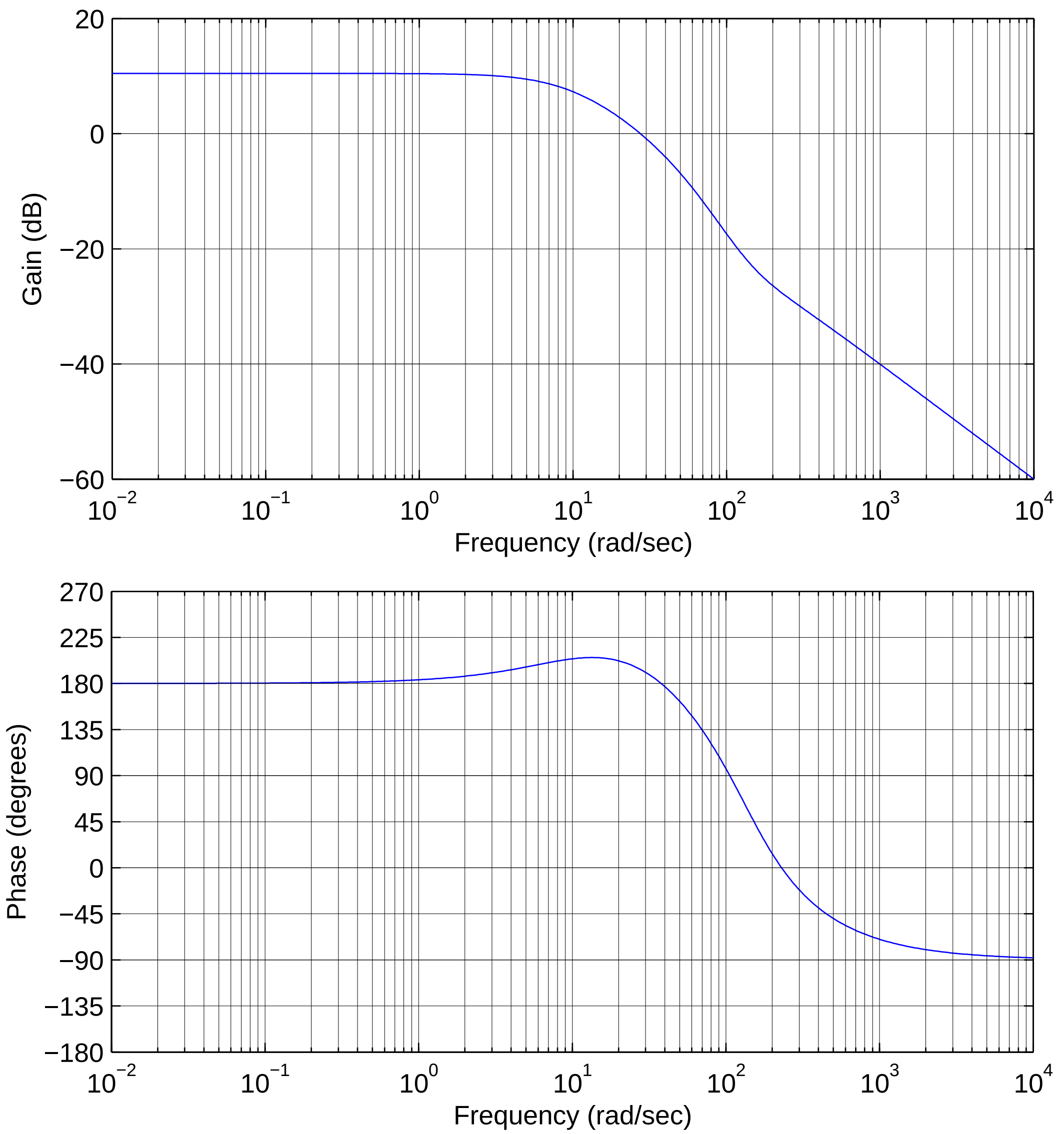

Extra copy of Fig. 2: Bode diagram for Question 3.

Engineering Tripos Part IIB

2017

Paper 4F1: Control System Design

Answers

1. (b)(iv) Closed-loop stable for $0 < k < 2$ (approx).

2. (c)(ii) largest $\delta_0 = 8.5$.

3. (a)(ii) One pole of $G(s)$ in the right half plane.

   (b)(ii) 2 RHP zeros around $s = 200$ and the RHP pole around $s = 10$.

   (c)(i) phase margin $= 54^o$.

   (c)(ii) gain margin $= 2.5$ dB.