EGT3
ENGINEERING TRIPOS PART IIB

Friday 4 May 2018   2.00 to 3.40

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Supplementary pages: two extra copies of Fig. 1 (Question 3)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) For a plant with transfer function $G(s)$ sketch the block diagram for a two-degree-of-freedom control system. Write down (but do not prove) the conditions which must apply to the transfer function $T_{r\to y}$ from reference input $r(t)$ to plant output $y(t)$ in any design. [15%]

(b) For an atomic force microscope, the transfer function relating the height of the cantilever beam $y(t)$ to the elongation of a piezo element $u(t)$ has been obtained in simplified form as:

$$G_1(s) = \frac{s^2+\omega_1^2}{s^2+\omega_2^2}$$

where $0 < \omega_2 < \omega_1$. Actuator dynamics have been modelled as a simple first-order lag:

$$G_2(s) = \frac{1}{\tau s+1}$$

where $\tau > 0$.

(i) Suppose proportional negative feedback with gain k is applied to $G(s) = G_1(s)G_2(s)$. Show that the closed loop system is stable if and only if

$$-\left(\frac{\omega_2}{\omega_1}\right)^2 < k < 0. \tag{1}$$

[Hint: you may find it helpful to make use of the Routh-Hurwitz criterion in the Information Data Book.] [20%]

(ii) Sketch the root-locus diagram for $G(s)$ for positive and negative k. [You may assume that the parameters of the system are such that there are no breakaway points on the real axis.]. [20%]

(iii) Show that the sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ have prescribed values at $s = j\omega_1$ and $s = j\omega_2$, for any stabilising controller of $G(s)$, and find these values. [15%]

(iv) Suppose a proportional feedback is chosen satisfying condition (1). Discuss the effect of disturbances and sensor noise on the control system in the range $0 \le \omega \le \omega_1$. [15%]

(v) Assuming $\omega_1 = \tau = 1$, design a two-degree-of-freedom control system to achieve a transfer function from reference input $r(t)$ to plant output $y(t)$ given by:

$$T_{r\to y} = \frac{s^2+1}{(s+1)^3}$$

where all controller coefficients are expressed as a function of ω_2. [15%]

2 (a) Discuss the role of the sensitivity function and the complementary sensitivity function in analysing the effect of uncertainty in a plant $G(s)$ which is stabilised by a feedback controller $K(s)$. [25%]

(b) Suppose that $G(s)K(s)$ is stable, minimum phase and has at least second order roll-off at high frequencies and let $S(s)$ denote the sensitivity function.

(i) Show that [25%]

$$\int_0^\infty \ln|S(j\omega)|d\omega = 0.$$

(ii) Suppose the following specifications are required:

A: $|S(j\omega)| < \varepsilon$ for $0 \le \omega \le 1$,

B: $|S(j\omega)| < 1.5$ for $1 \le \omega \le 10$,

C: $|G(j\omega)K(j\omega)| < \frac{1}{\omega^2}$ for all $\omega \ge 10$

where $0 < \varepsilon < 1$. Use C to obtain an upper bound on [25%]

$$\int_{10}^\infty \ln|S(j\omega)|d\omega.$$

[Hint: you may assume that

$$\int \ln(1-\omega^{-2})d\omega = \omega\ln(1-\omega^{-2}) + \ln\left(\frac{\omega+1}{\omega-1}\right).]$$

(iii) Hence find a positive number ε_0 such that the specifications in (ii) are infeasible for $\varepsilon < \varepsilon_0$. [25%]

3 Figure 1 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It is known that the system is closed-loop stable in the standard negative feedback configuration when

$$K(s) = K_1(s) = \frac{s+1}{s}.$$

(a) (i) Sketch on a copy of Fig. 1 the Bode diagram of $G(s)K_1(s)$. [5%]

(ii) Sketch the Nyquist diagram of $G(s)$. [5%]

(iii) Use the Nyquist stability criterion to determine the number of poles of $G(s)$ in the right half plane. [10%]

(b) (i) Sketch on a further copy of Fig. 1 the phase plot of the stable and minimum phase transfer function with the same gain (magnitude) plot as that of G(s). [10%]

(ii) Use your plot and your answer from (a)(iii) to determine the number of right half plane zeros of $G(s)$. Estimate the location of any right half plane poles and zeros of $G(s)$. [10%]

(iii) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(c) A feedback compensator $K(s) = K_1(s)K_2(s)$ is to be designed to achieve the following specifications for the return ratio $L(s) = G(s)K(s)$:

A: Velocity error constant $K_v = 10$ where $K_v = \lim_{s \to 0}(sL(s))$;

B: Phase margin of at least $60°$;

(i) By considering the cases of a lead and lag compensator separately, or otherwise, explain why it is not possible to achieve the specifications using a compensator $K_2(s)$ with one pole and one zero. [25%]

(ii) Design a compensator $K_2(s)$ to satisfy the specifications. Sketch the Bode diagram of $L(s)$ for your design on the copy of Fig. 1 which you used in (a)(i). [25%]

Two copies of Fig. 1 are provided on separate sheets. These should be handed in with your answers.

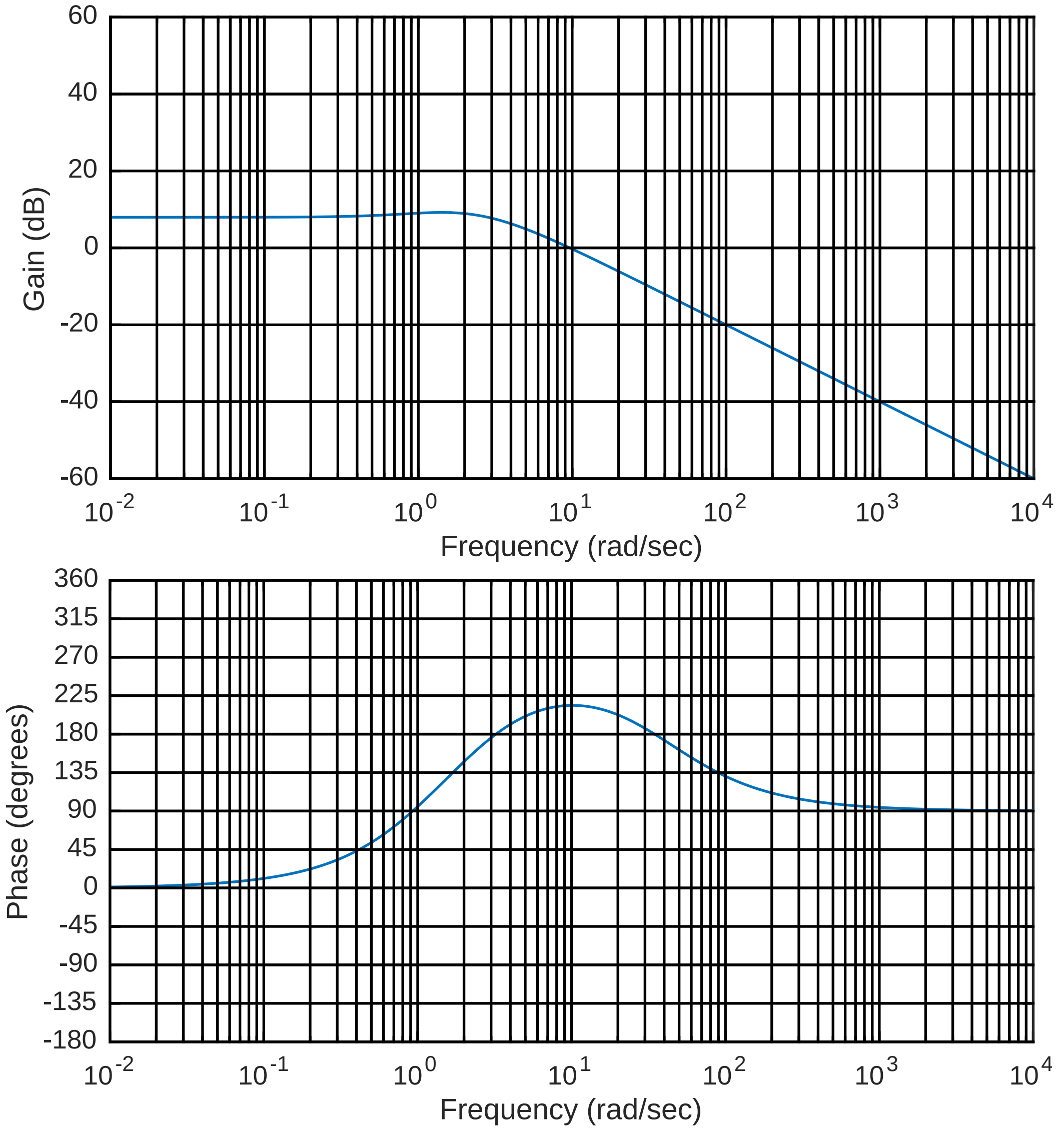
Gain (dB)
60
40
20
0
-20
-40
-60
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/sec)
Phase (degrees)
360
315
270
225
180
135
90
45
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/sec)

Fig. 1

Answers

Q1

(b)(iii) $S(j\omega_2) = 0,\ S(j\omega_1) = 1,\ T(j\omega_2) = 1,\ T(j\omega_1) = 0$

Q2

(b)(ii) 0.1

(b)(iii) $\varepsilon_0 \leq 0.0235$

Q3

(a)(iii) 2 RHP poles

(b)(ii) 1 RHP zero at $s = 40.$ 2 RHP poles at $s = 2$

END OF PAPER

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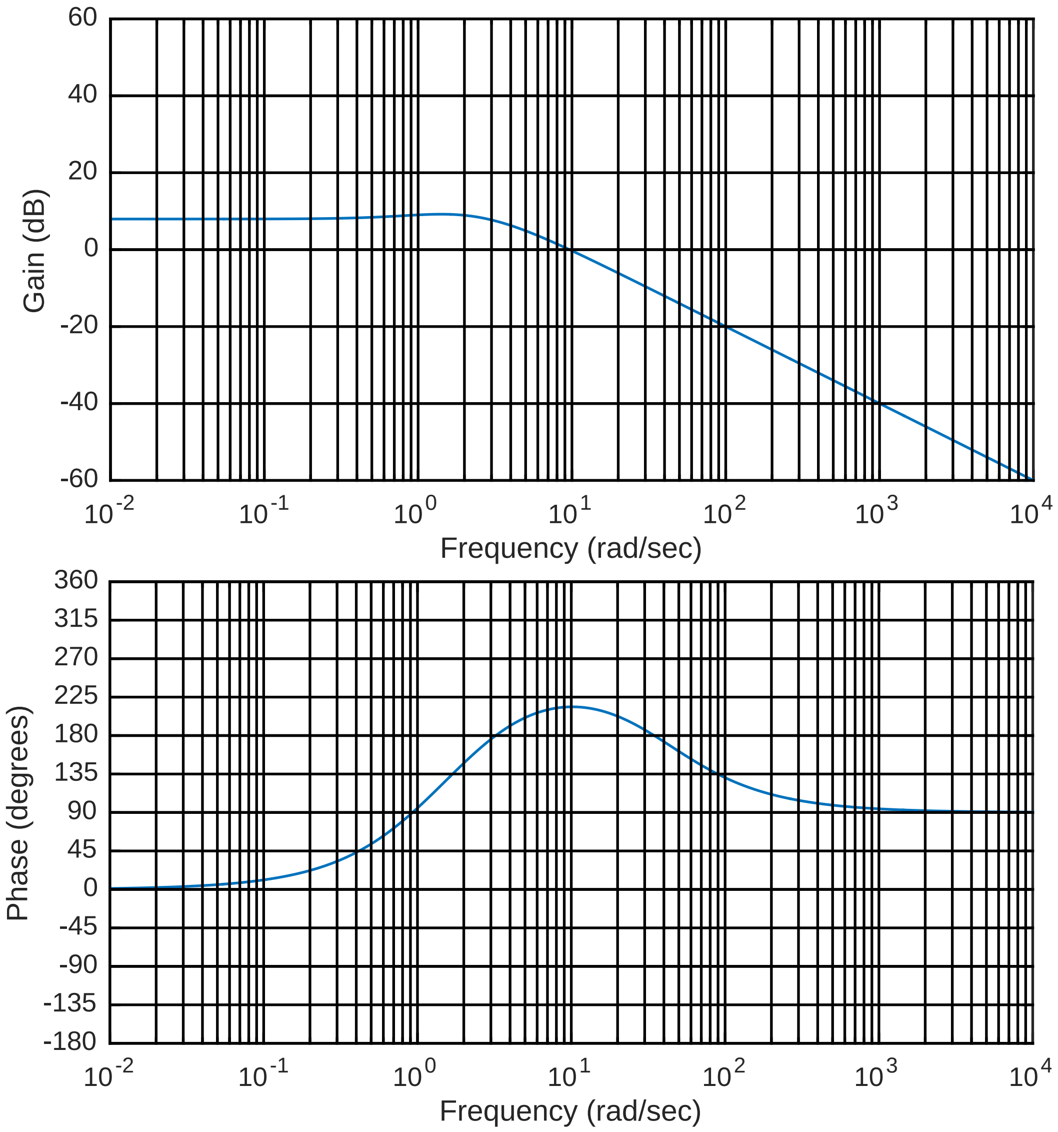

Extra copy of Fig. 1: Bode diagram for Question 3.

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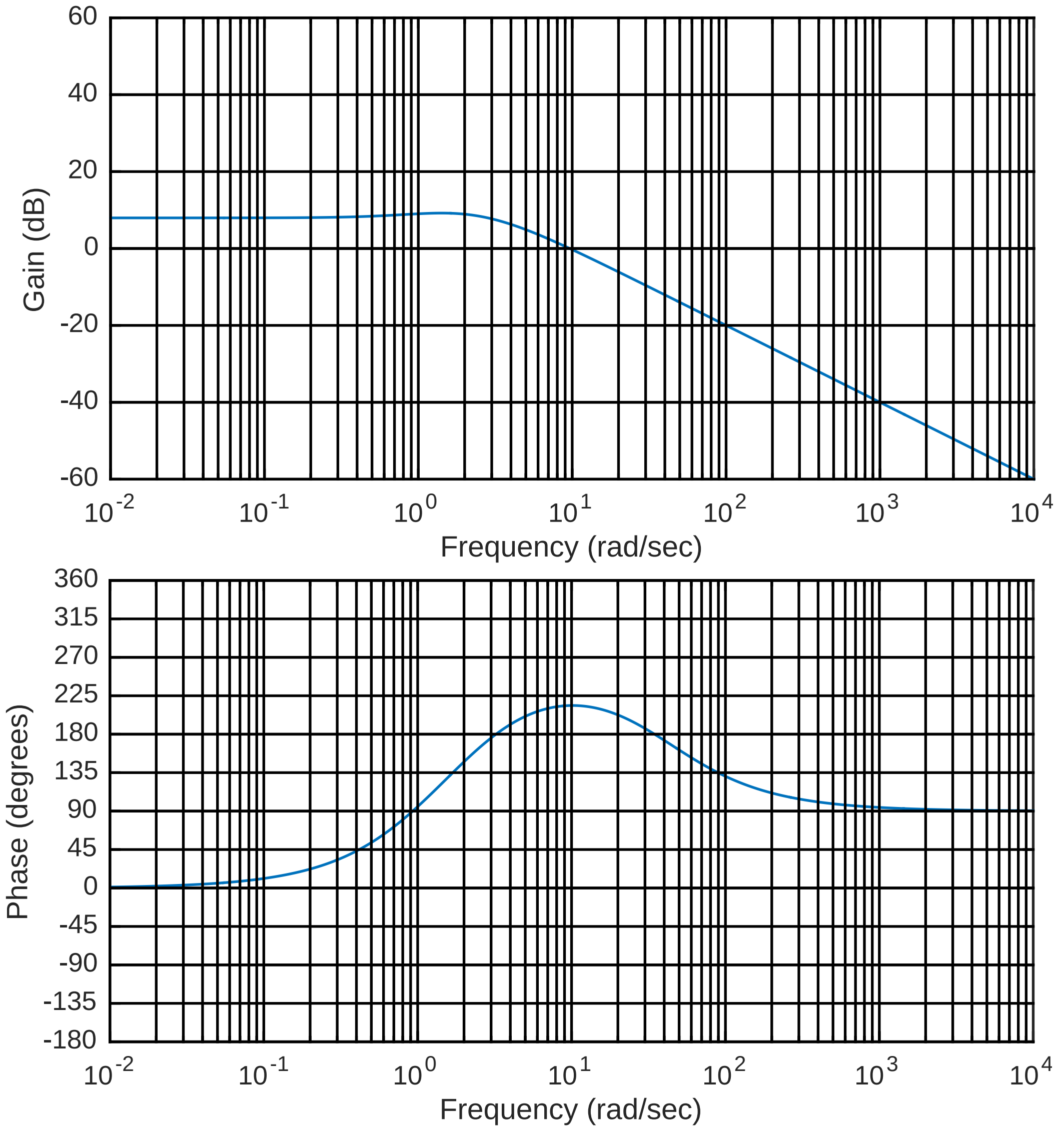

Extra copy of Fig. 1: Bode diagram for Question 3.

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