Version JL/2

EGT3
ENGINEERING TRIPOS PART IIB

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Monday 23 April 2018 2 to 3.40

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**Module 4F8**

**IMAGE PROCESSING AND IMAGE CODING**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 (a) Explain why phase is so important in images and describe what is meant by a *zero-phase* filter. What conditions on a frequency response, $H(\omega_1, \omega_2)$, are required for it to be zero phase? [10%]

(b) Write down the standard result for the impulse response of an ideal lowpass frequency response defined by

$$H_1(\omega_1, \omega_2) = \begin{cases} 1 & \text{if } |\omega_1| < \Omega_c \text{ and } |\omega_2| < \Omega_c \\ 0 & \text{otherwise} \end{cases}$$

and state the conditions on $\Omega_c$ needed to avoid aliasing. The original image is sampled with spacings $\Delta_1$ and $\Delta_2$ in the $u_1$ and $u_2$ directions respectively. [10%]

(c) Consider the zero-phase ideal frequency response $H_2(\omega_1, \omega_2)$ shown in Fig. 1, where $\Omega_s < \pi/\Delta_1$ and $\Omega_s < \pi/\Delta_2$, and $\Delta_1, \Delta_2$ are as in part (b). $H_2$ takes the value 1 in the shaded region and zero outside this region.
Show that the ideal impulse response for $H_2(\omega_1, \omega_2)$ takes the following form

$$h_2(n_1\Delta_1, n_2\Delta_2) \propto \operatorname{sinc}(n_1\Delta_1 + n_2\Delta_2)\frac{\Omega_s}{2} \operatorname{sinc}(n_1\Delta_1 - n_2\Delta_2)\frac{\Omega_s}{2}$$

and comment (without detailed calculations) on this result in comparison to the impulse result of the standard lowpass filter in part (b). [30%]

(d) Sketch the ideal impulse responses, $h_1$ for $H_1$ in part (b), and $h_2$ for $H_2$ in part (c) (assume that $\Omega_s = \sqrt{2}\Omega_c$). Describe, qualitatively, what effects filtering with $h_1$ and $h_2$ would have on the image in Fig. 2. [20%]

(e) The inverse Fourier transform of some desired zero-phase 2-D frequency response will not normally give an impulse response with finite support. Explain how the *windowing method* is used to create finite support filters and describe the effect of windowing on the ideal frequency response. Discuss two methods of forming 2-D window functions from 1-D window functions. [10%]

(f) Find the spectrum, $W(\omega_1, \omega_2)$ of the 2-D window function, $w(u_1, u_2)$, formed from the product of $w_1(u_1)$ and $w_2(u_2)$, where, for $i = 1, 2$

$$w_i(u_i) = \begin{cases} \cos^2\left(\frac{\pi u_i}{U_i}\right) & |u_i| < U_i \\ 0 & \text{otherwise} \end{cases}$$

Sketch the spectrum along the $\omega_1$ axis and comment on the properties of this window function. [20%]

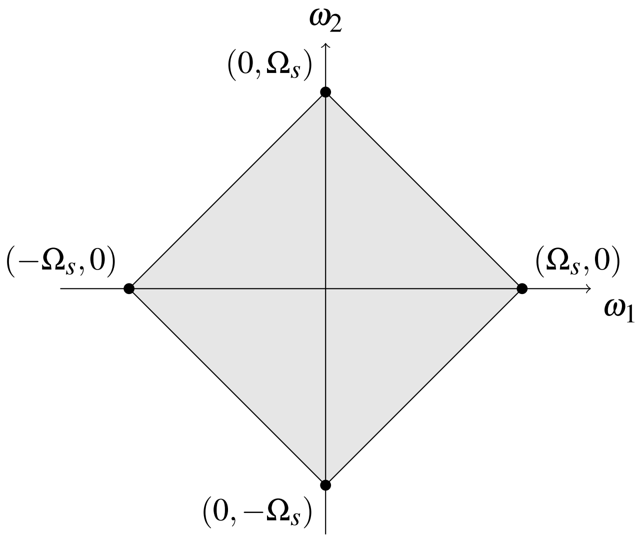


Fig. 1

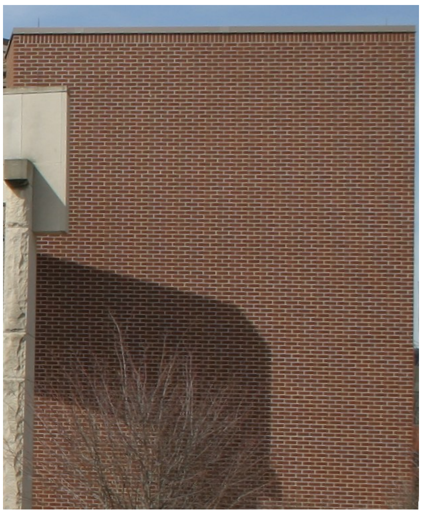

Fig. 2

2 (a) *Histogram equalisation* is a process whereby an image is visually improved without additional assumptions or data.

(i) Explain, qualitatively, the concept of *Histogram equalisation* in images and outline the situations where it would be most useful. [10%]

(ii) An $8 \times 8$ image has the form given in Fig.3, where the range of possible greyscale values is from 1 to 8. Sketch the histogram and discuss the distribution of grey levels. [10%]

(iii) Perform histogram equalisation on this image. Sketch the new equalised image and its histogram. How well has the process worked? [25%]

(b) Let $x(\mathbf{n})$, where $\mathbf{n} = (n_1, n_2)$, be our true underlying image and let $y(\mathbf{n})$ be our observed noisy image.

(i) Assuming $x(\mathbf{n})$ and $y(\mathbf{n})$ are *spatially stationary processes*, define the *cross correlation function* $R_{xy}(\mathbf{n})$ and the *cross power spectrum* $P_{xy}(\omega)$, where $\omega = (\omega_1, \omega_2)$. [10%]

(ii) The Wiener filter gives an estimate $\hat{x}(\mathbf{n})$ of the image $x(\mathbf{n})$ from the noisy measurements $y(\mathbf{n})$ via

$$\hat{x}(\mathbf{n}) = \sum_{\mathbf{q} \in \mathbf{Z}^2} g(\mathbf{q})\, y(\mathbf{n} - \mathbf{q})$$

where $\mathbf{Z}$ denotes the set of integers. By minimising an appropriate function, derive an expression which relates $R_{yy}$ (the autocorrelation of the measured image), $R_{xy}$ and $g(\mathbf{q})$, and indicate how one can use this expression to obtain the following form of the Wiener filter frequency response:

$$G(\omega) = \frac{P_{xy}(\omega)}{P_{yy}(\omega)}$$

where $P_{yy}(\omega)$ is the power spectrum of $y(\mathbf{n})$. Explain any assumptions that are made. [30%]

(iii) If the observed image can be modelled as the convolution of the true image and a point-spread function $h(\mathbf{n})$, plus additive noise $d(\mathbf{n})$, i.e. $y(\mathbf{n}) = \sum_{\mathbf{m} \in \mathbf{Z}^2} h(\mathbf{m})\, x(\mathbf{n} - \mathbf{m}) + d(\mathbf{n})$, give an expression for $P_{yy}(\omega)$ in terms of the spectrum of $h$ and the power spectra of the true image and the noise. What assumptions are made in deriving this expression? [15%]

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Fig. 3



(TURN OVER

3 (a) Explain how the characteristics of the *human visual system* can be used in the design of image compression systems. [20%]

(b) The basic Haar transform matrix, $T$, and a $2 \times 2$ block of image pixels, $X$, are given by

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Verify that $T$ is an orthonormal matrix and derive expressions for the coefficients of $Y$, the 2-D Haar transform of $X$, in terms of the pixels $a, b, c, d$. Show how $X$ can be obtained from the transform $Y$ and verify that the Haar transform preserves energy between the pixel and transform domains. [20%]

(c) A complete image is split into $2 \times 2$ blocks and $T$ is applied to each block. Explain how the result can be reordered into four subimages and describe the typical image characteristics selected by each subimage. Discuss the distribution of energies in these subimages and explain why this process is helpful for image compression. [20%]

(d) How does quantising the transformed image $Y$ with a given step size $Q_{step}$ compare with quantising the original image, $X$, with the same step size, in terms of energy distortion? Explain how we can extend the Haar transform to a multi-level transformation and discuss the mean bit rate reduction that can be achieved by this process. How many levels of the Haar transform are typically used? [20%]

(e) Visual artefacts will appear during transformation via a multi-level system such as that in part (d), due to the quantisation of the subimage coefficients. Describe the nature of these visual artefacts and discuss how the use of wavelets can reduce their visibility. [20%]

4 (a) Sketch a block diagram of a typical image coder and decoder. State the desirable properties of each block. Explain what is meant by *perfect reconstruction* in such a system and discuss where the main source of coding distortion occurs. [20%]

(b) Sketch a 2-band analysis filter bank, formed from filters $H_0(z)$ (lowpass) and $H_1(z)$ (highpass), and the equivalent reconstruction filter bank, formed from filters $G_0(z)$ and $G_1(z)$. Include appropriate downsamplers and upsamplers and explain their function. [20%]

(c) Show that if $\hat{y}(n) = y(n)$ when $n$ is even and $\hat{y}(n) = 0$ when $n$ is odd, then the $z$-transform of $\hat{y}_n$ is given by

$$\hat{Y}(z) = \frac{1}{2}[Y(z) + Y(-z)]$$

[15%]

(d) Using the expression in part (c), derive the *anti-aliasing* and *perfect reconstruction* conditions for the filters in the 2-band filter bank system of part (b). [25%]

(e) By considering the lowpass product filter $P(z) = H_0(z)G_0(z)$, outline the steps which are necessary to design a wavelet transform system that has good image compression properties. [20%]

**END OF PAPER**

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