ISSUED ON
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Part IB Paper 1: Mechanics

Examples Paper 1/3

Inertia Effects, Balancing

Straightforward questions are marked †
*More challenging questions are marked *.*

†1. A mass M is attached to a fixed pivot P by a light rod PQ, as shown in plan view in Fig. 1. The moment of inertia about its centre of mass G is $I = Mk^2$.

(i) What is the angular acceleration about P of the arm as a result of the steady torque T applied there ?

(ii) If, at a given instant after the application of T, the angular velocity of the body is ω, what are the components of the reaction at P (a) along the bar, and (b) perpendicular to the bar ?

(iii) At section AA in the bar what are the values of (a) the shear force, and (b) the bending moment ?

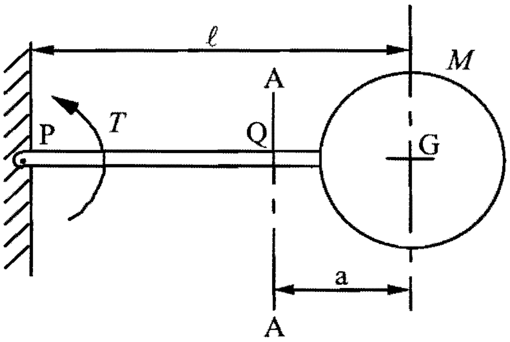

Fig. 1

†2. A uniform bar of mass m per unit length and of length ℓ is pivoted about a fixed vertical axis at one end. A torque T is applied to the bar about the axis of the pivot.

(i) What is the angular acceleration of the bar ?

(ii) What is the reaction at the pivot transversely to the bar ?

(iii) What are the magnitudes of the shear force and bending moment at a section distance x along the bar from the pivot ?

3*. Fig. 2 shows a flywheel with centre of gravity G attached by the arm PQ to a rotor which rotates at constant angular velocity Ω about a fixed centre 0. The arm is freely pivoted to the rotor at P. The effect of gravity can be ignored.

(i) Determine the components of the acceleration of G, along and normal to PQ, in terms of Ω, r, ℓ and θ and its derivatives.

(ii) What is the natural frequency of small oscillations of the flywheel about its equilibrium position if its mass is m and it has moment of inertia is I about its centre of mass G ? The mass of the arm is negligible.

(iii) If the amplitude of small oscillations of the arm is θ_0, obtain an expression for the tension and shear force at section A-A on the arm when $\theta = \theta_0$.

(iv) What is the value of the oscillating torque which the shaft must apply to the rotor to maintain Ω constant when $\theta = \theta_0$?

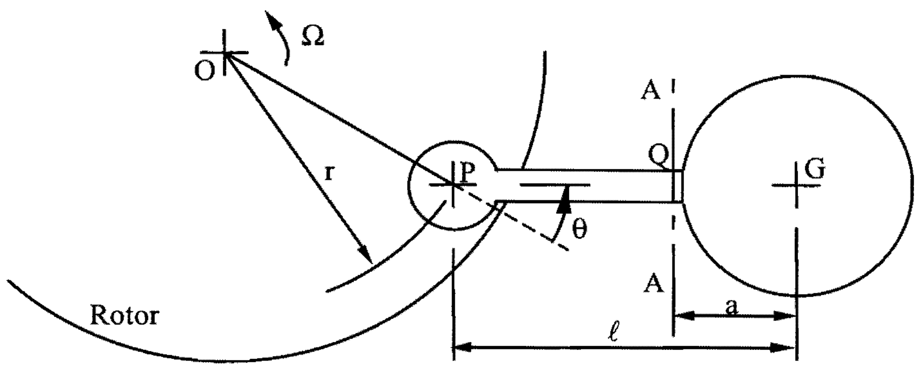

Fig. 2

4. A uniform bar of length $(a+b)$ is pivoted about a horizontal axis at a distance a from one end. The bar is held in a horizontal position and is then released.

(i) If $a < b$, find the initial angular acceleration of the bar.

(ii) Show that, provided a is less than $b/2$, the maximum sagging bending moment in the longer part of the bar initially occurs at a section

$$(4a^2 - ab + b^2) \;/\; 3(b - a) \text{ from the pivot.}$$

5. A rotor is made by bolting together 3 similar discs having unbalances of 5, 9 and 11×10^{-5} kg m.

At what relative angles would you assemble these discs if the rotor is to be 'statically balanced' ?

In what order should the discs be arranged so as to minimise the out of balance couple?

If the disc spacing is 25 mm and the speed of rotation is 6000 rpm, what will be the magnitude of the out-of-balance couple ?

6. A shaft rotates in two bearings, 1.5 m apart, and projects 0.3 m beyond each bearing. At each end of the shaft there is a pulley, one of mass 15 kg and the other of mass 40 kg, their centres of gravity being respectively 10 mm and 15 mm from the axis of the shaft. Midway between the bearings is a third pulley of mass 50 kg, its centre of gravity being 12.5 mm from the shaft axis. If the system is in static balance, find the forces (other than gravity) acting on the bearings when the shaft is rotating at 400 rpm.

* The shaft is to be balanced by adding masses to the outer pulleys at 150 mm radius. Find the magnitude and angular positions of the masses required to make the shaft run smoothly at all speeds.

For further practice try the following IB Mechanics Tripos questions:

2008 Q1; 2009 Q1, Q6; 2011 Q6; 2012 Q5

ANSWERS

1 (i) $T/[M(k^2 + \ell^2)]$ (ii) (a) $M\ell\,\omega^2$ $\leftarrow$ (b) $T\ell/(k^2 + \ell^2)$ $\uparrow$

(iii) (a) $-T\ell/(k^2 + \ell^2)$ (b) $T(k^2 + a\ell)/(k^2 + \ell^2)$ (positive, or 'hogging')

2 (i) $3T/m\ell^3$ (ii) $3T/2\ell$ (iii) $-\dfrac{3T}{2\ell}\left\{1-\left(\dfrac{x}{\ell}\right)^2\right\}$, and $T\left\{1-\dfrac{3}{2}\left(\dfrac{x}{\ell}\right)+\dfrac{1}{2}\left(\dfrac{x}{\ell}\right)^3\right\}$

3 (i) $\Omega^2 r\cos\theta + \ell(\Omega+\dot\theta)^2$ along PQ $\ell\ddot\theta + \Omega^2 r\sin\theta$ perpendicular to PQ

(ii) $\dfrac{1}{2\pi}\sqrt{\dfrac{\ell r\Omega^2}{(k^2+\ell^2)}}$ where $k^2 = I/m$

(iii) $T = m\Omega^2(\ell + r\cos\theta_0)$ $F = -m(\Omega^2 r\sin\theta_0 - \ell\omega^2\theta_0)$

(iv) $m\ell r\Omega^2\theta_0\left(1+\dfrac{\ell r}{k^2+\ell^2}\right)$

4 (i) $\dfrac{3g(b-a)}{2(b^2-ab+a^2)}$

5 5 at 0°, 9 at 80.4° and 11 at 233.8° with the 11×10^{-5} kg m disc in the middle; 0.942 Nm

6 751N, 2.04 kg on both discs, at -28° and +152° from the orientation of the centre disc.

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