Engineering Tripos Part IB SECOND YEAR

Part IB Paper 5: Electrical
Examples Paper 6 Electromagnetic Fields and Waves
Transmission Lines

1.

a) Show that a general solution of the wave equation, $\frac{\partial^2 A}{\partial t^2} = v^2 \frac{\partial^2 A}{\partial x^2}$, is A=~~f(x±vt).~~

b) Explain the physical significance of this solution and the difference between f(x+vt) and f(x-vt)

c) If $A = e^{j(\omega t - \beta x)}$ write an expression for the wave velocity in terms of β and ω

d) If $A = e^{j(\omega t - \beta x)}$ write an expression for the wave velocity in terms of wavelength λ and frequency f.

2. A pair of cables on the national grid has a shunt capacitance per unit length of 4.6 pF m^{-1} and a loop inductance per unit length of 2.4 μHm^{-1}.

a) Calculate the wave velocity and wavelength at f=50 Hz.

b) Calculate the characteristic impedance.

3. A loud-speaker which operates at up to 20kHz is at the end of a wire pair with a shunt capacitance per unit length of 10 pF m^{-1} and a loop inductance per unit length of 3μHm^{-1}.

a) Estimate how long the wire pair can be before transmission line effects need to be considered.

b) If the loud-speaker has a resistance of 8Ω what is the voltage reflection coefficient?

4.

a) Show from first principles that the voltage reflection coefficient of a transmission line of characteristic impedance Z_0 terminated by a load impedance $\overline{Z}_L$ is given by:

$$\overline{\rho}_L = \frac{\overline{Z}_L - Z_0}{\overline{Z}_L + Z_0}$$

b) A television antenna connection has an input resistance of 75Ω and is fed by a cable of characteristic impedance 50Ω. What fraction of the incoming signal power is reflected back up the cable?

5. The cable of an oscilloscope probe has a characteristic impedance of 75Ω. The input impedance at the oscilloscope is 1MΩ in parallel with 16 pF.

a) Show that the effect of the resistive component is negligible at 10 MHz.

b) Estimate the voltage reflection coefficient in magnitude and phase at 10MHz.

c) How do you eliminate any reflection?

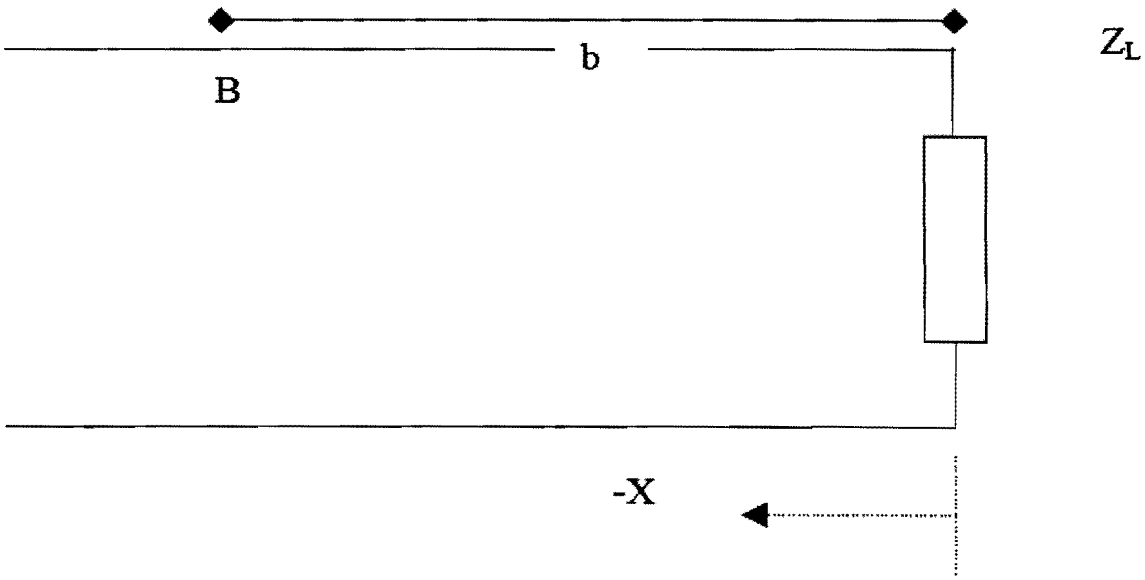
d) What is the consequent reduction in signal voltage at the oscilloscope?

6. A pair of logic gates has an output impedance of 70Ω and an input impedance of $5k\Omega$. The output of one is connected to the input of the other by a meter-length track with loop inductance per unit length $1\mu Hm^{-1}$ and shunt capacitance per unit length $20pFm^{-1}$.

After the line is switched, when will the reflections settle to less than 5% (by power) of the initial input?

7. The impedance of a line at a certain point is equal to the characteristic impedance only in the absence of reflections.

a) Assuming that the impedance of the line at any point is given by the ratio of the voltage to the current, derive an expression for the impedance $\overline{Z_b}$ at any point B, situated at a distance x=-b from the load.



b) Two cables, one having $Z_0=60\Omega$, the other $Z_0=80\Omega$, and carrying a signal at 1GHz, are joined by a short section of wire, with the correct impedance to eliminate reflections and a capacitance per unit length of 60 pF m^{-1}. Calculate:

b1) Z_0 of the joining section

b2) Its inductance per unit length

b3) Its length

8. A co-axial cable has an inner core of diameter 1mm, separated from an outer core of diameter 5mm by a material with $\varepsilon_r=2.2$ and $\mu_r=1$.

a) Derive an expression for its capacitance per unit length.

b) Derive an expression for its inductance per unit length

c) Find its characteristic impedance and the wave velocity.

9. A transmission line of characteristic impedance 50Ω and length λ/4 has an inductive load of j50Ω placed at the end.

a) Calculate the reflected voltage and the forward and reflected current in terms of the forward voltage.

b) Calculate the input impedance to the line.

Answers

2. a) $3 \cdot 10^8$m/s, 6000km.

   b) 722Ω

3. a)~570m,

   b)-0.97

4. b) 4%

5. b) 1; -8.6°,

   d) 0.5

6. 22ns

7. a) $\overline{Z}_b = \overline{Z}(-b) = Z_0 \dfrac{\overline{Z}_L + jZ_0 \tan(\beta b)}{Z_0 + j\overline{Z}_L \tan(\beta b)}$

   b1) Z_0=69.3Ω

   b2) $288 \cdot 10^{-9}$H/m

   b3) 6cm

8. c) 65Ω; $2 \cdot 10^8$ m/s

9 b) -50jΩ

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