Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 4 – Strength-limited Design, Process Selection, and Environmental Impact of Materials

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

You will need to look up data in the Materials Databook, and use the Cambridge Engineering Selector (CES) software.

Material Selection: Strength-limited Design

Questions 1-3 continue the application of the *Cambridge Engineering Selector (CES)* software, introduced in Examples Paper 1, to problems of material selection. As before, the problems can be solved using the Materials Databook or the *CES* software (though it is usually much easier using *CES*) – try both for some of the problems.

1. (a) Find materials for which the failure strength (elastic limit) σ_f > 300 MPa, and the density ρ < 3 Mg/m^3. Eliminate materials with poor ductility (e.g. apply a minimum elongation of 5%) and identify the cheapest. [Note: elongation data are not in the Databook, but use your judgement.]

(b) Compare the specific strength, σ_f/ρ, of steels, Ti alloys, Al alloys, Mg alloys, and CFRP (taking the highest strength in each case).

(c) Find metals and composites that are both stiffer and stronger than the highest strength Al alloys.

(d) State the performance index for maximum elastic energy storage per unit volume (Examples Paper 3, Q8a). Use this index to find materials which are suitable for efficient springs. Can you identify spring applications using these materials.

2. Explain briefly why the material property bubbles for metals are very elongated on the Young's Modulus – Strength chart. Use *CES* to explore whether the same is true for the polymers, and comment on the result. [Note: the Databook charts do not show the individual polymers, for clarity.]

3. A low temperature furnace operating at 250^{o}C uses solid shelves of rectangular cross-section for supporting components during heat treatment. The shelves are simply supported at the edges with the components located towards the centre, where the temperature is uniform. The designer wishes to minimise the mass, for ease of automated removal of the shelves, but failure of the shelves in bending must be avoided. The width b and length L of the shelves are fixed, but the thickness d can vary, up to a specified limit.

(a) For a specified load W applied at mid-span of the shelves, show that the maximum stress in bending is given by:

$$\sigma_{max} = \frac{3\,W\,L}{2\,b\,d^2}$$

Hence show that the performance index to be maximised for minimum mass is $M = \sigma_f^{1/2}/\rho$ where σ_f is the material strength, and ρ is the density. Explain why there is also a lower limit on the material strength.

(b) Use a Strength-Density property chart to identify a short-list of candidate materials (excluding ceramics and glasses - why is this?). Eliminate materials which cannot operate at the required temperature, and check whether the remaining materials would have a problem with the thickness limit (for which a minimum strength of 200 MPa is suggested). Find the cost/kg of the candidate materials.

(c) The designer is concerned about the cost, and asks for an alternative short-list based on minimum material cost. Modify your material performance index accordingly, and use a suitable material property chart (in *CES*) to identify a short-list of materials (applying the same secondary constraints as before).

4. A cable of cross-sectional area *A* and material density ρ is suspended over a fixed span *L*. The maximum allowable sag, δ, is specified, and the tensile stress in the cable must not exceed 0.8 of the yield stress, σ_y. Show that this design specification places a lower limit on the specific strength of the material used for the cable. Does this result depend on the cross-sectional area?
[You may assume that the sag is small, i.e. the tension in the cable is approximately constant, and equal to the horizontal reaction at the support.]

Effect of Shape in Lightweight Design

5. The Structures Databook provides section data for beams made of steel, aluminium and GFRP. Evaluate the shape factors for stiffness, for the smallest universal I-beam in steel, and the largest I-beams in aluminium and GFRP. How do these values compare with the maximum shape factors (for stiffness) for these materials given in the lecture notes?

* **6.** (a) Using a square solid section as reference shape, derive the expression for the shape factor for strength in bending, $\Phi_f = \dfrac{6\,I/y_{max}}{A^{3/2}}$, where y_{max} is the maximum distance of the cross-section from the neutral axis.

(b) A beam of fixed length *L* is to carry a specified load in bending, without failure. The cross-sectional area may be varied, with freedom to change the shape (characterised by the shape factor). Show that, to minimise the mass of the beam, the index $(\Phi_f \sigma_f)^{2/3}/\rho$ should be maximised.

(c) The design of the wings in a student aircraft project calls for the main spar to be as light as possible, but to remain fully elastic. The spar can be treated as a cantilevered beam, built-in at the fuselage, of prescribed length ℓ and subject to a uniform transverse loading per unit length w. Three candidate designs have been drawn up, as detailed in the table below.

Material	Section	ρ (kg/m^3)	σ_f (MPa)
Balsa wood	box section	130	8.0
CFRP	tube	1500	1400
Aluminium	I-section	2700	450

Details of the cross-sectional shapes are given in Figure 1. In each case the **shape** of the cross section is fixed, but the **size** (and thus the cross-sectional area A) can be varied by varying t.

(i) For the box section, find approximate expressions for the area and second moment of area, in terms of t. Hence find the value of the shape factor for failure in bending of the box section.

(ii) Find values for the shape factors for the tube and I-beam, using the following expressions for ϕ_B^f :

Tube: $\dfrac{3}{\sqrt{2\pi}}\sqrt{\dfrac{r}{t}}$ (r = tube radius); I-beam: $\dfrac{1}{\sqrt{2}}\sqrt{\dfrac{h}{t}}\dfrac{(1+3b/h)}{(1+b/h)^{3/2}}$ (b = breadth, h = height)

(iii) Use the results above to choose the best design of spar. What other factors should be considered in the design?

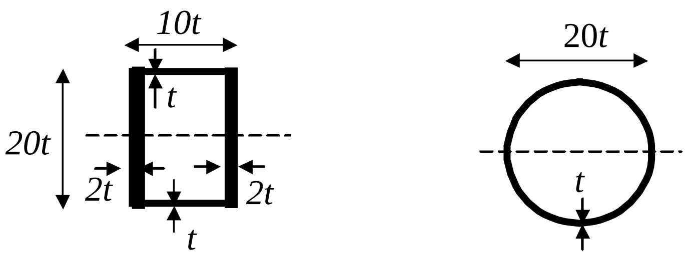

Figure 1

Multiple Constraints in Design

* **7.** As much as half the cost of a house is associated with the building material. A simply supported floor beam of fixed length L = 3m with a rectangular cross section is subject to a central load F = 2kN, as illustrated in Figure 2. The central deflection δ should be less than 5 mm, and the stress must not exceed σ_f anywhere in the beam. The breadth b is fixed and is equal to 100 mm, but the depth d of the beam can vary, with the constraint that d must not exceed 100 mm. Some pertinent formulae for this geometry are given below.

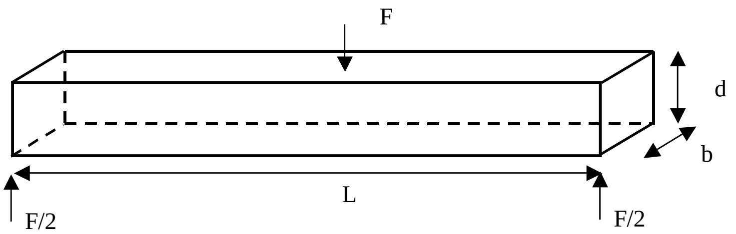

Figure 2

$$I = \frac{bd^3}{12}, \; \sigma_{max} = \frac{(d/2)M_{max}}{I}, \; M_{max} = \frac{FL}{4}, \; \delta = \frac{FL^3}{48EI}$$

First determine the mass required in each material listed in the table below, to satisfy both the stiffness and strength constraint. Check whether any materials are eliminated due to the size constraint. Then convert the required mass in each case to material cost, and hence identify the cheapest option. What other factors would you include to finalise your design?

	E GPa	ρ kg/m^3	σ_f MPa	C^* £/kg
GFRP	30	1900	400	10
Aluminium alloy	70	2700	400	2
Mild steel	210	7800	200	1
Nylon	3	1100	100	4
Soft wood	17	500	40	1

* Costs are very approximate.

Process Selection

8. "Shaker tables" are used for vibration testing. They consist of an electromagnetic actuator operating at frequencies up to 1000 Hz, driving the table to which the test object (such as an automobile component) is attached. To ensure that the natural frequencies of the table are higher than the range of test frequencies, the table is made from a material of high stiffness-to-weight ratio: magnesium alloy (ρ = 1750 kg/m^3).

A circular shaker table has a diameter of 2m and a thickness of 100 mm. The top surface and hub of the table are to be finished to a tolerance, T of ± 0.07 mm and a RMS roughness, R of 5 μm; the finish of the remaining surfaces is not critical. The market for shaker tables is small – each order is essentially a one-off. Suggest a possible process route, using the Process Attribute Charts in the Materials Databook.

9. In a cost analysis for casting a small aluminium alloy component, costs were assigned to tooling and overheads (including capital) in the way shown in Table 1 below. The costs are in units of the material cost of one component. Use the simple cost model presented in lectures to identify the cheapest process for batch sizes of 100 and 10^6.

Table 1

Process	Sand Casting	Investment casting	Pressure die	Gravity die
Material, C_m	1	1	1	1
Overhead, C_L (hr^{-1})	500	500	500	500
Tooling, C_c	50	11,500	25,000	7,500
Rate $\dot{n}$ (hr^{-1})	20	10	100	40

Environmental Impact of Materials

10. Drinking bottled water is often criticized as an unnecessary extravagance in a country with a clean supply of tap water, in part because of the environmental impact of transporting water to and around the UK.

As a test calculation, consider the life cycle energy of the following bottled water. The water is sold in 1 litre blow-moulded bottles made of PET (mass 40 g), with a PP cap (mass 1g). Filled bottles are transported 900 km to the UK from Switzerland, by large truck. It is refrigerated for 2 days, requiring 0.2 m^3 of refrigerator space per 100 bottles, and then sold. Assume that the energy expended in disposal is negligible.

(a) Use the data below (taken from CES) to estimate the life cycle energy associated with each of the following phases in the product life: material production, bottle manufacture, transport and use (refrigeration). Make the calculation for a batch of 100 bottles. Which phase dominates? Suggest some implications for the design of the bottle, and the impact of giving up bottled water altogether.

(b) The total energy consumption per person in the UK is estimated to be 125 kWh per day (Mackay D., *Sustainable Energy – without the hot air*). Roughly how many PET bottles would you need to recycle to save one day's average energy consumption?

Embodied energy, PET	84 MJ/kg
Embodied energy, PP	95 MJ/kg
Manufacturing energy, polymer moulding	19 MJ/kg
Recycling energy, PET	40 MJ/kg
Transport energy (large truck)	0.46 MJ/tonne.km
Refrigerator power rating (per m^3)	0.12 kW
Energy conversion efficiency (electrical to mechanical)	0.45 **

** NB 50% is associated with the initial generation of the electricity (from a typical European country mix of fossil fuel, nuclear and renewable); the conversion efficiency for the electric pump, from electrical to mechanical, is assumed to be 90%.

Answers

5. Stiffness shape factors: steel 20.8, aluminium 17.7, GFRP 14.6.

6. (b) $I \approx 3750t^4$, $A = 92t^2$, 2.55, (c) 3.78 for the tube, 4.3 for the I-beam, (d) CFRP.

7. Weight: GFRP, aluminium; Cost: aluminium and steel, but wood could be better with a slight re-design.

8. Sand cast then machine.

9. (i) sand cast (C = 26.5 units); (ii) pressure die cast (C = 6 units).

10. (a) Material: 346 MJ; Manufacture: 78 MJ; Transport: 43 MJ; Use: 9.2 MJ.
(b) about 255 bottles.

Suggested Tripos Questions
(some incorporate material from previous Examples Papers, and from the Easter Term)

2006 Q12
2007 Q7(b,c)
2009 Q10(a), 12 (note on Q12: treat "fatigue strength" and "endurance limit" in exactly the same way as "strength" – more details in the Easter Term).
2010 Q9
2011 Q11
2012 Q10(b)
2013 Q12(b)

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