ISSUED ON
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Engineering Tripos Part IB

Part IB Paper 5: Electrical
Examples Paper 7 -Electromagnetic Fields and Waves
Electromagnetic Waves

1. Glass has a relative permittivity ε_r=2.3 and a relative permeability μ_r=1 at optical wavelengths.

a) Calculate the velocity of light in glass and its intrinsic impedance.

b) If the electric field is $E_x = 10\cos(\omega t - \beta z)\frac{V}{m}$ in the x-direction, what is the magnitude and direction of H?

2. A laser beam has a diameter of 1mm. At what power does the beam cause electrical breakdown in the light path, if the dielectric strength of air is $3 \cdot 10^6$ V/m ?

3. Gauss's law states that, for an arbitrary surface, $\int_S \underline{D} \bullet d\underline{S} = Q$, where $\underline{D}$ is the electric flux density and Q the charge enclosed. Starting from the equation for the electrostatic field in free space, due to a point charge:

a) Evaluate the flux through a spherical surface of diameter d centred on a point charge +Q.

b) Imagine that the sphere is flattened, as shown in Figure 1. Evaluate the flux through the area which remains spherical, and the flux through the circular flattened area.

c) Imagine that the flattened sphere of part b) has no charge inside. What would then be the flux emerging from it?

d) Two point charges, +Q and –Q, are placed a distance d/2 apart, as in Figure 2. Show that the net electric field along the boundary is perpendicular to the boundary.

e) Using the results from parts b), c), d) show that, for the surface defined by the exterior of the spheres in Figure 2, the net flux is zero, hence satisfies Gauss's law.

Figure 1

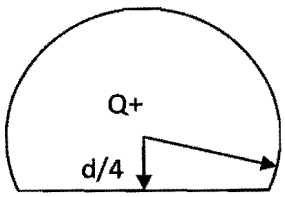
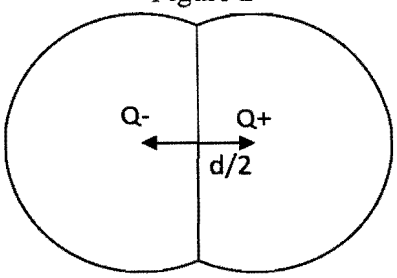


Figure 2



4. A travelling wave $E_i e^{j(\omega t - \beta z)}$ is reflected from a plane metallic sheet with negligible resistance at z = 0.

a) What is the sum of the incident and reflected waves? Show that it can be expressed as a standing wave.

b) Sketch the waveform of the electric field at $\omega t = 0$, $\pi/4$ and $\pi/2$.

5. Consider a plane wave of light travelling through material 1, with refractive index n_1 and relative permittivity ε_1, incident upon a planar interface to material 2, with refractive index n_2 and relative permittivity ε_2. If the angle of incidence is θ_i to the normal to the plate, draw a detailed diagram showing the reflected and refracted waves. Derive from the diagram the relationship between the angle of refraction, θ_t, and θ_i.

6. Because of the large losses encountered in sea water by high frequency waves, low frequency radio waves are used to communicate with submarines. To illustrate this, consider an airplane flying over the ocean surface, for which $\sigma = 4S/m$, ε_r=81 and μ_r=1. The airplane transmits a signal at 1MHz using a trailing, long-wire antenna. Assume that this transmitted wave is in the form of a uniform plane wave, with an electric field intensity of 1000 V/m and normal incidence to the ocean surface. If the submarine requires a minimum signal level of 10μV/m for adequate reception, determine the maximum communication depth of the submarine. The coordinate system is in Figure 3, with the ocean surface as the yz plane.

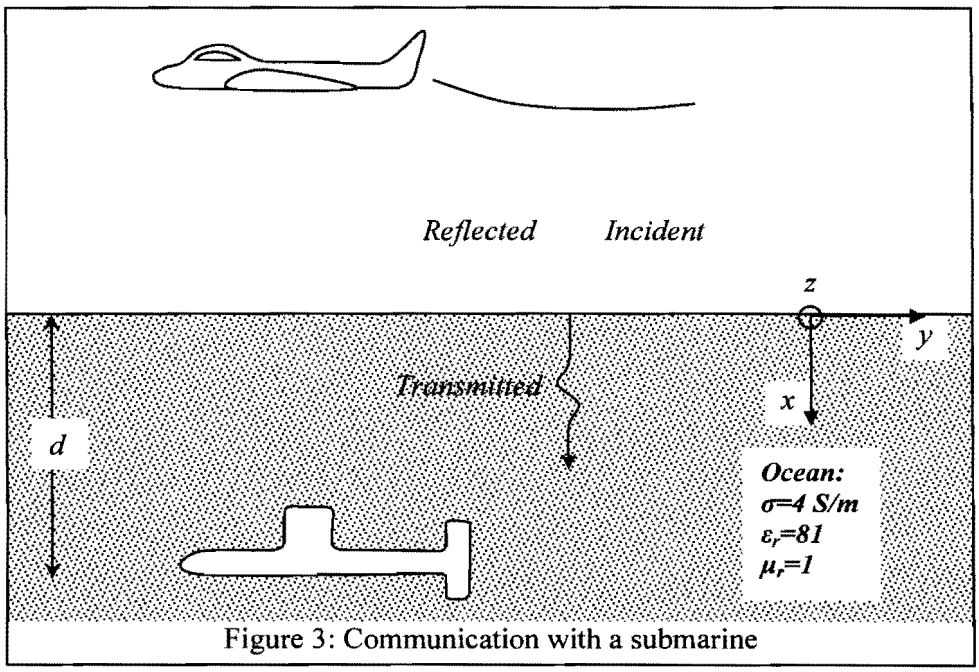


Figure 3: Communication with a submarine

7. A low frequency radio antenna radiates 100W isotropically.

a) What is the power intensity 10km from the antenna?

b) If a portable radio's antenna has an effective area of $0.01m^2$, what power does it receive?

8. The intensity radiated from a dish antenna varies as $\cos^2(5\theta)$, where θ is the angle from the antenna centre-line. If the gain of the antenna is 4.04 and it radiates a total of 50W, what is the intensity 10 degrees off the centre-line at a distance of 2km?

9 A satellite antenna has a gain of 4000 and transmits a 10W radio wave to the ground.

a) What is the intensity at ground level if the satellite is in geostationary orbit 36000km above ground?

b) If the receiving antenna has an effective area of 0.1m^2 and if both antennas interface without reflection to a cable with 75Ω characteristic impedance, what current flows into the transmitter and what current out of the receiver?

Answers:

1. a) $2\cdot10^8$m/s,248Ω; b) $H_{yF} \approx 0.04\cos(\omega t - \beta x)\ \frac{A}{m}$
2. 9.3kW
3. a) Q b)3Q/4; Q/4 (flat surface) c) 0
6. 3.41m
7. a) 79nW/m^2; b) 0.79nW
8. $1.66\ \mu\text{W/m}^2$
9. a) $25\ \text{pW/m}^2$; b) 0.37A rms (transmitter), 0.06 μA rms (receiver)