ISSUED ON
23 APRIL 2014

Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 5 – Fracture and Weibull Statistics

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

Fracture

†1. Define the following four terms

(i) a stress concentration factor.

(ii) a stress intensity factor K – how does this differ from (i)?

(iii) a strain energy release rate G – how does this differ from (ii)? Does the strain energy release rate have any meaning if a crack is not propagating?

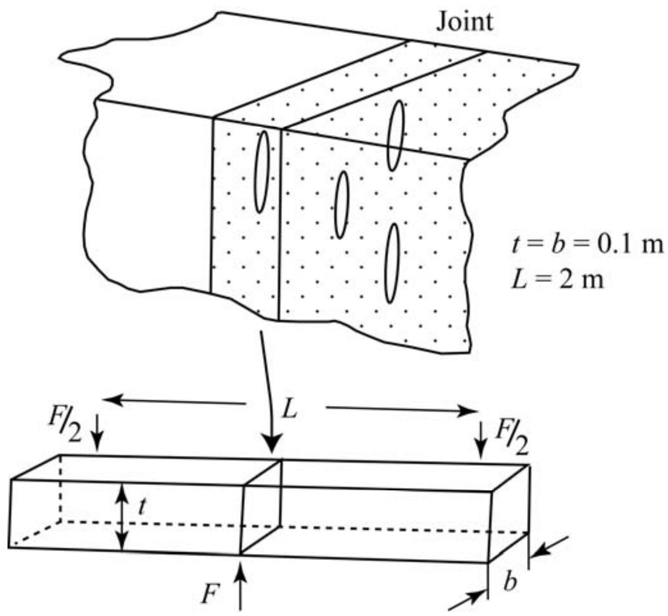
(iv) a critical stress intensity factor K_{IC} – how does this differ from (ii)?

Which of the above are **materials** parameters?

†2. Two wooden beams are butt-jointed using an **epoxy** adhesive. The adhesive was stirred before application, entraining air bubbles which, under pressure in forming the joint, deform to flat, penny-shaped discs of diameter 2 mm, as shown below.

(a) Show that the maximum tensile stress at the surface is given by $\sigma = \dfrac{3FL}{2bt^2}$

(b) If the beam has the dimensions shown, estimate the maximum load F that the beam can support before fast fracture occurs. Assume $K = \sigma\sqrt{\pi a}$ for the disc-shaped bubbles, but decide whether a, the representative crack dimension, is best taken as the radius or diameter of the bubble, by making the comparison with what you know of the corresponding expression for cracks in semi-infinite plates.



3. (a) Explain why the parameter $\dfrac{K_{IC}^2}{\pi\sigma_f^2}$ plotted on the K_{IC} - σ_f chart in the data book (page 19) is a measure of the applicability of the fracture toughness property K_{IC} to cracked specimens; σ_f is the strength of the material which, for metals, equals the yield stress σ_y. Use the property chart to decide whether you should use K_{IC} to estimate the failure load for an internal crack of length 1 mm in a large specimen of (i) alumina, (ii) CFRP?

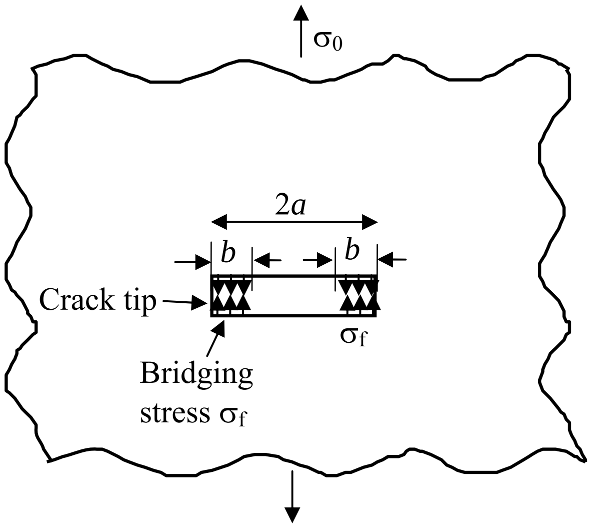
(b) Fibres in a composite material can bridge the tip of the crack, carrying a tensile stress which tends to prevent the crack opening and so reduces the stress intensity factor at the crack.

(i) To investigate this, first calculate the remote applied stress needed to give fast fracture in an epoxy of fracture toughness 2 MPa m$^{1/2}$, at the tip of a crack of total length 3 mm in an infinite solid (i.e. using $K = \sigma\sqrt{\pi a}$, where a is the half-length of the crack).

(ii) Assume that fibres bridge the crack over a region b = 0.3 mm at the tip of a crack of length $2a$ = 3 mm, with a uniform stress across the bridged zone σ_f = 120MPa, as illustrated in the figure below. Find the (negative) stress intensity factor (SIF) associated with this loading, using the appropriate expression for the stress intensity factor:

$$K = -2\sigma_f\sqrt{\frac{a}{\pi}}\cos^{-1}\left(\frac{a-b}{a}\right)$$

(iii) Use the principle of superposition, that the total SIF at the crack tip is the sum of the SIF's due to the remote load (as per part (i)) and the bridging stress (as per part (ii)), to make a new estimate of the remote stress required to give fast fracture in the composite in the presence of the bridging stress. Assume that fast fracture occurs when the total stress intensity factor is 2 MPa m$^{1/2}$.



4. (a) Why are ceramics much weaker in tension than compression? Why is a statistical approach needed for tensile loading?

(b) Explain, with reference to the fracture mechanisms, why metals which fracture in a ductile manner have a considerably higher toughness G_{IC} than ceramics.

Weibull Statistics

***5.** (a) When a brittle solid of volume V_0 is subjected to a uniform tensile stress σ we can write

$$P_s(V_0) = \exp\left(-\left(\frac{\sigma}{\sigma_0}\right)^m\right)$$

where $P_s(V_0)$ is the probability that the solid will survive against failure, and σ_0 and the Weibull modulus m are constants. Sketch the variation of $P_s(V_0)$ with σ/σ_0 as a function of m.

(b) Use the above equation to show that, for a volume V of the same material, weakest link theory dictates

$$P_s(V) = \exp\left(-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right)$$

(c) In order to test the strength of a ceramic solid, cylindrical specimens of length 25 mm and diameter 5 mm are subjected to axial tension. The tensile stress σ which causes 50% of the specimens to break is found to be 120 MPa. Cylindrical ceramic components of length 50 mm and diameter 11 mm are required to withstand an axial tensile stress of 40 MPa. Given that the Weibull modulus $m = 5$, calculate the survival probability.

6. The Ansell Adams photo below shows stalactites (calcite needles hanging down from caves) in New Mexico. Their failure due to self weight loading is to be modelled using Weibull statistics. The geometry of the stalactites is idealised as a cone of length L and semi-angle α. The cone angle is assumed small so that the base radius equals αL. The stalactite density is ρ.

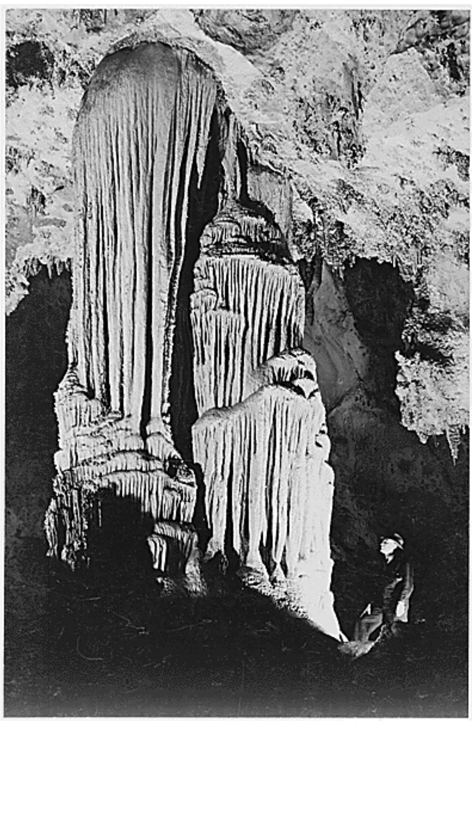
(a) Show that the variation of tensile stress σ with height x is given by $\sigma = \frac{1}{3}\rho g x$.

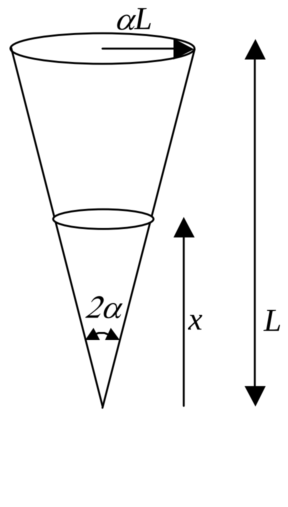
(b) Use the databook expression for Weibull statistics with a varying stress to show that the probability of survival $P_s(L)$ for a stalactite of length L is given by

$$P_s(L) = \exp\left(-\left(\frac{\rho g}{3\sigma_0}\right)^m \frac{\pi\alpha^2 L^{m+3}}{(m+3)V_0} \right)$$

where V_0 and σ_0 are the reference volume and stress and m is the Weibull modulus. Explain, using your understanding of the origin of Weibull statistics, why there is a dependency on cone angle α, even though the stress variation up the stalactite is independent of α.

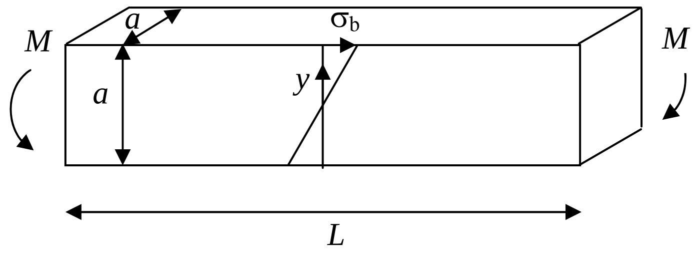
(c) Comment on possible practical difficulties with the sample tests.



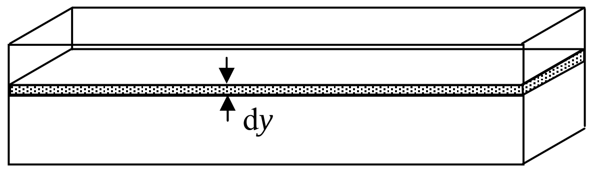


***7.** Data from bending tests on beams of silicon nitride are to be used to design an identical specimen loaded in tension. The test specimens are of length L and square cross-section of side a. The bend specimens are subjected to a uniform bending moment M with the maximum bending stress (at the top and bottom of the beam) equal to σ_b, as illustrated.

Sketch of bend specimen



Element slice for use in part (a)



(a) By considering slices of the specimen of length L, breadth a and height dy, (where the co-ordinate y is measured from the neutral axis) show that the probability of survival P_{sb} for the bend specimens is given by the expression

$$-\ln\left(P_{sb}\right)=\frac{a^2L\sigma_b^m}{2V_0\sigma_0^m\left(m+1\right)}$$

where V_0 and σ_0 are a reference volume and stress respectively and m is the Weibull modulus of the material. **Hints:** You should only integrate over the upper half of the beam (why?). Don't substitute in values sooner than you have to for part (b) below.

(b) The beam is loaded in tension under a uniform stress σ_t that gives the same probability of failure as σ_b. Derive an expression for the applied stress σ_t in terms of σ_b and m. For the bending tests, 50% of the beams broke when or before the maximum bending stress σ_b in the beam reached 500 MPa. Estimate the stress σ_t that gives the same probability of failure using $m = 10$ for silicon nitride.

Answers

1. (iii) Yes. Only K_{IC} is a materials parameter, the others are geometry/loading parameters.
2. This geometry is similar to a centre-notched crack, where $K = \sigma\sqrt{\pi a}$ with a as the crack half-length; take a as the radius here. $F = 7.7$ kN with $K_{IC} = 1.3$ MPa m$^{1/2}$ (p13, Data book).
3. (a) As the Data book states, the group given is a measure of the diameter of the process zone at the crack tip. (i) no problem (ii) questionable, depending on the exact laminate. (b) (i) 29 MPa, (ii) -3.37 MPa m$^{1/2}$ (iii) 78 MPa.

5. (c) Probability of survival = 0.9728

7. (b) $\sigma_t = \sigma_b / \left(2(m+1)\right)^{1/m}$, σ_t = 367 MPa

Suggested Tripos Questions:

Fracture:
2012 Q9

Weibull Statistics:
2010 Q11
2011 Q9
2013 Q10

NB Ignore 1A Tripos questions on diffusion/creep – no longer covered.

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Easter Term 2014