Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Examples Paper 5

(Starter questions are marked "s", Elementary ones †, and Tripos standard *)

sQ1 A refrigerator extracts heat at a rate of 800 W from a cold space while rejecting heat to the atmosphere at a rate of 1.2 kW. Calculate the power input and the COP?

sQ2 Calculate the efficiency of a reversible heat engine operating between temperature limits of 1000K and 300K.

sQ3 Give two examples of (a) irreversible processes, and (b) reversible processes. What defines a reversible process?

sQ4 A quantity of Argon gas (γ = 1.67), initially at 1 bar and 300 K, is compressed adiabatically and reversibly (and hence isentropically) to half its initial volume. Calculate the final pressure and temperature.

The Second Law and Entropy

Q5 Fill in the blanks as appropriate for the following systems (a-f), all of which are undergoing cyclic processes and exchanging heat with two reservoirs. Identify each of the possible devices as an engine, refrigerator, heat pump or other device.

System:	**a**	**b**	**c**	**d**	**e**	**f**
Source temperature, °C	327		300	60	327	140
Sink temperature, °C	27	70	100	60	827	
Heat from source, kJ/h	10 000	7000			6000	6000
Heat to sink, kJ/h	5500			1440		5000
Power output, kW			29.3	–0.4		0.3
Thermal efficiency, %			40			
COP_R		3.5				
COP_P					2.2	
Rev., irrev., impossible		rev				

Q6 A gas-powered refrigerator works by taking in heat from a heat exchanger at 500°C and from a cold space at −10°C, while rejecting heat to the environment at 35°C. The refrigerator is a cyclic device, and there is no work input or output. Use the Clausius inequality to find the minimum rate at which heat must be supplied by the heat exchanger in order to extract heat at a rate of 1 kW from the cold space.

*Q7 A cyclic heat pump takes 600 MJ (mega joules) of heat at 27°C while rejecting heat to a system whose temperature rises 1°C for each MJ of energy supplied to it. Initially this system is also at 27°C.

(a) Explain why there is a limit on the final temperature of the system. State whether it is an upper or lower limit, and under what circumstances it would be reached.

(b) Calculate the limit on the final temperature of the system. What is the work input to the heat pump when this limit is achieved?

Q8 (a) A mass, m, of iron is heated reversibly from temperature T_1 to T_2. During this process it may be assumed that the change in volume is negligible. Show that the change in entropy is given by:

$$\Delta S = mc_v \ln\left(\frac{T_2}{T_1}\right)$$

where c_v is assumed constant.

(b)* A system comprises two blocks of iron, one of mass 1 kg and initially at 20°C, the other of mass 2 kg and initially at 400 °C. The blocks are brought into thermal contact so that there is heat transfer between them, but there is no heat exchange with the surroundings.

(i) Use the First Law to determine the final temperature of the system.

(ii) Calculate the increase in entropy of the 1 kg block.

(iii) Calculate the decrease in entropy of the 2 kg block.

(iv) Show that the process is irreversible.

(v) Explain why the result derived in part (a) for a reversible process is still valid for this irreversible process.

Take c_v = 450 J/kgK for iron.

(c)* Describe how the system described in part (b) can be returned to its initial state by means of a reversible heat pump. What other process is also required? Explain why the original process in (b) must have been irreversible, even though the initial state can be restored.

*Q9 With reference to sketches of suitable cyclic devices, show that violation of the Clausius statement of the Second Law implies violation of the Kelvin-Planck statement and *vice versa* and hence that the two statements are equivalent. (Hint: you may wish to review example 5.2 in the notes first.)

Q10 Starting from the appropriate "Tds" equation, show that lines of constant specific volume on a T-s diagram for a perfect gas have gradients that are proportional to the absolute temperature. Sketch carefully two such lines, distinguishing between high and low specific volume.

Q11 A certain gas has a specific heat capacity at constant volume given by $c_v = (\alpha + \beta T)$ where $\alpha = 200$ J K^{-1} kg^{-1}, $\beta = 0.1$ J K^{-2} kg^{-1} and T is the temperature in Kelvin. When it is expanded reversibly and adiabatically from a temperature of 1000K to a larger volume, its temperature falls to 700K. When it is expanded adiabatically from the same initial state into an evacuated space such that the final volume is the same, its temperature falls instead to 950K.

(a) Sketch the two processes on a T-S diagram.

(b) Explain why the gas cannot be an *ideal* gas.

(c) Calculate for each process the entropy change per unit mass of gas.

Q12 (a) Calculate the work done and change in entropy when 0.1 kg of helium, confined in a cylinder by a piston, expands adiabatically from a volume of 0.1 m^3 and a pressure of 8 bar to a volume of 0.8 m^3 and a pressure of 0.4 bar.

(b) Sketch the process on a T-S diagram and include the lines of constant volume that pass through the initial and final states.

(c) By reference to the sketch, determine which type of process will give the maximum work output for any adiabatic expansion from the same initial conditions to the same final volume. Calculate this maximum work output.

Reciprocating Internal Combustion Engines

Q13 The air-standard Otto cycle provides an approximate model for spark ignition engines. The following data apply to the gas engines that you have tested in the laboratory, when operating at 250 rpm.

	National	**Hornsby-S.**
Cylinder diameter	0.178m	0.184m
Piston stroke (distance between TDC and BDC)	0.381m	0.381m
Compression ratio (V_{max} / V_{min})	4.5	4.5
Firing strokes per minute	107	107
Atmospheric pressure	101325 Pa	101325 Pa
Atmospheric temperature	293 K	293 K
Indicator calibration	54 kPa/mm	54 kPa/mm

In the following calculations, assume that the air/fuel mixture and the products of combustion all have the same properties as air, and that the combustion process is equivalent to a heat input of 1460 kJ/kg.

(a) For the engine that you tested calculate:

(i) The maximum and minimum volumes during the cycle.

(ii) The mass of air in the cylinder, assuming that conditions at the start of the compression process are the same as those of the atmosphere.

(iii) The pressure and temperature at the end of the compression process, assuming this to be isentropic. Calculate also the compression work.

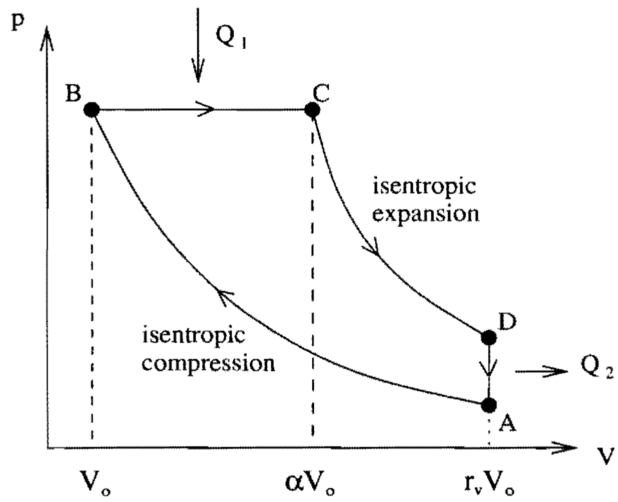
(iv) The pressure and temperature at the end of the combustion process, assuming this to occur at constant volume.

(v) The pressure and temperature at the end of the power stroke, assuming this to be isentropic. Calculate also the work done during this expansion process.

(vi) The work done per cycle, the thermal efficiency and the power output. Confirm that the value of efficiency agrees with that given by the expression for η_{otto} derived in lectures.

(b) Compare the values of pressure calculated above with those you have measured in the laboratory experiment on the indicator diagram. Compare also the measured power output and efficiency based on the work determined from the indicator diagram (i.e. the *indicated efficiency*) with that calculated above.

Q14 The figure below shows the p-V diagram for an air-standard Diesel cycle.



(a) For this cycle, find expressions for:

(i) The temperature at the end of the compression, T_B, in terms of T_A and r_v.

(ii) The temperature after "combustion", T_C, in terms of T_A, r_v, and α.

(iii) The temperature after the expansion , T_D, in terms of T_A, r_v, and α.

(b) Using your answers to (a), find expressions for the heat input, Q_1, and heat rejected, Q_2, per unit mass of air. Hence show that the cycle efficiency is given by:

$$\eta_{diesel} = 1 - \frac{1}{r_v^{\gamma-1}}\left(\frac{\alpha^\gamma - 1}{\gamma(\alpha - 1)}\right) .$$

(c) Noting that the term in brackets is always greater than unity, show that the Diesel cycle is always less efficient than an Otto cycle with the same compression ratio.

(d) Why do "diesel" engine cars usually have a better fuel economy than similar petrol engine cars?

ANSWERS:

sQ1	400W	2.0			
sQ2	70%				
sQ4	3.18 bar	477.3 K			
Q5	(a)	1.25 kW	45%	Irreversible	Engine
	(b)	−6.3°C	9000 kJ/h	−0.556 kW	Refrigerator
	(c)	Impossible (2nd Law)			
	(d)	Irreversible	Brake or similar dissipative device		
	(e)	11 000 kJ/h	−1.388 kW	Reversible	Heat Pump
	(f)	Impossible (1st Law)			
Q6	0.284 kW				
Q7	(a)	Lower limit	(b) 2216.7 K	1317 MJ	
Q8	(b)	(i) 546.5 K	(ii) 280.3 J/K	(iii) −187.6 J/K	
Q11	(c)	0 J/kgK	86 J/kgK		
Q12	(a)	72 kJ	146.5 J/K	(c) 90 kJ	

Q13	(a)	National	Hornsby-Stockport
	(i)	0.01219, 0.00271 m^3	0.01303, 0.00289 m^3
	(ii)	0.0147 kg	0.0157 kg
	(iii)	832 kPa, 535 K, 2.56 kJ	832 kPa, 535 K, 2.73 kJ
	(iv)	3986 kPa, 2563 K	3986 kPa, 2563 K
	(v)	485 kPa, 1404 K, 12.26 kJ	485 kPa, 1404 K, 13.09 kJ
	(vi)	9.70 kJ, 17.3 kW, 45.2%	10.4 kJ, 18.5 kW, 45.2%

Q14 (a) (i) $T_B = T_A r_v^{\gamma-1}$ (ii) $T_C = T_A \alpha r_v^{\gamma-1}$ (iii) $T_D = T_A \alpha^{\gamma}$

SUGGESTED TRIPOS QUESTIONS:

Part 1B Paper 4:	2007 Q1*	2006 Q2	2005 Q2 (a) & (b)		2003 Q3
Part 1A Paper 1:	2012 Q4, Q5	2010 Q3	2008 Q6	2007 Q6	2004 Q3

* NOTE: Replace "PER" with "COP" in this question

H. Babinsky
A.J. White
Easter 2014