ISSUED ON
7 MAY 2014

Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 6 – Fatigue Fracture, Oxidation & Tribology

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

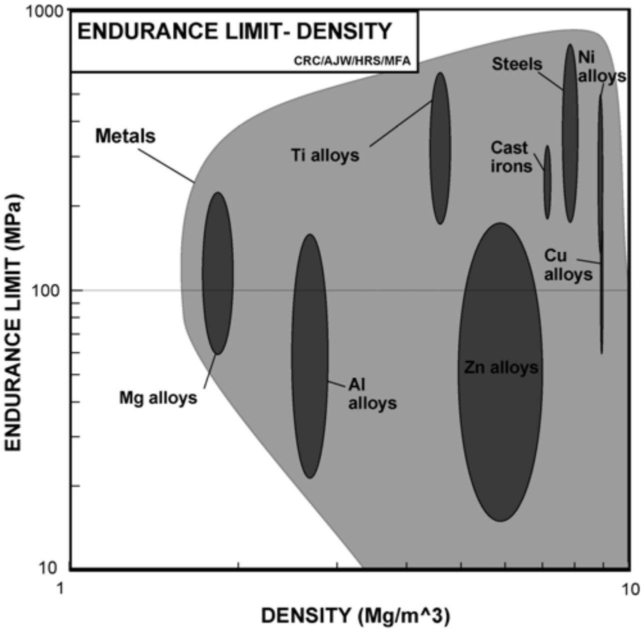
You will need to use the Materials Data Book

Fatigue Fracture

1. Consider the design of a bicycle crank. It is assumed that the design is governed by fatigue failure, and that to avoid this the maximum stress amplitude must be kept below the endurance limit σ_e of the material. The crank is modelled as a beam in bending of square cross section $a \times a$, where a is free to vary. The length of the beam and the applied moment are fixed.

† (a) Derive an expression for the material index which should be maximised to minimise the mass. Hence choose a shortlist of materials using the property chart below, commenting on your choice. Use the most advantageous material properties within the range given.

(b) At present an aluminium crank weights 0.2 kg. A cyclist is prepared to pay extra for a lighter component made of titanium. Use the property chart to estimate the mass of the corresponding titanium component. Hence find the perceived value per kg needed to make the switch to titanium? Assume that the as-manufactured costs per kg of aluminium and titanium are 10 and 40 £/kg, respectively. [Hint: remember that lines of the appropriate slope on the property chart correspond to contours of constant mass.]

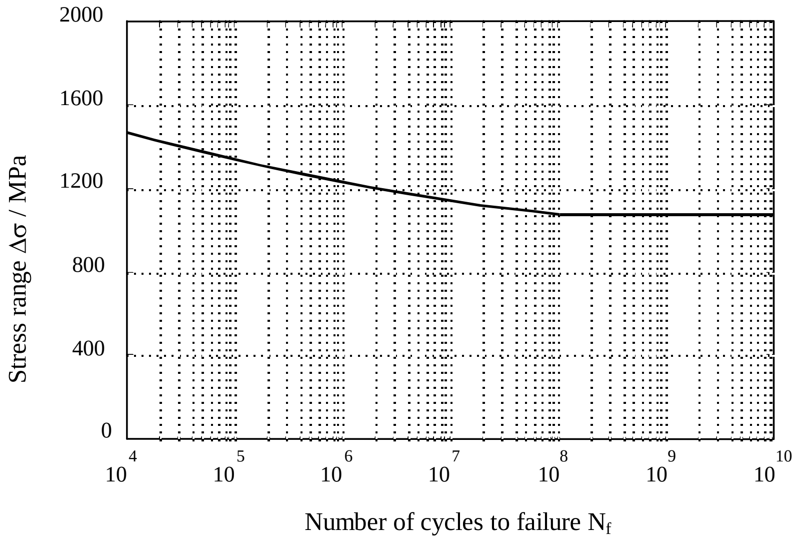


†**2.** Some uncracked bicycle forks are subject to fatigue loading. Appropriate S-N data for the material used are given in the figure below, for a zero mean stress.

(a) The loading cycle due to road roughness is assumed to have a constant stress range $\Delta\sigma$ of 1200 MPa and a mean stress of zero. How many loading cycles will the forks withstand before failing?

(b) Due to constant rider load the mean stress is 100 MPa. Use Goodman's rule to estimate the percentage reduction in lifetime associated with including this mean stress. The ultimate tensile stress σ_{ts} of the steel used equals 1100 MPa.

(c) What practical changes to the forks could you make to bring the stress range below the fatigue limit, and so avoid fatigue?



3. (a) An aluminium alloy for an airframe component was tested in the laboratory under an applied stress which varied sinusoidally with time about a mean stress of zero. The alloy failed under a stress range $\Delta\sigma$ of 280 MPa after 10^5 cycles. Under a range of 200 MPa, the alloy failed after 10^7 cycles. Assuming that the fatigue behaviour of the alloy can be represented by

$$\Delta\sigma\left(N_{\mathrm{f}}\right)^{\alpha} = C$$

where α and C are materials constants, find the number of cycles to failure for a component subjected to a stress range of 150 MPa.

(b) An aircraft using the airframe components has encountered an estimated 4×10^8 cycles at a stress range of 150 MPa. It is desired to extend the airframe life by another 4×10^8 cycles by reducing the performance of the aircraft. Find the **decrease** in stress range necessary to achieve this additional life. For this high-cycle fatigue you may use Miner's Rule [in the Data Book, p 7].

*__4.__ (a) Explain the “leak-before-break” criterion and how proof testing is used to ensure the safety of pressure vessels.

(b) A cylindrical steel pressure vessel of 7.5 m diameter and 40 mm wall thickness is to operate at a working pressure of 5.1 MPa. The design assumes that small thumb-nail shaped flaws in the inside wall will gradually extend through the wall by fatigue.
If the fracture toughness of the steel is 200 MPa $m^{1/2}$ would you expect the vessel to failure in service by leaking (when the crack penetrates the thickness of the wall) or by fast fracture? Assume $K = \sigma\sqrt{\pi a}$, where a is the length of the edge-crack and σ is the hoop stress in the vessel. **Note that you will need to convert from pressure to hoop stress.**

(c) During service the growth of a flaw by fatigue is given by

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\left(\Delta K\right)^4$$

where $A = 2.44 \times 10^{-14}$ $(\mathrm{MPa})^{-4}$ m^{-1}. Find the minimum pressure to which the vessel must be subjected in a proof test to guarantee against fast fracture in service in less than 3000 loading cycles from zero to full load and back.

Oxidation and Aqueous Corrosion

5. Explain the following observations:

(a) Mild steel support trays in furnaces and mild steel tools left in the garden corrode rapidly, while mild steel radiators in central heating systems only corrode slowly.

(b) Look up the free energy of formation of oxides of iron, aluminium and gold in the Data Book (p37). Relate this information to the observations that iron oxidises at high temperatures, while aluminium and gold **do not** oxidise visibly at high temperatures.

(c) Stainless steel is resistant both to oxidation at high temperatures and to aqueous corrosion in pure water.

(d) Lumps of magnesium or zinc alloys attached to mild steel structures (for example pipelines) prevent the structures suffering aqueous corrosion.

(e) A copper roof secured with copper nails has a longer life than an iron roof secured with iron nails and both last much longer than an iron-nailed copper roof. (Hint: look up the standard electrode potentials of oxygen, copper, and iron.)

6. The kinetics of oxidation of mild steel at high temperature are parabolic such that the mass change per unit area is related to time by

$$(\Delta m)^2 = k\, t$$

where $k = 1.75 \times 10^{-8}$ kg^2 m^{-4} s^{-1} at 500°C. A mild steel bar is used in a furnace for one year at 500°C. Find (i) the mass gained per unit area, and (ii) the corresponding depth of metal lost from the surface of a mild steel tie bar in a furnace at 500°C after 1 year. You may assume that the oxide scale is predominantly FeO. You will need to look up appropriate atomic weights and densities.

Hint: To find the depth of metal lost, first calculate how many kmols of oxygen atoms are needed to give the calculated increase in mass.

Tribology

*__7.__(a) Explain, with the help of a sketch, the details of what happens when the surfaces of two metals come into contact.

(b) The mild (medium carbon) steel body of a car weighing 200 kg has been compressed into a cube with a density half that of mild steel. (The cube also contains some air.) Calculate the nominal area of contact between the car and the relatively hard metal chute that it sits on. Using the lower value of yield stress for medium carbon steel given in the data book, estimate the true area of contact between the surfaces.

(c) The car is now tipped into a furnace. During sliding the asperity junctions grow, so that the true area of contact is twice that calculated in part (b). Calculate the friction coefficient between the surfaces if the shear strength of the oxide film separating the surfaces is about 100 MPa. Hence find the minimum angle of elevation of the chute required. Assume that the angle of elevation is small so that the normal reaction on the cube can be approximated by the weight of the cube.

(d) How would this minimum chute angle change with the weight of car or the degree of compaction of the cube?

(e) How does the hardness of the chute prevent wear of the chute?

(f) Explain how boundary additives put onto the chute would reduce the friction coefficient between the car and the chute.

(g) Explain why the friction coefficient is likely to increase significantly if the chute were made of rubber.

Answers

1. (a) $\sigma_e^{2/3}/\rho$. Mg, Ti, Al & Fe; (b) ~£60/kg
2. (a) 2×10^6 cycles; (b) about 90%; (c) increase the bending resistance (second moment of area) of the forks by using fatter tubes or use material with a higher fatigue limit.
3. (a) 5.13×10^8 cycles; (b) the range **decreases** by 13 MPa.
4. (b) Vessel leaks; (c) 9.5 MPa
6. 0.33 mm.
7. (b) Nominal area = 0.137 m^2, true area of contact = 2.14mm^2. Note the different units.
   (c) $\mu \approx 0.22$, $\sin^{-1}(0.22)=13°$, (d) No change. μ is independent of the nominal area and the weight.

Suggested Tripos Questions:

Fatigue Fracture:

2009 Q9

2011 Q12

2013 Q12

Oxidation and Aqueous Corrosion:

2010 Q10

2011 Q8

2013 Q7

Tribology:

2012 Q7

AE Markaki / AJ Kabla

Easter Term 2014