Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Examples Paper 6

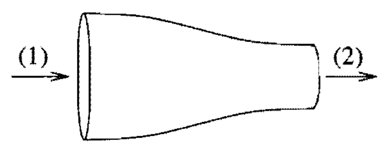
(Starter questions are marked “s”, Elementary ones †, and Tripos standard *)

Steady Flow Processes

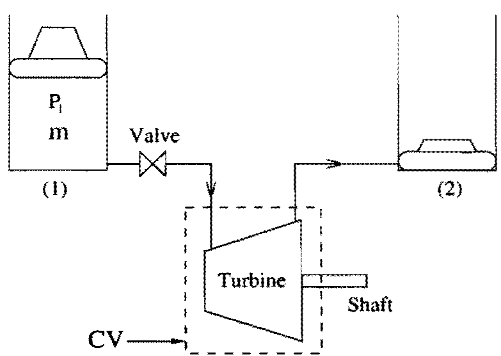
sQ1 Air flows down a pipe of diameter 200mm with a mass flow rate of 5 kg/s. If the pressure and temperature are 5 bar and 15°C respectively, calculate the air velocity.

sQ2 Calculate the change in enthalpy when 0.1 kg of helium is heated from 15°C to 95°C.

sQ3 The figure below shows a converging nozzle that is used to accelerate a flow of air from 50 m/s at (1) to 150 m/s at (2). The upstream air temperature is 25°C. Assuming heat transfer to the air is negligible, use the steady flow energy equation (SFEE) to calculate the temperature at (2).



Q4 A novel facility for testing a gas turbine is shown in the figure below. Cylinders (1) and (2) are maintained at constant pressures p_1 and p_2 (where $p_1 > p_2$) by means of weighted pistons. The volume of the turbine and pipework is negligible compared to that of the cylinders. In the initial state shown, cylinder (1) contains a mass m of gas and cylinder (2) is empty. The valve is opened and all the gas is transferred to cylinder (2) whilst the turbine produces shaft work, W_x.



(a) Taking the mass m of gas as a closed system, find an expression for the work done on this system by piston (1) in terms of m, p_1 and the specific volume of the gas in this cylinder, v_1. Write down a similar expression for the work done by the system on piston (2).

(b) Neglecting changes in kinetic and potential energy, and assuming there is no heat transfer, apply the First Law to the system to show that

$$-W_x = m\{(u_2 + p_2 v_2) - (u_1 + p_1 v_1)\}$$

where u is the specific internal energy of the gas.

(c) Verify that the same result is obtained by applying the SFEE to the control volume shown in the figure and integrating over time. Explain why enthalpy appears in the SFEE.

†Q5 Air flows steadily and isothermally at 27°C along a horizontal pipe of constant cross-sectional area 100 cm^2. At point A the pressure is 3 bar and the mean velocity is 160 m/s. At point B (which is downstream of A) the pressure is 2 bar. Calculate the velocity at point B and find the heat transfer per kg of air between the two points.

Q6 Two streams of carbon dioxide enter a steady flow device. Heat is transferred to the flow at a rate of 500 kW. There is no shaft work and changes in kinetic and potential energy may be neglected. The first stream has a mass flow rate of 1 kg/s and a temperature of 25°C. The second stream has a temperature of 200°C. After mixing, all the CO_2 exits from the device at 500°C. Find the mass flow rate of the second stream, assuming that CO_2 behaves as a perfect gas.

Q7 Nitrogen gas flowing at a rate of 0.5 kg/s enters an insulated pipe of 50 mm diameter. Some distance downstream, the flow encounters a partial blockage. The pressure and temperature upstream of the blockage are 200 kPa and 50°C respectively.

(a) Calculate the velocity upstream of the blockage.

(b) If there is a pressure drop of 30 kPa across the blockage, find the downstream temperature and velocity.

(c) Sketch the process on a T-s diagram and include constant pressure lines at the upstream and downstream pressures. Explain why the entropy increases.

(d) Calculate the force exerted by the flow on the pipe. In which direction does it act?

*Q8 Between the inlet (i) and exit (e) of a flow device, fluid properties may be related by integrating the "Tds" equation:

$$Tds = dh - vdp\ ,$$

provided the flow is in thermodynamic equilibrium at both i and e.

(a) Use the above to show that for steady reversible flow the heat addition per unit mass of fluid passing through the device is:

$$q = (h_e - h_i) - \int_i^e v\,dp$$

(b) Hence show that for steady reversible flow the shaft work per unit mass of fluid passing through the device is:

$$-w_x = \int_i^e v\,dp + \tfrac{1}{2}(V_e^2 - V_i^2) + g(z_e - z_i)$$

(c) A water pump passes a mass flow rate of 50 kg/s. The inlet and exit pipes have the same diameter, but the exit is situated 2 m above the inlet. The pressure at inlet is 0.1 bar, and that at exit is 40 bar. Calculate the minimum power input to the pump. Why, in practice, is the actual power input greater than this?

Non-steady flow

*Q9 A compressor takes in air at 1 bar, 20°C and low velocity, and delivers it to an insulated storage tank of 10 m^3 capacity, as shown below. Initially the tank contains atmospheric air and at the end of the process the pressure is 5 bar. Flow through the compressor is reversible and adiabatic, and the volume of the compressor and pipework is negligible compared to that of the tank.

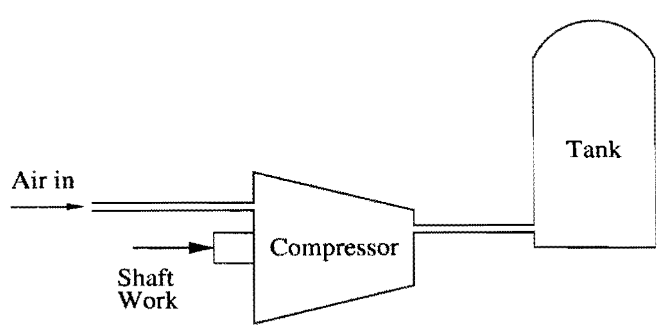
(a) Explain why the pressure and temperature inside the tank are related by:

$$T / p^{(\gamma-1)/\gamma} = \text{constant}$$

and calculate the final temperature in the tank.

(b) Calculate the initial and final masses of air in the tank.

(c) Select an appropriate control surface and apply the non-steady energy equation to find the compressor shaft work input.



Joule Cycles and Jet Engines

sQ10 Air enters the compressor of an industrial gas turbine at 15°C and 1 bar. At exit from the compressor, the pressure is 15 bar. Assuming the flow is steady, reversible and adiabatic, and neglecting changes in kinetic energy, calculate the temperature at exit from the compressor and the shaft work input per kg of air.

†Q11 (a) Sketch the air standard Joule cycle on a T-s diagram.

(b) Show that the compressor work input per kg of air is given by:

$$w_C = c_p T_1 (r_p^{(\gamma-1)/\gamma} - 1) = c_p T_1 (r_t - 1)$$

where T_1 is the compressor inlet temperature, r_p is the cycle pressure ratio and r_t is the compressor temperature ratio.

(c) Show that the turbine work output per kg of air is given by:

$$w_T = c_p T_3 (1 - 1/r_t)$$

where T_3 is the temperature at turbine inlet. Hence write down an expression for the net work output.

(d) Show that for fixed T_1 and T_3, there is a maximum net work output per kg of air which occurs when $r_t = (T_3 / T_1)^{1/2}$. Hence find the pressure ratio for maximum work output when $T_1 = 280$K and $T_3 = 1500$ K.

*Q12 A two-engined aircraft is preparing for takeoff on a day when the temperature and pressure are 0°C and 1 bar. The compressor pressure ratio in each engine is 25:1, and the turbine inlet temperature is 1500°C. Flow through the turbine, compressor and final nozzle may be assumed isentropic, and the pressure in the exhaust jet is the same as that of the flow entering the engine.

Assume that the kinetic energy of the flow is negligible everywhere except in the exhaust jet, and that the mass flow rate of fuel is small compared to that of air. You may also neglect pressure losses in the combustor and mechanical losses in the turbine / compressor shaft. Treat the working fluid as air throughout the engine.

(a) Sketch the "cycle" on the T-s diagram.

(b) Calculate the temperature at exit from the turbine.

(c) Find the velocity of the jet leaving the engine.

(d) If each engine has a mass flow rate of 42 kg/s, calculate the total thrust. (Assume the aircraft is stationary.)

ANSWERS:

sQ1	26.3 m/s				
sQ2	41.5 kJ				
sQ3	15.1 °C				
Q4	(a)	$W_1 = p_1 v_1 m$	(–ve work)	$W_2 = p_2 v_2 m$	(+ve work)
Q5	240 m/s	16 kJ/kg			
Q6	0.425 kg/s				
Q7	(a)	122 m/s			
	(b)	320.4 K	142.5 m/s		
	(d)	48.7 N downstream			
Q8	(c)	200.5 kW			
Q9	(a)	464 K			
	(b)	$m_1 = 11.9$ kg	$m_2 = 37.5$ kg		
	(c)	2.45 MJ			
Q10	624 K	340 kJ/kg			
Q11	(d)	$r_{p,\max} = 18.9$			
Q12	(b)	1361 K			
	(c)	1150 m/s			
	(d)	96.6 kN			

SUGGESTED TRIPOS QUESTIONS:

Part 1A Paper 1: 2011 Q4, Q6 2010 Q6, 2009 Q4, Q6 2008 Q4
2007 Q3, 2006 Q3, 2005 Q6

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