Part IA Paper 1: Mechanical Engineering

MECHANICS

EXAMPLES PAPER 4

Questions marked with a † are of a straightforward nature: those marked * of Tripos standard.

URLs for some web pages related to examples paper questions may be found at
www.eng.cam.ac.uk/~hemh/IAexamples.htm

Moment of momentum

1. Figure 1 shows a frictionless horizontal plane upon which a particle A of mass m slides. This particle is attached to an similar particle B by a light inextensible string, which passes through a small frictionless hole. The string is of sufficient length that particle B always remains clear of the plane. Particle B moves in a vertical line at all times.

Particle A is a distance r from the hole when it is projected with a speed V_1 perpendicular to the string. Some time later the particle has reached a distance of $2r$ from the hole and its speed is now V_2, again perpendicular to the string.

(a)† Sketch, in plan view, a curve to represent the path of A and show on your sketch vectors to represent the initial and final velocities V_1 and V_2. Use principles relating to moment of momentum to find speed V_2 find in terms of V_1.

(b) Find an expression for V_1 in terms of the gravitational acceleration g and r.

(c) for these two positions do you expect the string tension to be equal to mg ?

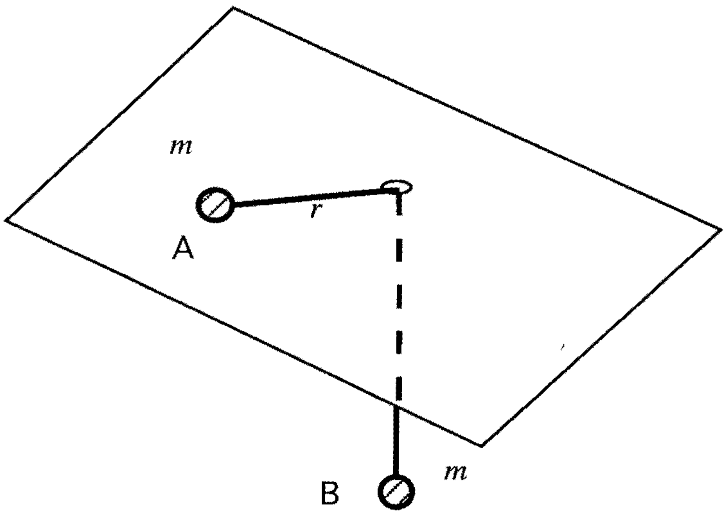

Figure 1

2.* In the motion of a particle, under what circumstances is

(a) total mechanical energy conserved

(b) moment of momentum conserved about an axis?

(note that moments taken about an *axis* and about a *point* are different. You should discuss this with your supervisor)

A smooth conical vessel with a cone angle of 70° and a height of height 2 m is fixed with its vertex downwards and its axis vertical, as shown in Fig. 2. A particle is projected with horizontal velocity v_1 on the inner surface at a pojnt A which is at height 1 m measured vertically above the vertex. Some time later the particle reaches the lip of the vessel at point B.

Sketch, in plan view, a curve to represent the path of the particle from A to B and show on your sketch vectors to represent the initial and final velocities. Use principles relating to moment of momentum and mechanical energy to show that if v_1 is approximately equal to $5{\cdot}1\ \mathrm{m\ s^{-1}}$ the particle will *just* remain inside the vessel at B. Will v_1 be different if a cone with a different cone angle is used ?

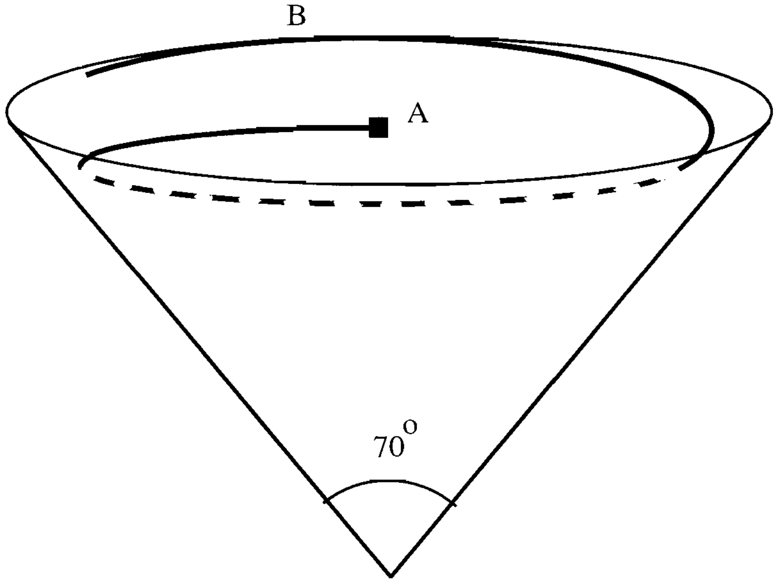

Figure 2 (not to scale)

Satellite motion

3. A satellite of mass 2000 kg is in elliptical orbit about the earth. At its perigee (point of closest approach) it has an altitude of 1100 km and a speed of 7900 $\mathrm{ms^{-1}}$. The earth's radius is 6400 km and g at the earth's surface is 9.81 $\mathrm{ms^{-2}}$.

(a) What is the energy of the satellite and how much energy is needed to put the satellite into orbit ?

(b) If the *burn* of the launching rocket is 10min, what is the mean power required ?

(c) What is the altitude and speed of the satellite at its apogee (the point on its orbit furthest from the earth) ?

(d) What is the eccentricity of its elliptical orbit and the length of the minor axis ?

4*. An artificial satellite of mass m is transferred from one circular orbit at speed v_p to another at speed $v_p/7$ by exerting a short-duration thrust (impulse I_P) tangentially at P, see Fig. 3, which is not to scale.

(a) Using the equations of motion for the satellite in its two circular orbits, determine the ratio of the radii r_2/r_1 of the two orbits.

(b) The impulse I_P at P increases the speed of the satellite from v_p to v_1. By considering the motion from P to A, determine the speed of arrival v_2 at A in terms of v_p.

(c) Determine the magnitude of the impulse I_P in terms of m and v_p. Another short duration thrust (impulse I_A) is required at A to prevent the satellite returning to P. Determine the magnitude and direction of this impulse in terms of m and v_p.

(d) What is the shape of the path traced from P to A? What happens to the satellite if no impulse is delivered at A?

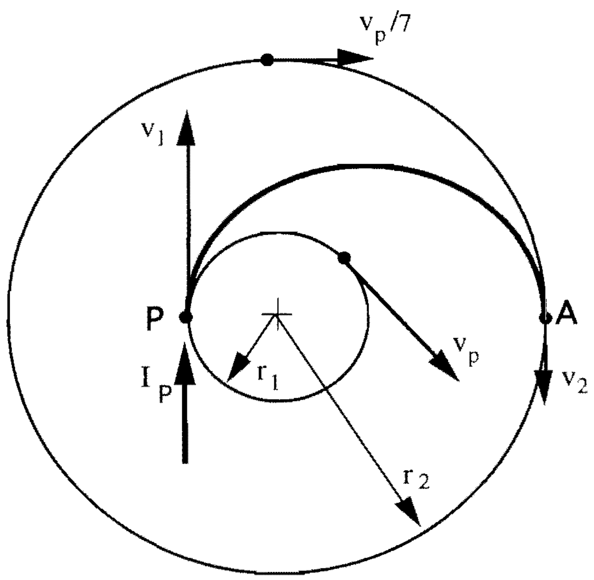

Figure 3

Moment of inertia and angular acceleration of a planar rigid body about a fixed pivot

5. (a)† Find from first principles the mass moment of inertia about A of a thin bar AB of length l and mass m. What is the moment of inertia about a point one third of the way along the bar?

(b)† The bar above is free to rotate in a vertical plane about a fixed horizontal frictionless pivot at A. The bar is initially held horizontally and released from rest. What is the initial angular acceleration of the bar?

(c) A particle of mass $2m$ is attached to the bar at B. The bar is again released from the horizontal rest position. What is the initial angular acceleration of the bar?

6. (a)† Determine from first principles an expression for the polar moment of inertia of a uniform thin disc of mass m and radius a about an axis passing through the centre of the disc. What is the moment of inertia of this disc about an axis perpendicular to the plane of the disc and passing through a point on the circumference of the disc?

(b)† The disc in part (a) is now rigidly fixed to a light horizontal shaft of radius b, as shown in Fig. 4. The shaft is supported in rigid frictionless bearings. A light string is wrapped around the shaft and pulled vertically downwards with a tension F. Assuming the string does not slip, calculate the angular acceleration of the disc.

(c)* A mass of m is instead attached to the free end of the string so as to provide the tension F. The disc is held stationary with the string taut and then released. Assuming the string does not slip, calculate the angular acceleration of the disc.

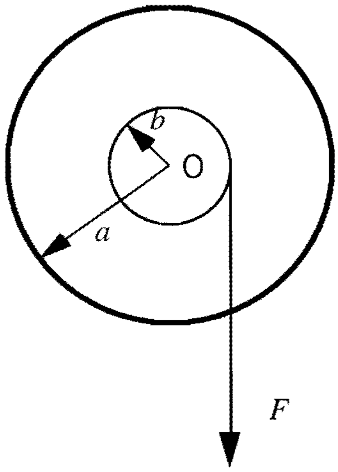

Figure 4

Variable mass dynamics

7. The Space Shuttle has a mass of 2000 tonnes on take-off. It ejects fuel a constant rate (measured relative to the spacecraft) of 9000 kgs^{-1} for the first 30 s of its flight. The initial acceleration of the Shuttle at take-off is $1.2\,g$, where $g = 9.81\ ms^{-1}$.

(a) What is the speed relative to the Shuttle that the fuel is being ejected?

(b) Assuming the mass of the Shuttle remains constant, what is its acceleration and speed after 30 s?

(c)* Calculate the acceleration and speed of the Shuttle after 30 s, assuming the mass does not remain constant.

8.* Coffee beans are being dropped from rest onto the scale pan of an electronic balance from fixed height h and at a constant rate $\dot{m}$. They do not bounce. The flow of beans is cut off at source when the balance reads the exact weight required. Show that the weight of beans in the air at that instant will exactly compensate for the false reading of the balance caused by the change in momentum of the falling beans.

ANSWERS

1. (a) $v/2$ (b) $\sqrt{(8gr/3)}$ (c) No, because B is accelerating

2. No

3. $-4{\cdot}47 \times 10^{10}$ J ; $8{\cdot}09 \times 10^{10}$ J ; 135 MW ; 5663 ms^{-1} 4062 km ; 0.165 ; 17,716 km

4. (a) $r_2/r_1 = 49$; (b) $v_2 = v_p/35$; (c) $I_P = \frac{2}{5}mv_p$; $I_A = \frac{4}{35}mv_p$

   in the same direction as v_2

5. (a) $ml^2/3$; $ml^2/9$ (b) $3g/2l$ (c) $15g/14l$

6. (a) $ma^2/2$; $3ma^2/2$ (b) $2bT/ma^2$ (c) $2bg/(a^2 + 2b^2)$

7. (a) 4800 ms^{-1} (b) 1.2 g ; 353 ms^{-1} (c) 1.5 g ; 402 ms^{-1}

Past Tripos Paper questions:

	EP1	EP2	EP3	EP4	
2009	Q7	Q11(a)	Q8,Q11	Q9	
2008	Q7	Q8(a)	Q8	Q9,Q11	
2007	Q11	Q9(a)	Q9	Q7,Q8	
2006	Q9	Q8(a)	Q8	Q7	
2005	Q8	.	Q12	Q9,Q10	from 2005 onwards the "short" and "long" style of questions began
2004	.	Q8(a)	Q8	Q6,Q7	
2003	.	.	Q6,Q7	Q8	
2002	.	.	Q7	Q6,Q8	
2001	.	.	Q6	Q7,Q8	from 2001 onwards there were only three questions in Mechanics (Q6,7,8)
2000	Q6,Q8	.	Q5	Q7	prior to 2001 there were four questions in Mechanics (Q5,6,7,8)
1999	Q7(a)(b)	Q5(a)	Q5,Q7(c)	Q8	
1998	Q5	.	Q6,Q7	Q8	
1997	Q5	.	Q5,Q6	Q8	
1996	Q7(a)(b)	.	Q6,Q7	Q5	
1995	Q7	Q5(a)(b)(c)	Q5	Q6,Q8	
1994	Q5	Q7(a)	Q7	Q8	
1993	Q5	.	Q8	Q6	

It's also at
<http://www2.eng.cam.ac.uk/~hemh/IAexamples.htm>

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Michaelmas 2013