P2: Structural Mechanics, Examples paper 1

Straightforward questions are marked †.
Tripos standard questions are marked *.

Forces, Moments and Couples

† 1. Figure 1 shows three stationary weights supported on strings, each tied to a small light ring R. The strings supporting weights W_1 and W_2 pass over frictionless pulleys at A and B, respectively. Draw a free body diagram for the ring R, and a corresponding triangle of forces.

If $W_3 = 100$ N, use the triangle of forces to solve the following:

(i) If $\alpha = 30°$ and $\beta = 60°$, find W_1 and W_2.

(ii) If $\beta = 45°$ and $W_2 = 141.4$ N, find α and W_1.

(iii) If $W_1 = 60$ N and $W_2 = 80$ N, find α and β.

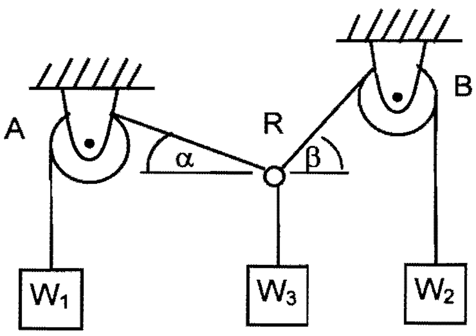


Fig. 1

† 2(a) Figure 2(a) shows a stationary, smooth (i.e. frictionless) sphere of radius R and weight W in contact with a smooth vertical plane and with a plane inclined at 60° to the vertical. Draw a free-body diagram showing the three forces acting on the sphere. By resolution of forces, find the magnitudes of the normal reactions N_1 and N_2 exerted on the sphere by the vertical plane and by the inclined plane.

Draw a triangle of forces to check your results.

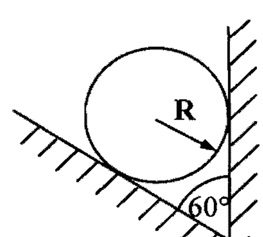


Fig. 2(a)

(b) Figure 2(b) shows two smooth spheres each of radius R and weight W at rest inside a smooth vertical cylindrical tube of diameter 3 R which has open ends. The spheres and tube are supported on a smooth horizontal table.

(i) Find the angle between the tangent plane to the spheres at E, and the vertical.

(ii) By considering the equilibrium of sphere A, find the magnitudes of the forces exerted on sphere A by the tube, and by sphere B.

(iii) By considering the equilibrium of sphere B, find the force exerted on sphere B by the tube.

(iv) Find the minimum weight of the tube to prevent it from toppling over. (Hint: draw a free-body diagram for the tube in the position shown, assuming that it is on the point of toppling about point C, and lifting off the table at D).

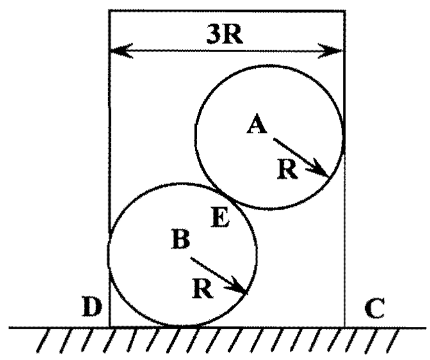


Fig. 2(b)

3. Figure 3 shows a uniform rod AB of weight W and a uniform rod AC of weight 2W. The rods are freely hinged together at A and rest, in a vertical plane, with B and C on a smooth horizontal table. Equilibrium is maintained by a light inextensible string attached to the mid-points of the rods. Find the magnitude of the reaction which the table exerts on rod AC, the tension T in the string, and the direction of the force that rod AB exerts on rod AC at A.

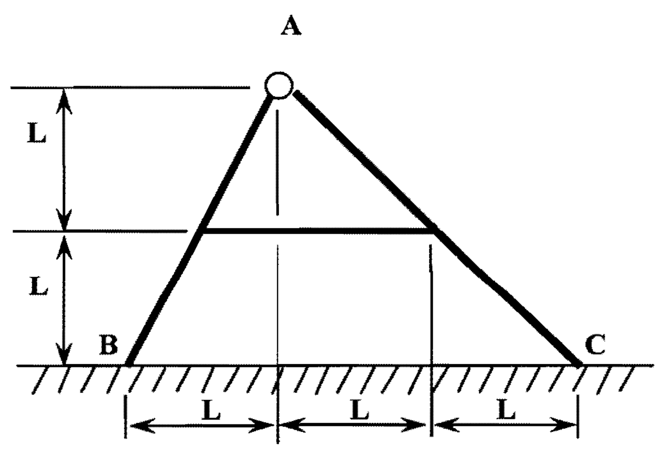


Fig. 3

4. In Fig. 4 the uniform horizontal bar AB of weight 100 N is pinned to the wall at A. The string BDC passes over a light pulley and is attached to the block C of weight 250 N, as shown. The pulley is free to rotate about E.

(a) By considering the combined system of bar, block, string and pulley together, and writing *one* suitable equilibrium equation, find the vertical reaction at A.

(b) By drawing suitable free-body diagrams, find the reaction between the block C and the bar AB.

(c) Consider an imaginary cut through the bar at F, mid-way between A and B. By considering the equilibrium of piece AF find the vertical force and moment acting here on the material to the left.

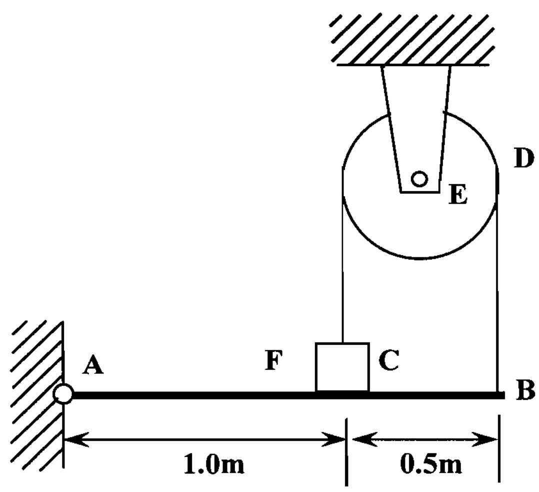


Fig. 4

5. Find the tension in the cable BCD of the structure shown in Fig. 5. Both the hinge at A and the pulley at C are frictionless, and the self-weight of the structure is negligible in comparison with the loads shown.

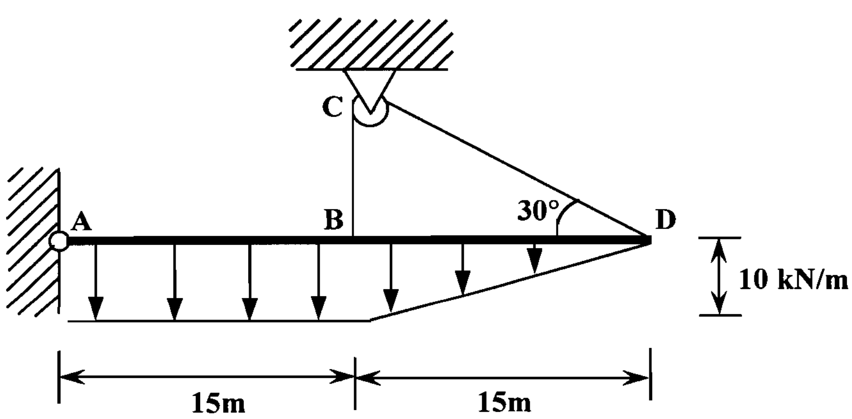


Fig. 5

Friction

6. Figure 6 shows a block ABCD of weight W resting on a plane inclined at an angle of 15° to the horizontal. The angle of friction is 30°. **Use graphical methods to:**

(i) Find the minimum value of the force that must be applied in the direction CD to pull the block down the plane.

(ii) Find the minimum value of the force that must be applied in the direction DC to pull the block up the plane.

* (iii) If a string is attached to the block at C, what is the minimum tension in the string required to pull the block up the plane? What is then the inclination of the string?

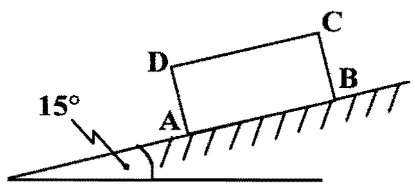


Fig. 6

7(a) An undergraduate rests the ends of a ruler on one finger of each hand, with the right hand closer to the end than the left hand, a side view is shown in Fig. 7(a). The hands are then slowly brought together. At which hand does slip first occur?

Explain your answer by drawing a free-body diagram of the stationary ruler with one finger slipping and one stationary. If $\mu_s = \mu_d$, predict the subsequent behaviour.

What happens in reality (try it!)? Why?

* (b) A uniform bar of length L and weight W rests on a level floor so that there is a uniform contact pressure between floor and bar. A *horizontal* force P is applied at one end, normal to the bar, see the plan view shown in Fig. 7(b). If the coefficient of friction between bar and floor is μ show that the force P needed to move the bar slowly is $(\sqrt{2}-1)\mu W$.

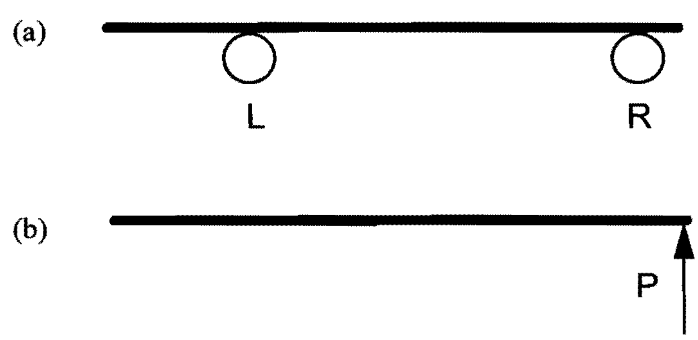
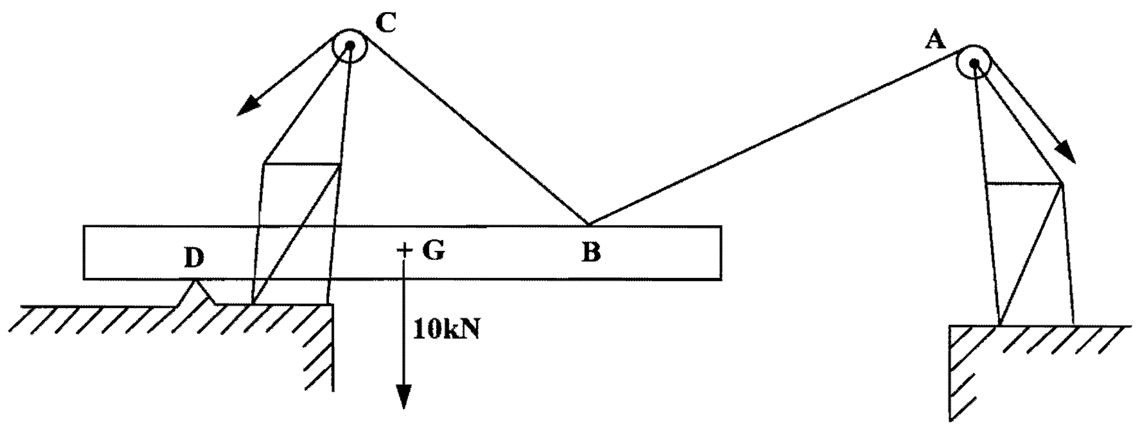


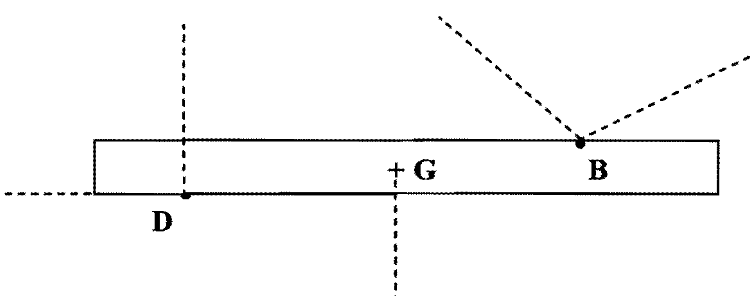
Fig. 7

8. Figure 8(a) shows a uniform girder of weight 10 kN which is being slowly dragged across a gap from left to right by means of two cables. The coefficient of friction at D is 0.25.

Figure 8(b) shows the girder and the points where forces act upon it. Show the forces on Fig. 8(b), treating the reaction at D as a single resultant force. Use a graphical method to find the approximate tensions in AB and BC measuring from the figures; great accuracy is not needed.



(a)



(b)

Fig. 8

9. A cylinder of weight W is attached by a horizontal string to a rough plane, as shown in Fig. 9. For a ramp angle of 30° this cylinder is in equilibrium, though just on the point of slipping.

(i) Find the angle of limiting friction for the cylinder on the plane.

(ii) Determine the magnitude of the least upward vertical force applied at A that would cause the cylinder to slip up the plane.

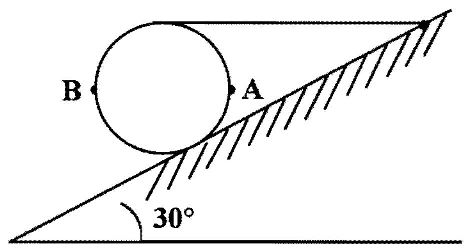


Fig. 9

10. Determine the range of cylinder mass m for which the system shown in Fig. 10 is in equilibrium. The coefficient of friction between the 50 kg block and the incline is 0.20 and that between the cord and the cylindrical support surface is 0.30. (Ref.3/Pr.6-99)

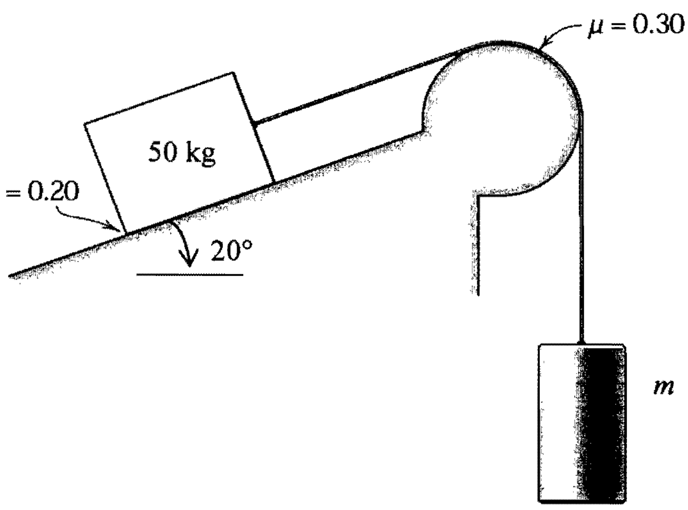


Fig. 10

Suitable Tripos Questions

2005	Q3a
2006	Q1
2007	-
2008	Q1a, Q1b, Q3
2009	Q1a, Q6a
2010	-
2011	Q1a, Q5b
2012	Q1a, Q3a, Q6ab
2013	Q1, Q3a

Answers

1. (i) $W_1 = 50$ N; $W_2 = 86.6$ N
   (ii) $\alpha = 0$; $W_1 = 100$ N
   (ii) $\alpha = 36.9°$, $\beta = 53.1°$

2 (a) $N_1 = W/\sqrt{3}$; $N_2 = 2\,W/\sqrt{3}$
  (b) (i) 60°
      (ii) $N_1 = W/\sqrt{3}$; $N_2 = 2\,W/\sqrt{3}$
      (iii) $N_3 = N_1 = W/\sqrt{3}$
      (iv) 2 W/3

3. 3 W/2; W; 26.6° to horizontal (upwards and left to right)

4. (a) 90 N
   (b) 120 N
   (c) 40 downwards; 48.8 Nm anti-clockwise

5. 87.5 kN

6. (i) 0.3 W
   (ii) 0.82 W
   (iii) $W/\sqrt{2}$; 45°

7 (a) R

8. 5.1 kN; 4.1 kN

9. (i) 15°
   (ii) $W/\sqrt{3}$

10. 4.33 kg < m < 47.1 kg

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