Part IA: Dimensional Analysis

EXAMPLES PAPER 1

Voluntary/optional "warm-up" questions are marked with a V.

Straightforward questions are marked with a †, Tripos standard with a *.

V1. Using the unit conversion factors given in lectures and in the appendices of the "Guide to Units", complete the following:

(a) 6 ft = ... m
(b) 2 kg = ... lb
(c) 70 mph = ... m/s (speed)

†2. Using the unit conversion factors given in lectures and in the appendices of the "Guide to Units", complete the following:

(a) 1 year = ... s
(b) 1 ft^3/min = ... m^3/hour (volumetric flow rate)
(c) 1 kg/m^3 = ... lb/ft^3 (density)
(d) 1 lbf/in^2 = ... N/m^2 (pressure or stress)

The lbf (pound force) is defined in Appendix D of the "Guide to Units".

V3. Using the definitions given in the "Guide to Units", find the dimensions (in the M-L-T-Θ system) of the following quantities:

(a) Energy
(b) Power
(c) Thermal conductivity

†4. Which of the following equations <u>appear</u> to be dimensionally inconsistent? In other words, in which equations do the constants have dimensions?

(a) The formula used by heating contractors to determine the heating requirements of a room:

$$Q = 0.04V + W + 0.33A$$

Q = heat supply for room per °F temperature difference between inside and outside expressed in Btu/hour °F (a British Thermal Unit, Btu, is a measure of energy),
V = volume of room expressed in ft^3,
W = area of windows expressed in ft^2,
A = area of external walls expressed in ft^2.

(b) The 'White' formula for the tension left in a straight weld joining two steel plates, on account of the shrinkage of the weld-metal:

$$T = 0.2\,\frac{Q}{v}$$

T = tension,
Q = electrical power input to the welding arc,
v = velocity of welding arc along the weld-line.

[Continued overleaf]

(c) The 'Chezy' formula for the mean flow velocity of water in a sloping pipe, the cross-section of which is not necessarily circular:

$$u = C\sqrt{\frac{AS}{L}}$$

u = mean flow velocity,
A = cross-sectional area of pipe,
S = slope of pipe,
L = 'wetted perimeter' of cross-section,
C = constant.

V5. You are considering a dimensional analysis problem that has N variables containing M dimensions. What is the minimum number of dimensionless groups you can form if

(a) $N = 3, M = 2$ (b) $N = 5, M = 3$ (c) $N = 3, M = 3$?

V6. Find as many dimensionless groups (that are independent of each other) as you can from the following sets of variables:

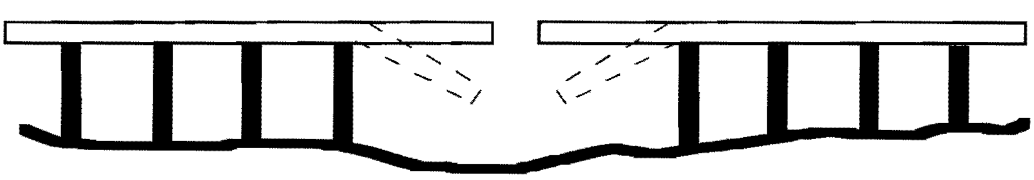
(a) Power P, mass m, speed V, length L

(b) Pressure p, density ρ, speed V, gravitational acceleration g, height h

7. When a circular disc of material is rotating about a central axis that is perpendicular to the plane of the disc, it will 'burst' under the effects of its own inertia loading if a critical angular velocity ω_c (rad/s) is exceeded. In tests carried out on gas turbine discs, it is found that the value of ω_c is dependent only on the maximum stress σ (N/m^2) that the material can withstand, the density ρ (kg/m^3) of the material and the radius R of the disc.

What is the form of the relationship governing ω_c, ρ, σ and R? Use the elimination method to perform any dimensional analysis required.

8. A construction company is required to build a long road bridge across a marsh. There are many short spans but there is to be one long span in the middle. The figure below shows a stage of construction when all the short spans have been completed and the two arms of the long span are being extended so that they meet at the centre. The ground conditions are such that no supporting structure can be built.



The chief engineer is concerned that the individual arms of the incomplete long span may collapse under their own weight before they can be joined together, in the manner indicated by the broken lines above.

The bridge will be constructed using girders made from steel. The steel has a density ρ of 7843 kg/m^3 and can withstand a maximum stress σ of 400 MN/m^2.

A simple model of one half of the long span is constructed out of aluminium. The half-span is 500 mm. The aluminium has a density ρ of 2720 kg/m^3 and can withstand a maximum stress σ of 70 MN/m^2. Using a centrifuge, it is found that the scale model collapses when the acceleration reaches 400 m/s^2. (The use of the centrifuge allows the acceleration due to gravity g applied to the model to be varied.)

What is the half-span of the largest steel bridge that can be constructed in this way using this particular design? Use the elimination method to perform any dimensional analysis required.

9. The aerodynamic lift (force) F that acts on an aircraft to support its weight when at cruise conditions depends on the following quantities:
   (i) the speed of the aircraft relative to the air (the "air-speed") v,
   (ii) the speed of sound in the air c,
   (iii) the area of the wings A,
   (iv) the density of the air ρ.

   Use Buckingham's Pi Theorem to indicate the minimum expected number of dimensionless groups for this problem.

   A Boeing 747 cruises at an air-speed of 252 m/s in air for which the speed of sound is 298 m/s and the density is 0.388 kg/m^3.

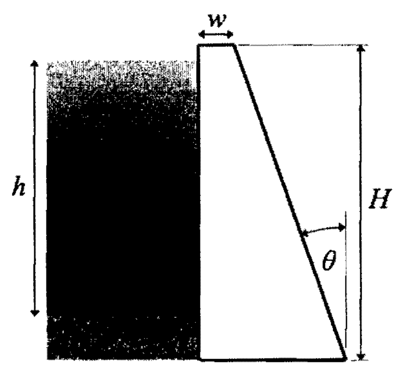
   A $^1/_{100}$th scale model of a Boeing 747 is tested in a wind tunnel where the speed of sound is 340 m/s and the density of air is 1.225 kg/m^3. Cruise conditions are simulated in the wind tunnel tests.

   (a) What is the air-speed of the model?

   (b) The aerodynamic lift on the model is measured as 1.2 kN. What is the aerodynamic lift on the full-size aircraft at cruise conditions?

*10. As part of his work for the famous Dambusters raid during the World War 2, Barnes Wallis used dimensional analysis to determine the mass of explosive needed to breach the Mohne Dam.

   The figure below shows an idealisation of the geometry of a gravity dam.



   If it can be assumed that the duration of the explosion of a bouncing bomb is much shorter than the time taken for the pressure waves caused by the explosion to propagate across the dam, then the explosive energy E required to breach the dam depends only on the geometrical parameters shown in the figure above and its breadth b, the density ρ of the material from which the dam is constructed and the acceleration due to gravity g.

   (a) Use dimensional analysis to find the relationship between the relevant parameters. Hence show that for *geometrically similar* dams the dependent dimensional group must have a fixed value.

(b) In a war-time experiment in Oxfordshire, Barnes Wallis found that explosive of mass 0.087 kg was needed to breach a scale-model of the Mohne Dam with $h = 1.1$ m. (The mass of explosive can be assumed to be linearly proportional to E.)

For his Dambusters recreation for television (*Dambusters: Building the Bouncing Bomb* which aired on Channel 4 on 2 May 2011), CUED's Dr Hugh Hunt found that explosive of mass 82 kg was needed to breach a geometrically similar dam with $h = 6.1$ m.

In both cases the models were made from material of the same density as that used in the Mohne Dam and the same explosive was used.

Are these findings consistent?

(c) How much of the same explosive would be needed to breach the Mohne Dam, for which $h = 15$ m?

*11. An oil tanker is damaged and spills oil of density $\rho_o = 800$ kg/m^3 onto the ocean surface; the sea density ρ_s is 1025 kg/m^3. Following the damage the spilt oil emerges from the tanker at a steady rate Q of 1 m^3/s and spreads over the ocean surface in a radially expanding surface layer of pure oil.

Model experiments are to be performed in a centrifuge at 1:1000 scale to establish the time taken t for pollution to spread over a certain distance D. The sea water is modelled by oil of density 900 kg/m^3 and the spilt oil by liquid of a specially selected density. The centrifuge creates an artificial gravity of 100 times that at the sea surface.

(a) Explain why the time t is the dependent variable in this problem.

(b) Under what conditions will the model test M have dynamical similarity to the real case R?

(c) In order to model the spilt oil, what density and flow rate should the fluid have in the scaled experiment?

(d) The pollution is observed in the model to travel a certain distance D_M in 1 s after the vessel ruptures. What duration does this correspond to in the full-scale case?

Suitable Past Tripos Questions:

Realistically, you should not attempt these questions until you have covered the Engineering topics with which they are concerned in the relevant lecture course.

IA 2010, Paper 1, Q1(c); IA 2009, Paper 2, Q11(d); IA 2008, Paper 1, Q5(e);

IA 2007, Paper 1, Q5(e); IA 2006, Paper 1, Q6(b)(i); IA 2005, Paper 1, Q4;

IA 2004, Paper 1, Q4

Older questions that will be harder to find:

IA 2003, Paper 2, Q6(b), Q10(a); IA 2002, Paper 2, Q3; IA 2001, Paper 1, Q2(d);

IA 2000, Paper 1, Q10; IA 1998, P1, Q1

ANSWERS

1. (a) 1.829 (b) 4.409 (c) 31.31
2. (a) 31.5×10^6 (b) 1.70 (c) 0.0624 (d) 6894
3. (a) ML^2T^{-2} (b) ML^2T^{-3} (c) $MLT^{-3}\Theta^{-1}$
4. (a) and (c) have constants with dimensions
5. (a) 1 (b) 2 (c) 0
6. (a) $\dfrac{PL}{mV^3}$ (b) $\dfrac{p}{\rho V^2}$ and $\dfrac{gh}{V^2}$ (other equivalent results are possible)
7. $\omega_c R\sqrt{\rho/\sigma}$ = constant
8. 40.4 m (dimensional analysis gives $\rho gs/\sigma$ = constant)
9. 2 dimensionless groups (a) 288 m/s (b) 2.9 MN

   (dimensional analysis gives $\dfrac{F}{\rho v^2 A}$ depends on $\dfrac{c}{v}$ or other equivalent results)
10. (a) Dimensional analysis gives $\dfrac{E}{\rho g h^4}$ depends on $\left\{\dfrac{H}{h},\dfrac{b}{h},\dfrac{w}{h},\theta\right\}$ (or other equivalent results)

    (b) Yes, they are consistent.

    (c) ~3000 kg
11. Dimensional analysis gives $\dfrac{tQ}{D^3}$ depends on $\left\{\dfrac{\rho_o}{\rho_s},\dfrac{gD^5}{Q^2}\right\}$ (or other equivalent results)

    (b) For DS $\left(\dfrac{\rho_o}{\rho_s}\right)_R=\left(\dfrac{\rho_o}{\rho_s}\right)_M$ and $\left(\dfrac{gD^5}{Q^2}\right)_R=\left(\dfrac{gD^5}{Q^2}\right)_M$

    (c) 702.4 kg/m³ and 3.16×10^{-7} m³/s

    (d) 316 s

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