Part IA Paper 1: Mechanical Engineering

MECHANICS

EXAMPLES PAPER 1

Questions marked with a † are of a straightforward nature: those marked * of Tripos standard.

URLs for some web pages related to examples paper questions may be found at
www.eng.cam.ac.uk/~hemh/IAexamples.htm

Force, weight and gravity

†1. What is the weight in newtons of a person whose mass is 75 kg? If the person were to jump out of an aeroplane what would be their weight during free fall?

†2. At what altitude h above the north pole is the weight of an object reduced to one half of its value on the earth's surface? Assume the earth to be a sphere radius R and express h as fraction of R.

†3. Two steel spheres, each of diameter 200 mm, are just touching. At what distance from the centre of the earth will the force of mutual attraction between them be equal to the weight (i.e. the force exerted by the earth) of one of them?

Mass of the earth $= 5{\cdot}98 \times 10^{24}$ kg, density of steel $= 7840$ kg m^{-3}.

Kinematics of a particle: (a) using cartesian coordinates

†4. A particle moves on a plane such that its cartesian coordinates of position x and y are given by the equations

$$x = R\sin\psi \quad \text{and} \quad y = R(1 - \cos\psi)$$

where R is a constant and ψ varies with time t.

Write down expressions for the cartesian components of velocity of the particle $\dot{x}$ and $\dot{y}$ in terms of R, ψ and the rate of change $\dot{\psi}$.

If $\dot{\psi}$ is constant, what can you say about the speed of the particle? Sketch the path of the particle.

†5. A particle moves along a curve whose parametric equations using cartesian coordinates are $x = e^{-t}$ and $y = 2\cos 3t$, where the distances are in metres and t is the time in seconds. Find:

(i) vector expressions for its velocity and acceleration for any time t,

(ii) the magnitude and direction of its velocity and acceleration at $t = 0$, and

(iii) the component of its velocity and acceleration in the direction $(\underline{i} - 3\underline{j})$ at $t = 0$.

6. A particle moves round a circle having a fixed centre C and radius R, such that the radius vector CP rotates with angular velocity $\dot{\psi}\underline{k}$ and angular acceleration $\ddot{\psi}\underline{k}$. Taking an origin O on the circumference of the circle as shown in Fig. 1, find the vector expressions in terms of ψ, $\dot{\psi}$, and $\ddot{\psi}$ for the velocity and acceleration of P in cartesian coordinates (x, y), using $\underline{i}$ and $\underline{j}$.

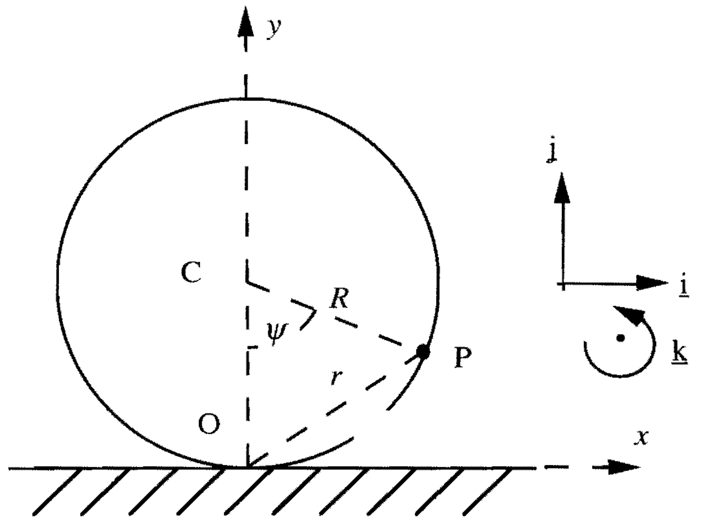

Fig. 1

Kinematics of a particle: (b) using polar coordinates

†7. For the hydraulic cylinder in the position shown diagrammatically in Figure 2, the cylinder is rotating anticlockwise about a fixed point O such that $\dot{\theta}$ = 1.2 rad s^{-1} and $\ddot{\theta}$ = – 0.8 rad s^{-2} ; and the piston is contracting such that $\dot{r}$ = – 0.15 m s^{-1} and $\ddot{r}$ = 2.0 m s^{-2}. Determine the position, velocity and acceleration of point A on the piston in terms of the unit vectors $\underline{e}_r$ and $\underline{e}_\theta$. What are the *magnitudes* of the velocity and acceleration?

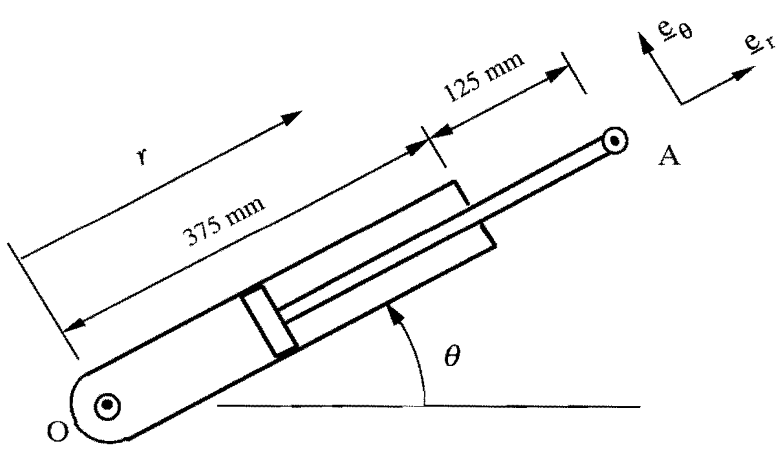

Fig. 2

8. For the particle moving round the circle shown in Fig. 1, find the vector expressions in terms of ψ, $\dot{\psi}$, and $\ddot{\psi}$ for the velocity and acceleration of P in polar coordinates (r, θ), using $\underline{e}_r$ and $\underline{e}_\theta$. Take the origin for polar coordinates at O so that r is the length OP and θ is the anticlockwise angle between the x axis and OP.

Hint: Start by determining the directions of the unit vectors $\underline{e}_r$ and $\underline{e}_\theta$.

Kinematics of a particle: (c) using intrinsic coordinates

†9. A particle is travelling along a curved path. At a given instant the magnitudes of the velocity and the acceleration are 5 m s^{-1} and 10 m s^{-2}, respectively. The acceleration vector makes an angle of $25°$ relative to the velocity vector.

Determine:
(i) the magnitude of the component of the acceleration along the path, and
(ii) the instantaneous radius of curvature of the path.

Hint: Draw a clear diagram showing the direction of the acceleration vector relative to the velocity vector for a particle moving along a curved path.

10. A taut string CP is unwrapped from a fixed drum, centre O and radius R, with uniform angular velocity $\dot{\psi}\underline{k}$. The end of the string P, initially an contact with the drum at A, traces out a planar curved path AB as shown in Fig. 3. (This path is known as an 'involute' and is the most commonly used profile for gear teeth — you will learn why this is the case later in the course).

Write down expressions for the position vector of P relative to O in terms of the unit vectors $\underline{e}_t$ and $\underline{e}_n$ as shown. By differentiation find expressions for the velocity and acceleration of P in this coordinate system. From the acceleration verify that the radius of curvature of the path of P is equal to CP. Would you expect this?

Hint: Determine the rate of rotation of the unit vectors $\underline{e}_t$ and $\underline{e}_n$.

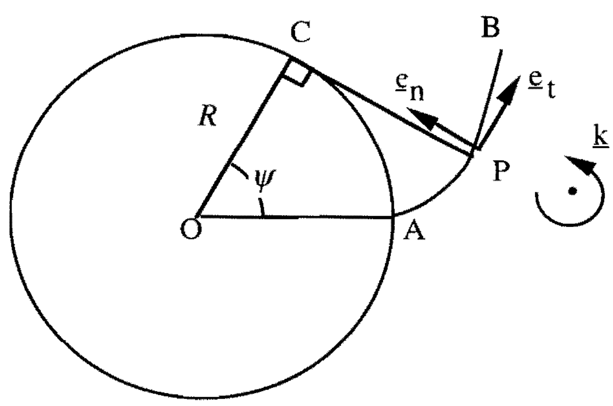

Fig. 3

Kinematics of a particle: (d) switching between cartesian, polar and intrinsic coordinates

11. For the particle moving round the circle shown in Fig. 1, find the vector expressions in terms of ψ, $\dot{\psi}$, and $\ddot{\psi}$ for the velocity and acceleration of P in intrinsic coordinates (s, ψ), using $\underline{e}_t$ and $\underline{e}_n$.

Hint: Start by determining the directions of the unit vectors $\underline{e}_t$ and $\underline{e}_n$.

By considering the relationship between the three pairs of unit vectors $(\underline{e}_t, \underline{e}_n)$, $(\underline{i}, \underline{j})$ and $(\underline{e}_r, \underline{e}_\theta)$ used to describe the motion of the particle in Fig. 1, express $\underline{e}_t$ and $\underline{e}_n$ both in terms of $\underline{i}$ and $\underline{j}$ and in terms of $\underline{e}_r$ and $\underline{e}_\theta$.

Three coordinate systems have been used to describe the same particle's velocity and acceleration (cartesian in Question 6, polar in Question 8 and intrinsic in this question). To demonstrate the equivalence, substitute the expressions for $\underline{e}_t$ obtained above into the expression for the velocity of the particle obtained in intrinsic coordinates above to confirm that you obtain the velocity expressions found in cartesian coordinates in Question 6 and in polar coordinates in Question 8.

Note: Similar substitutions can be made into the expression for the acceleration above to demonstrate equivalence.

*12. A tracking radar detects an aircraft due south at a range of 10 km. The radar points continually at the aircraft. It measures a range that is decreasing at a constant rate of 200 m s^{-1} while the radar is rotating clockwise at a constant rate of 0·6 deg s^{-1}.

(i) What is the course (direction) and speed of the aircraft?

(ii) What is the component of the aircraft's acceleration along its path?

(iii) What is the value of the instantaneous radius of curvature of the path of the aircraft?

ANSWERS

1. 736 N

2. $(\sqrt{2} - 1)R$

3. $85{\cdot}3 \times 10^9$ m

4. $\dot{x} = R\psi\cos\psi$; $\dot{y} = R\psi\sin\psi$; constant

5. (i) $\underline{v} = (-e^{-t}\underline{i} - 6\sin 3t\,\underline{j})$ m s^{-1} ; $\underline{a} = (e^{-t}\,\underline{i} - 18\cos 3t\,\underline{j})$ m s^{-2}

   (ii) 1 m s^{-1} at 180° to the x-direction; $5\sqrt{13}$ m s^{-2} at 273.2° to the x-direction

   (iii) $\dfrac{-1}{\sqrt{10}}$ m s^{-1} ; $\dfrac{55}{\sqrt{10}}$ m s^{-2}

6. $\underline{v} = R\psi(\cos\psi\,\underline{i} + \sin\psi\,\underline{j})$

   $\underline{a} = R(-\psi^2\sin\psi + \ddot{\psi}\cos\psi)\underline{i} + R(\psi^2\cos\psi + \ddot{\psi}\sin\psi)\underline{j}$

7. $\underline{r} = 0.5\,\underline{e}_r$ m

   $\underline{v} = -0.15\,\underline{e}_r + 0.6\,\underline{e}_\theta$ m s^{-1}

   $\underline{a} = 1.28\,\underline{e}_r - 0.76\,\underline{e}_\theta$ m s^{-2}

   $v = 0.62$ m s^{-1} ; $a = 1.49$ m s^{-2}

8. $\underline{v} = R\psi\left(\cos\frac{\psi}{2}\,\underline{e}_r + \sin\frac{\psi}{2}\,\underline{e}_\theta\right)$

   $\underline{a} = R\left(-\psi^2\sin\frac{\psi}{2} + \ddot{\psi}\cos\frac{\psi}{2}\right)\underline{e}_r + R\left(\psi^2\cos\frac{\psi}{2} + \ddot{\psi}\sin\frac{\psi}{2}\right)\underline{e}_\theta$

9. $9{\cdot}1$ m s^{-2} ; $5{\cdot}9$ m

10. $R\,\underline{e}_t - R\psi\,\underline{e}_n$; $R\psi\,\psi\,\underline{e}_t$; $R\,\psi^2\,\underline{e}_t + R\psi\,\psi^2\,\underline{e}_n$

11. $\underline{v} = R\psi\,\underline{e}_t$

    $\underline{a} = R\,\ddot{\psi}\,\underline{e}_t + R\,\psi^2\,\underline{e}_n$

    $\underline{e}_t = \cos\psi\,\underline{i} + \sin\psi\,\underline{j} = \cos\frac{\psi}{2}\,\underline{e}_r + \sin\frac{\psi}{2}\,\underline{e}_\theta$

    $\underline{e}_n = -\sin\psi\,\underline{i} + \cos\psi\,\underline{j} = -\sin\frac{\psi}{2}\,\underline{e}_r + \cos\frac{\psi}{2}\,\underline{e}_\theta$

12. (i) $332{\cdot}4°$; 226 m s^{-1} (ii) $-0{\cdot}97$ m s^{-2} (iii) $12{\cdot}1$ km

For further practice try the following Tripos questions from:

Paper 1 of Part IA: 1993 Q5, 1994 Q5, 1995 Q7, 1996 Q7 parts (a) and (b) only, 1998 Q5, 2000 Q6.

HEMH/DC