ISSUED ON

11 OCT 2013

Engineering Tripos Part IB SECOND YEAR

Part IB Paper 2: Structures

Examples Paper 2/1

Thin-walled structures

*Straightforward questions are marked by †; Tripos standard questions by *.*

Stresses

† 1. The pressurised fuselage of a particular aircraft may be treated as a circular cylinder of diameter 4.0 m, with wall thickness 2 mm; in cruising conditions the differential pressure is 0.6 bar. Find the corresponding hoop and longitudinal stresses due to pressurisation.

As a fail-safe measure, the fuselage is reinforced by hoopwise straps spaced at 0.85 m intervals along its length. Each strap is designed to take (in an emergency) a force equivalent to three times the hoop force in a 0.85 m bay length of fuselage skin. What is the magnitude of the strap design force?

2. (a) † Show that the membrane stress σ (tension +ve) in a thin-walled sphere, of radius r and wall thickness t, under external differential pressure p is $-pr/2t$. (Hint: consider the equilibrium of forces acting on half the spherical shell.)

   (b) † In January 1960, Jacques Piccard, in the bathyscaphe *Trieste*, reached a water depth of 10910 m (in the Challenger Deep of the Pacific Marianas Trench). The pressure vessel of the bathyscaph consisted of an alloy steel sphere of external diameter, 2.18 m, and wall thickness, 120 mm. Use thin-walled theory to estimate the magnitude of the compressive membrane stress in the sphere.

   (c) The wall thickness of the *Trieste* is, in fact, not very thin. Sketch the circular cross-section revealed by cutting the thick-walled sphere in half, showing the areas over which (i) the pressure and (ii) the membrane stresses act. Assuming that the distribution of membrane stress is uniform, calculate the average value of the membrane stress.

   (d) A strain-gauge attached to the inner surface would show that the membrane stress at the inner wall is higher than the average membrane stress calculated. Which of the assumptions of thin-walled theory is being violated?

3. A calculation has shown that a particular member in a structural framework will have to carry a maximum shear force of 12 kN, and a maximum bending moment of 6 kNm, about its major axis of bending. The member is to be a 120 × 60 × 5 mm rectangular hollow section. Calculate the maximum expected (i) longitudinal stress, and (ii) shear stress on a cross-section, due to these loads.

* 4. Figure 1(a) shows one cross section of a steel box-girder bridge. The box is thin-walled, the top flange plate AB is 12 mm thick, and the wall thickness elsewhere is 10 mm. All plate thickness are smeared thicknesses to account for the presence of longitudinal stiffeners (not shown).

A uniform length of bridge, spanning 68 m, is simply supported, as shown in Figure 1(b). In addition to the self-weight, an exceptional load of 1500 kN is concentrated at the centre of the bridge.

(a) † Plot a shear force and bending moment diagram for the bridge, marking salient values.

(b) What is the maximum longitudinal stress anywhere in the span. Where does it occur?

(c) What is the maximum shear stress on a cross-section? Where does it occur?

In practice, what is the effect of longitudinal stiffeners? In addition, there would be transverse diaphragms: why is this, and do they significantly affect the self-weight if equally spaced, say, every 8.5 m?

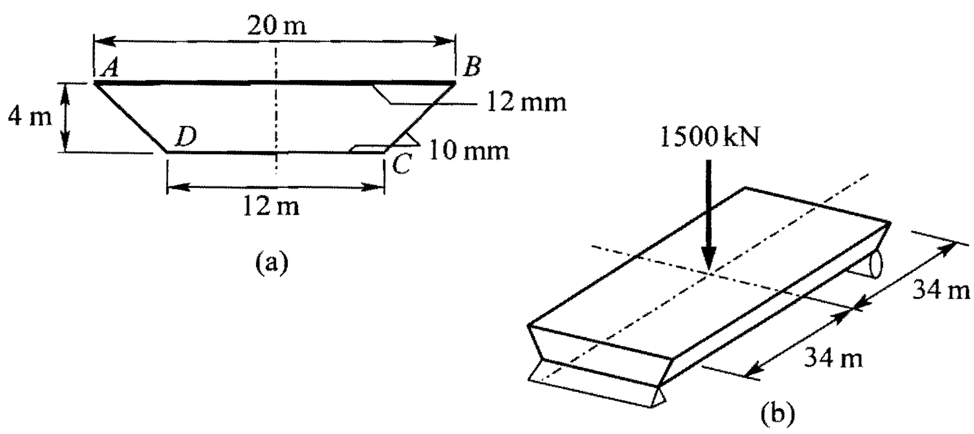

Figure 1

Strains

† 5. A pair of undergraduates are curious to know the pressure in an unopened soft drinks can. They fix a strain gauge hoopwise, mid-way along a new can, and find a strain change of -1250×10^{-6} when the can is opened. They then determine that the can diameter is 66 mm, and that the cylinder wall thickness is 0.12 mm.

The students know that the can is made from an aluminium alloy. What will be their estimate of the original differential pressure?

* 6. A new submersible to reach the bottom of the Challenger Deep (see question 2) is shown schematically in Figure 2. It consists of a cylinder, of median diameter 1.0 m, length 2 m and wall thickness 50 mm, with two hemispherical ends of median diameter 1.0 m. It is made of a ceramic, Alumina. For Alumina, assume a Young's modulus of 390 GN/m^2, and a Poisson's ratio of 0.33.

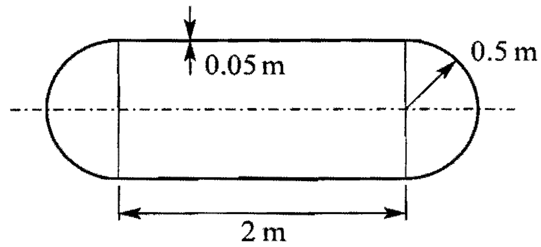

Figure 2

(a) To avoid bending stresses in the walls where the hemispherical ends meet the cylinder, it is necessary to ensure that the cylinder and hemispheres contract radially by equal amounts when the vessel is pressurised. Calculate the required wall thickness of the hemispherical ends to ensure this. Why is it particularly important to avoid bending stresses?

(b) What will be the reduction in the (i) median diameter, (ii) total length and (iii) enclosed volume of the vessel when it reaches the bottom of the Challenger Deep?

7. The long, thin-walled cylindrical tank shown in section (Figure 3) just fits between two rigid immovable end-walls when the gauge pressure is zero. The tank is made from linear-elastic material having Young's modulus, E, Poisson's ratio, ν, and coefficient of thermal expansion, α.

   Estimate the *total* force exerted on each rigid wall when the interior gauge pressure in the tank is p.

   What minimum change in temperature would then reduce the wall force to zero?

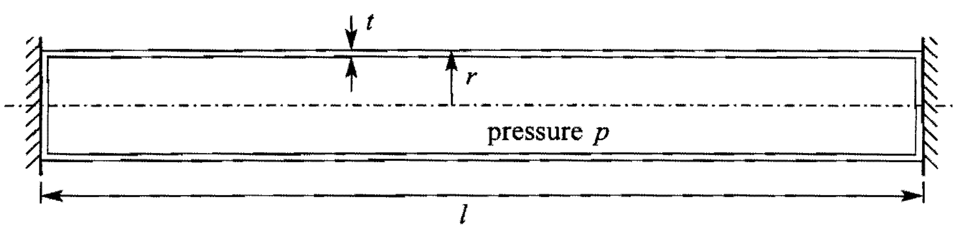

Figure 3

Torsion

8. A steel shaft of length 10 m is required to transmit a torque of 400 kNm without the shear stress exceeding 100 N/mm^2.

   (a) Calculate the minimum required diameter, if the shaft is of solid section. What is the angle of twist using this diameter?

   (b) If, instead, a hollow shaft is used with an external/internal diameter ratio of 4:3, what then is the required external diameter? What is the percentage saving in weight, as compared with the solid shaft?

   (c) What is the ratio of (i) angle of twist and (ii) torsional rigidity (GJ) in the two cases?

* 9. The load on the bridge described in question 4 is now moved to the edge of the bridge, as shown in Figure 4.

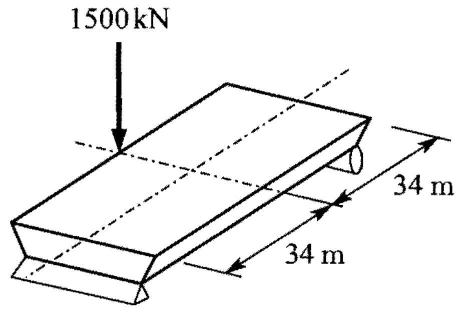

Figure 4

(a) Replace the load shown with an equivalent vertical force and couple acting at the centre of the bridge. What torque is carried by each half of the span?

(b) What is the torsional rigidity of the box section, and is this an under- or over-estimate? Estimate the angle of rotation of the central cross-section. Sketch a graph of the angle of rotation along the span, relative to the supported ends.

(c) What is the maximum shear stress on a cross-section (i) due to the couple alone? Where does it occur? (ii) Due to *all* of the load? Where does it occur?

(d) What is the maximum longitudinal stress due to the total applied load, and where does it occur?

In practice, a section like this would have longitudinal stiffeners to prevent buckling. How would these affect your estimates in parts (c) and (d)?

ANSWERS

1. 60 N/mm^2; 30 N/mm^2; 306 kN.
2. (b) Not less than -486 N/mm^2, depending on radius used. (c) -514 N/mm^2.
3. (i) 120 N/mm^2; (ii) 13.0 N/mm^2.
4. (a) $S_{\max} = 1990$ kN; $M_{\max} = -46.6$ MNm. (b) 79.6 N/mm^2. (c) 26.4 N/mm^2.
5. 0.381 MN/m^2 = 3.8 bar.
6. (a) 20 mm. (b)(i) 2.29 mm; (ii) 3.22 mm; (iii) 11.53×10^{-3} m^3.
7. $p\pi r^2(1-2\nu)$; $-pr(1-2\nu)/2E\alpha t$.
8. 273 mm, 5.2°; 310 mm, 44%; hollow/solid 0.88, 1.13.
9. (a) 7500 kNm. (b) 332×10^6 kNm^2; 44×10^{-3} deg. (c)(i) 5.9 N/mm^2; (ii) 32.3 N/mm^2; (d) 79.6 N/mm^2.

Suitable Tripos questions - will be listed in Examples Sheet 2/2.

K.A. Seffen
October 2013