

## Part IB Paper 3: MATERIALS

## Examples Paper 1: TEACH YOURSELF PHASE DIAGRAMS

This examples paper forms part of an interactive online resource 'Teach Yourself Phase Diagrams' which is available at:

[www-materials.eng.cam.ac.uk/typd](http://www-materials.eng.cam.ac.uk/typd)

You should work through the resource online, answering the relevant examples paper questions when prompted. This structured resource uses sections of illustrated text and animations (in PDF) to lead you through the core concepts and applications of phase diagrams in Engineering. A complete PDF of the content without the animations may be downloaded from the website – this may be useful for later revision, but follow the animations first time through. Each section builds on what has come before, so you must complete each set of exercises to consolidate your understanding, before proceeding to the next topic.

Some numerical answers are provided at the end of this examples paper. Fully worked solutions will be made available online in due course.

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October 2013

**This online resource is a work-in-progress. All feedback would be welcome:  
email [hrs@eng.cam.ac.uk](mailto:hrs@eng.cam.ac.uk)**

**PART 1: Introduction**

1. A 1.5 kg sample of  $\alpha$ -brass contains 0.45 kg of Zn, and the rest is Cu. The atomic weight of copper is 63.5 and zinc 65.4. Write down the concentration of copper in  $\alpha$ -brass, in wt%,  $W_{Cu}$ . Find the concentrations of copper and zinc in the  $\alpha$ -brass, in at%,  $X_{Cu}$  and  $X_{Zn}$ .
2. A special brazing alloy contains 63 wt% gold (Au) and 37 wt% nickel (Ni) (which is written Au-37wt% Ni). The atomic weight of Au (197.0) is more than three times that of Ni (58.7). At a glance, which of the two compositions, in at%, is more likely to be correct?  
(a)  $X_{Au} = 0.34$ ,  $X_{Ni} = 0.66$   
(b)  $X_{Au} = 0.66$ ,  $X_{Ni} = 0.34$
3. An alloy consists of  $X_A$  at% of A with an atomic weight  $a_A$ , and  $X_B$  at% of B with an atomic weight  $a_B$ . Derive an equation for the concentration of A in wt%. By symmetry, write down the equation for the concentration of B in wt%.
4. On heating, pure copper starts to melt at 1083°C. While it is melting, solid and liquid copper co-exist. Using the definition of a phase, are one or two phases present? Explain why.
5. Three components A, B and C of an alloy dissolve completely when liquid but have no mutual solubility when solid. They do not form any chemical compounds. How many phases, and of what compositions, do you think would appear in the solid state?



## PART 2: Simple Phase Diagrams

6. Use the Pb-Sn diagram in Figure 1 to answer the following questions.

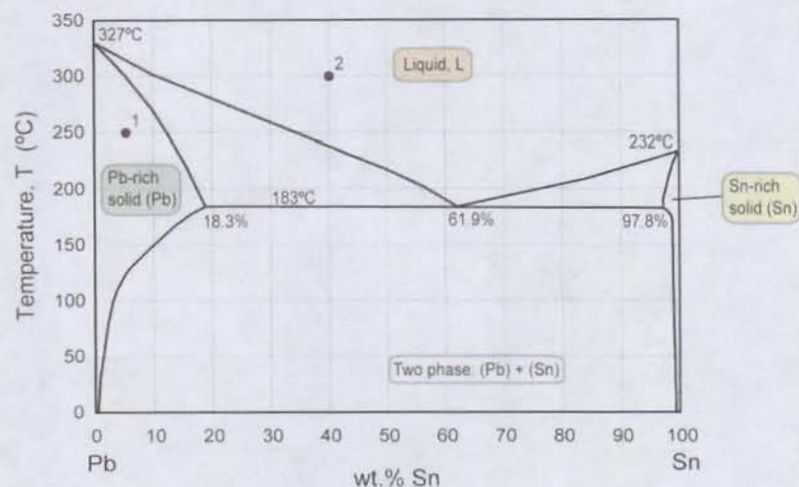


Figure 1: Pb-Sn phase diagram

- What are the values of the state variables (composition and temperature) at constitution points 1 and 2, and what phases are present?
- Mark the constitution points for Pb-70wt% Sn and Pb-95wt% Sn alloys at 210°C. What phases are present in each case?
- The alloy at constitution point 1 is cooled very slowly to room temperature, maintaining equilibrium. Identify the temperature of the phase boundary at which a change in the phases occurs. What phase(s) is/are present below the boundary?
- The alloy at constitution point 2 is cooled slowly to room temperature. Identify the temperatures at which phase changes occur, and the phase(s) before and after each change.

7. Use the Pb-Sn diagram in Figure 2 to answer the following questions.

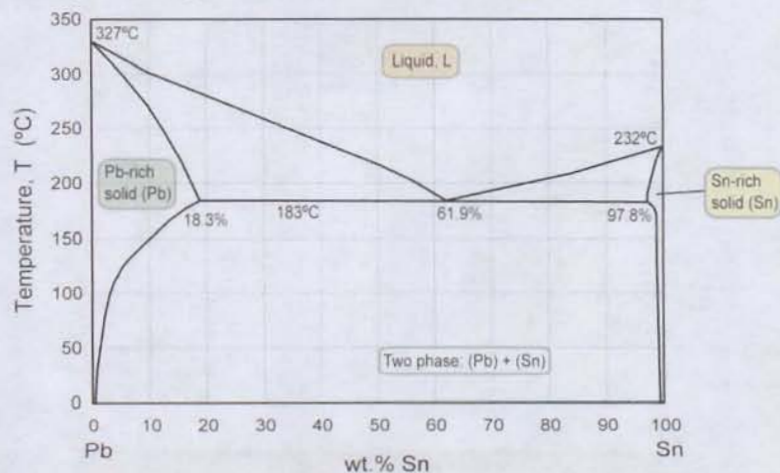


Figure 2: Pb-Sn phase diagram

- The constitution point for a Pb-25wt% Sn alloy at 250°C lies in a two-phase field. Construct a tie-line on the figure and read off the two phases and their compositions.
- The alloy is slowly cooled. Identify the phases and their compositions: (i) at 200°C; (ii) at 150°C.
- Indicate with arrows on the figure the lines along which the compositions of the two phases move during slow cooling from 250°C to 200°C. The overall composition of the alloy stays the same, of course. How can this be maintained when the compositions of the phases change?



8. Derive the lever rule for a general mixture of two phases,  $\alpha$  and  $\beta$  (Figure 3).

Let the composition of the alloy be  $W_B$  (wt% of alloying element), the compositions of the phases be  $W_B^\alpha$  and  $W_B^\beta$ , and the weight fractions of the phases be  $F^\alpha$  and  $F^\beta$ .

[Hints: find an expression conserving the mass of the alloying element between the alloy and the two phases, then define  $a$ ,  $b$  and  $\ell$  in this notation, and use the overall conservation of mass:  $F^\alpha + F^\beta = 1$ ].

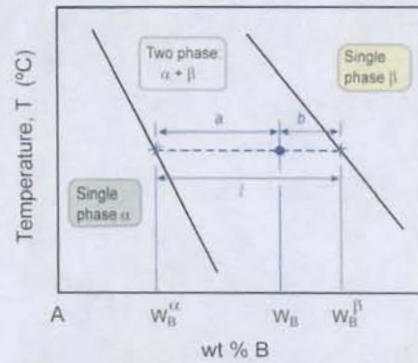


Figure 3: The lever rule

9. Using Figure 2, consider the Pb-Sn alloy with composition  $W_{Sn} = 25\%$ . What are the approximate proportions by weight of the phases identified in Question 7, at  $250^\circ\text{C}$ ,  $200^\circ\text{C}$  and  $150^\circ\text{C}$ ?

10. Use Figure 4 to answer the following:

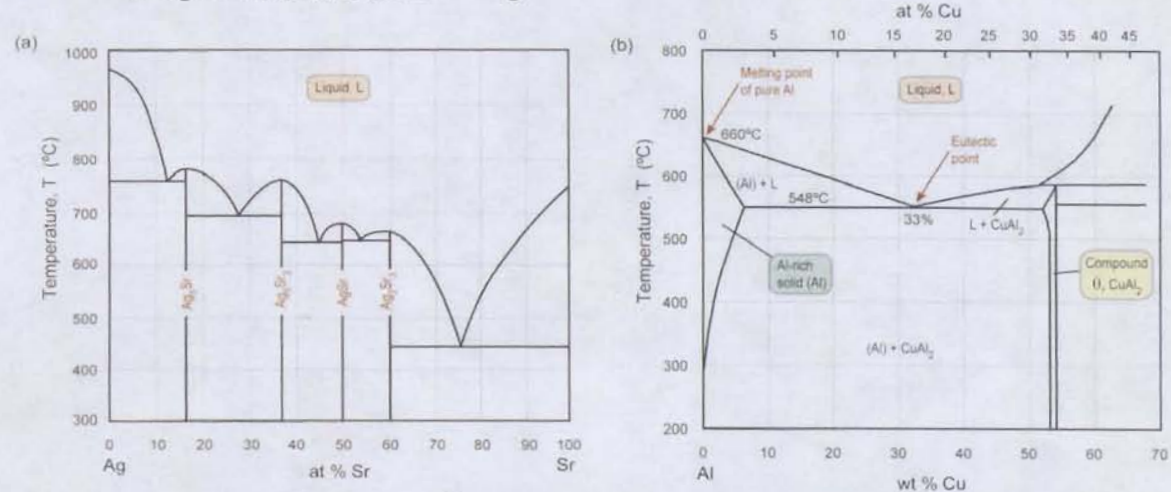


Figure 4: Phase diagrams showing intermediate compounds:  
(a) the silver–strontium Ag-Sr system; (b) part of the aluminium–copper Al-Cu system.

- (a) For an Ag-90at% Sr alloy at  $600^\circ\text{C}$ :
  - (i) Plot the constitution point on the phase diagram;
  - (ii) Identify the phases present, and find their compositions in at%;
  - (iii) The temperature is slowly reduced to  $500^\circ\text{C}$ . Will the phase compositions and proportions change?
- (b) For an Ag-30at% Sr alloy at  $600^\circ\text{C}$ :
  - (i) Plot the constitution point on the phase diagram;
  - (ii) Identify the phases present, and find their compositions in at%;
  - (iii) Will the proportions change if the temperature is reduced to  $500^\circ\text{C}$ ? Why is this?
- (c) The atomic weight of Ag is 107.9 and that of Sr is 87.6. Calculate the compositions of the four intermetallic compounds in the Ag-Sr system in *weight%*.
- (d) For an Al-4wt% Cu alloy:
  - (i) Calculate the composition in at% Cu (atomic masses of Al and Cu: 26.98 and 63.54 respectively);
  - (ii) At  $550^\circ\text{C}$ , identify the phase(s) present, and find their compositions (in wt%) and proportions by weight;
  - (iii) Repeat for  $250^\circ\text{C}$ .

**PART 3: The Iron-Carbon Phase Diagram**

11. Use Figure 5 to answer the following:

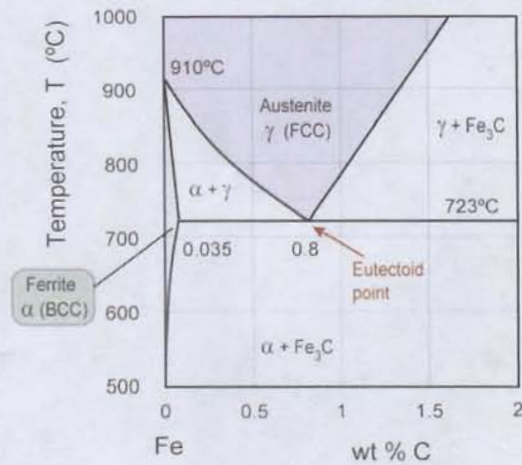


Figure 5: Expanded view of part of the Fe-C phase diagram, showing the eutectoid point.

- (a) For a Fe-0.4wt% C alloy at 900°C and 600°C:
- Plot the constitution points on the phase diagram;
  - Identify the phases present, and find their compositions in wt%;
  - If the temperature is slowly reduced from 900°C to 600°C, at what temperatures are phase boundaries crossed? Identify the phases present after each boundary is crossed.
- (b) How does slow cooling from 900°C to 600°C differ for a Fe-0.8wt% C alloy?

**PART 4: Interpreting More Complex Phase Diagrams**

12. The two phase diagrams (or parts of diagrams) below both have a eutectic point. Mark the eutectic point on each figure, and find the eutectic temperature and composition *in wt%* in each system. (Note: the atomic masses of Cr and Ni are 52.00 and 58.71 respectively).

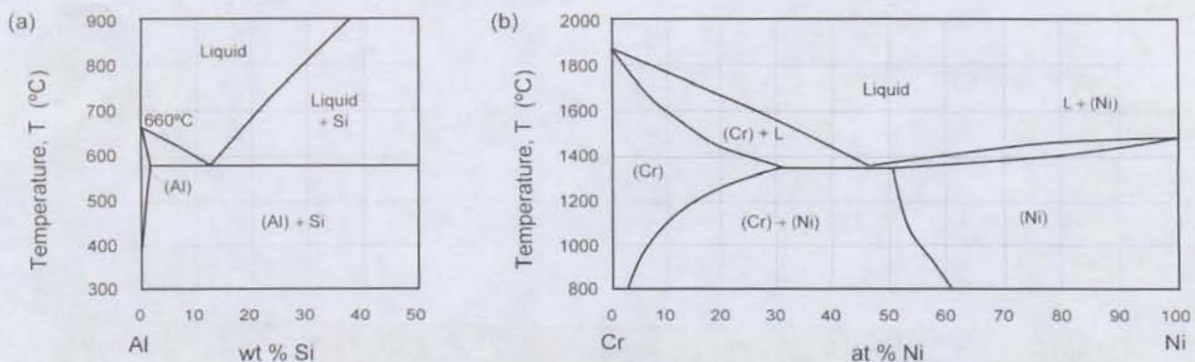


Figure 6: aluminium-silicon Al-Si and chromium-nickel Cr-Ni phase diagrams



13. The phase diagram for the copper-zinc system (which includes *brasses*) is shown in Figure 7. Use the diagram to answer the following questions.

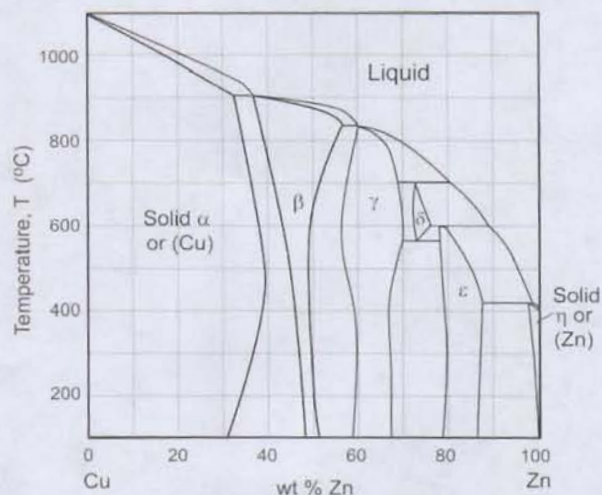


Figure 7: copper-zinc Cu-Zn phase diagram

The two common commercial brasses are: 70/30 brass:  $W_{Cu} = 70\%$ , and 60/40 brass:  $W_{Cu} = 60\%$ . Locate their constitution points on the diagram at 200°C.

- What distinguishes the two alloys?
- What roughly is the melting point of 70/30 brass?
- What are the phases in 60/40 brass at 200°C? Find their compositions and proportions.

14. The copper-tin system (which includes *bronzes*) is shown in Figure 8.

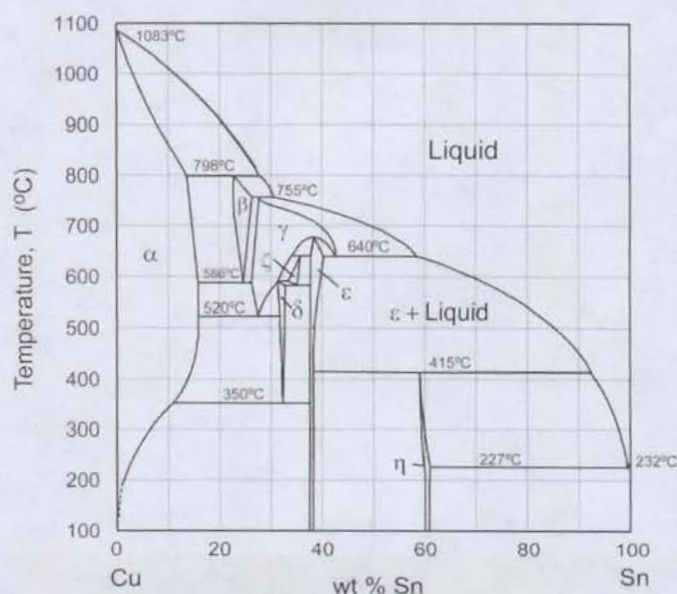


Figure 8: copper-tin Cu-Sn phase diagram

- Shade the single phase regions;
- Highlight the four eutectoids in this system, and write down their compositions and temperatures.

15. Figure 9 shows the Ti-Al phase diagram (important for the standard commercial alloy Ti-6% Al-4% V).

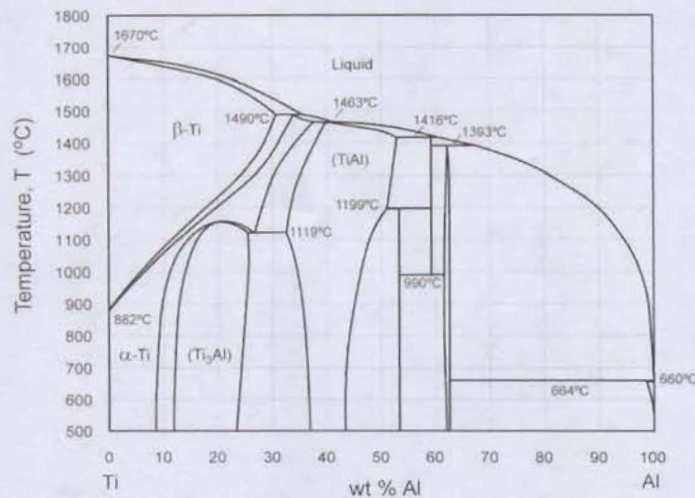


Figure 9: titanium-aluminium Ti-Al phase diagram

(NB: this is a more up-to-date and complicated version than that in the Materials Databook)

- Shade all single-phase fields and highlight three compounds with well-defined compositions. Find the formula for these three compounds (atomic weights of Ti and Al are 47.90 and 26.98 respectively). (Note that two other single phase fields are shown with a compound formula in brackets – i.e. these fields are solid solutions showing a spread of composition around that of each compound).
- Ring the five peritectic points, one peritectoid point, and two eutectoid points.
- On heating, over what temperature range does a Ti-6wt% Al alloy change from  $\alpha$ -Ti (HCP) to  $\beta$ -Ti (BCC)? Over what temperature range does it melt?

16. Use the phase diagram for the  $\text{SiO}_2$ - $\text{Al}_2\text{O}_3$  system in Figure 10 to answer the following:

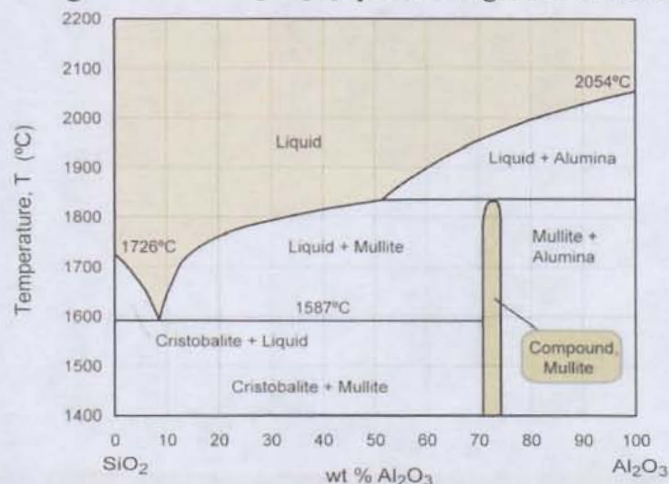


Figure 10: Phase diagram for the binary ceramic silica-alumina ( $\text{SiO}_2$ - $\text{Al}_2\text{O}_3$ ) system

- The intermediate compound mullite may be considered as having the formula  $\text{SiO}_2(\text{Al}_2\text{O}_3)_x$ . Find the approximate value of  $x$ . The atomic masses of Si, Al and O are 28.1, 26.9 and 16.0 respectively.
- Use the lever rule to find the equilibrium constitution of a 50 wt%  $\text{Al}_2\text{O}_3$  alloy at 1700°C. Is it valid to measure directly from the diagram in this case? Why?



**PART 5: Phase Transformations and Microstructure Evolution (for Slow Cooling)**

Use the Pb-Sn phase diagram in Figure 11 to answer questions 17-20.

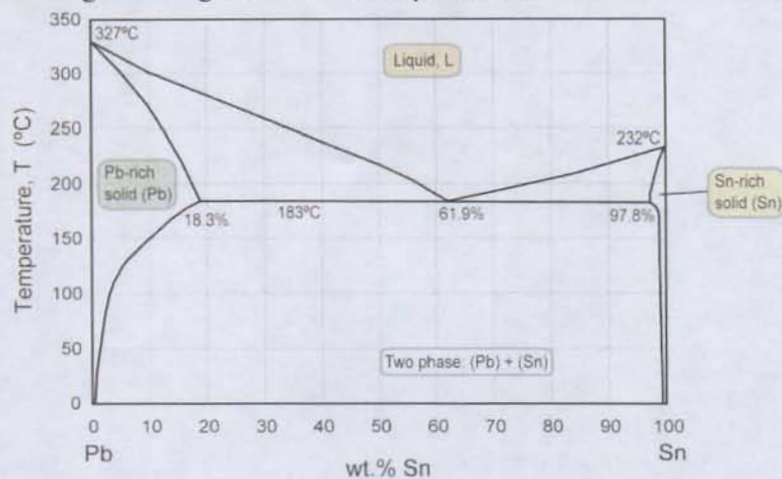


Figure 11: Pb-Sn phase diagram

17. Find the proportions and compositions of the phases formed after solidification of a Pb-10wt% Sn alloy: (i) at 250°C, and (ii) at room temperature (20°C).
18. Find the proportions and compositions of the solid phases formed on solidification at the eutectic point in the Pb-Sn system.
19. Not all alloys in the lead-tin system show a eutectic reaction on slow cooling: pure lead, for example, does not. Examine the Pb-Sn phase diagram and identify the composition range for which you would expect a eutectic reaction to be possible.
20. (a) A eutectic reaction was defined in the text. Define what happens on heating a solid of eutectic composition. Over what temperature range does melting occur?  
(b) For a Pb-35wt% Sn alloy, identify the temperatures at which melting starts and finishes on heating, and describe how the proportion of liquid evolves during melting.

21. Figure 12 shows part of the phase diagram for the Al-Si system (the basis of aluminium casting alloys). Use this figure to answer the following:

- (a) Describe the solidification of an Al-20wt% Si alloy, sketching the microstructure at key temperatures (e.g. onset of solidification, just above and just below the eutectic). Estimate the phase proportions in two-phase regions, and sketch the microstructures accordingly.
- (b) How would the final microstructure differ for Al-Si alloys of:
  - (i) Al-1wt% Si?
  - (ii) Eutectic composition?

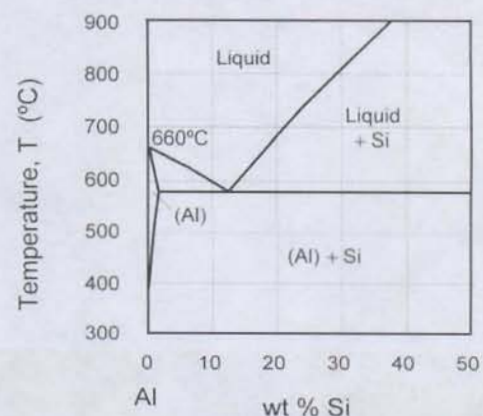


Figure 12: aluminium-silicon Al-Si phase diagram

22. Use Figure 13 to answer the following:

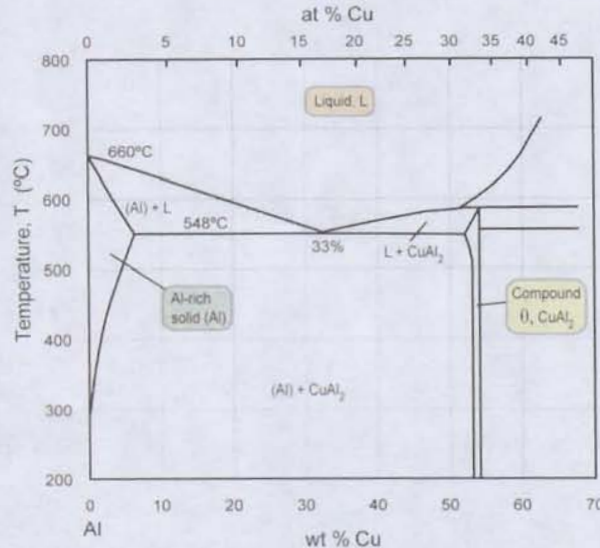


Figure 13: Phase diagram for part of the aluminium–copper Al-Cu system.

Al-10wt% Cu alloy was slow cooled and the length scales measured by various techniques in the final microstructures, as follows: primary grain size =  $100\mu\text{m}$ ; phase spacing in the eutectic =  $1\mu\text{m}$ ; precipitates within the primary grains  $0.05\mu\text{m}$ . Identify the phases involved in each of these microstructural features, and explain the differences in length scale.

23. Pure iron cooled slowly contains 100% ferrite; the eutectoid composition contains 100% pearlite. Estimate the carbon content of the hypo-eutectoid steel shown in Figure 14, and sketch the structure of a 0.2wt% carbon steel after slow cooling to room temperature.

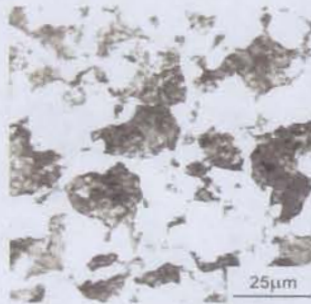


Figure 14: Optical micrograph of the ferrite-pearlite microstructure in a hypo-eutectoid carbon steel. (Image courtesy of ASM Micrograph Center, ASM International, 2005)

24. *Hyper-eutectoid* steels contain  $> 0.8\text{wt}\% \text{ C}$ . A high carbon steel containing  $1.0\text{wt}\% \text{ C}$  is cooled slowly from  $1000^\circ\text{C}$  in the austenite field. Refer to the Fe-C phase diagram in Figure 15 to answer the following:

- At what temperature does a phase transformation begin, and what new phase then appears? Why do you think this phase tends to nucleate and grow along the austenite grain boundaries?
- What happens to the compositions of the phases on cooling to just above  $723^\circ\text{C}$ ?
- What phase transformation takes place at  $723^\circ\text{C}$ ?
- What are the final phases at room temperature, their compositions and proportions? Sketch the expected final microstructure.



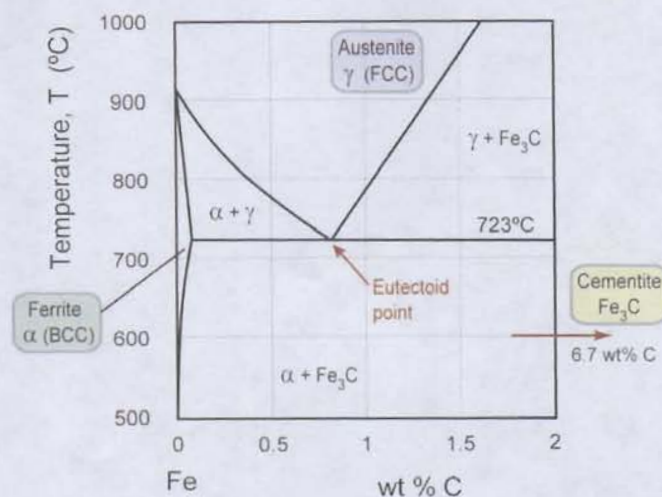


Figure 15: The Fe-C phase diagram for compositions of carbon steels below 1000°C, including the eutectoid point.

### Final Problem

- \* 25. Figure 16 shows the copper-antimony Cu-Sb phase diagram.

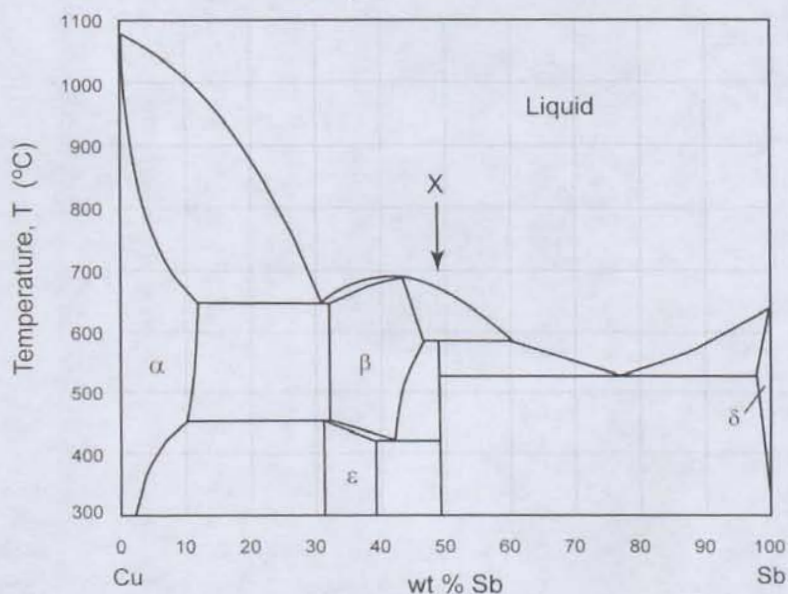


Figure 16: copper-antimony Cu-Sb phase diagram

- Find the chemical formula for the compound marked X (atomic weights of Cu and Sb are 63.54 and 121.75 respectively).
- The Cu-Sb system contains 2 eutectics, 1 eutectoid, 1 peritectic and 1 peritectoid. Mark them all on the figure, write down the temperature and composition of each point, and identify the phases involved in each reaction, on cooling.
- An alloy containing 95 wt% Sb is cooled slowly to room temperature from the melt. Describe the phase changes that occur during cooling, using schematic sketches of the microstructure at key temperatures to illustrate your answer.
- Sketch a temperature-time curve for the 95 wt% Sb alloy over the range 650 to 450°C and account for the shape of the curve.

**Numerical Answers**

1.  $W_{Cu} = 70\%$ ,  $X_{Cu} = 71\%$ ,  $X_{Zn} = 29\%$
6. (a) Pb-5wt% Sn, 250°C; Pb-40wt% Sn, 300°C; (c) 125°C; (d) 235°C, 183°C.
7. (a) 250°C: Pb-rich solid (12wt% Sn) and Liquid (34wt% Sn).
9. (a) 250°C: 41%, 59%; (b) 200°C: 82%, 18%; (c) 150°C: 83%, 17%
10. (c)  $Ag_3Sr$  : 14.0wt% Sr;  $Ag_5Sr_3$  : 32.8wt% Sr;  $AgSr$  : 44.8wt% Sr;  $Ag_2Sr_3$  : 54.9wt% Sr  
(d) (i)  $X_{Cu} = 1.74\text{at\% Cu}$
11. (a) 900°C: 0.4wt%C ; 600°C: effectively 0wt%C, 6.7wt%C.
12. 12 wt% Si, 50 wt% Ni.
15. (a)  $TiAl_2$  ( $\approx 53$  wt% Al);  $Ti_2Al_5$  ( $\approx 59$  wt% Al);  $TiAl_3$  ( $\approx 62$  wt% Al).  
(c)  $\approx 980 - 1010^\circ\text{C}$ ;  $\approx 1650 - 1660^\circ\text{C}$ .
16. (a)  $x \approx 1.6$
25. (a)  $Cu_2Sb$

**Relevant IB Tripos Questions from previous years**

- 2004: Q.4  
2005: Q.6  
2006: Q.1, 6(b)  
2007: Q.1  
2008: Q.1(a,b)  
2009: Q.2  
2010: Q.2(c), 4  
2011: Q.2  
2013: Q.4