ISSUED ON

25 OCT 2013

Engineering **SECOND YEAR**

Part 1B Paper 4: Thermofluid Mechanics

FLUID DYNAMICS

Examples Paper 2

Starter questions are marked 'S', straightforward questions are marked '†', Tripos standard are marked '*'.

Pipe flow and network analysis

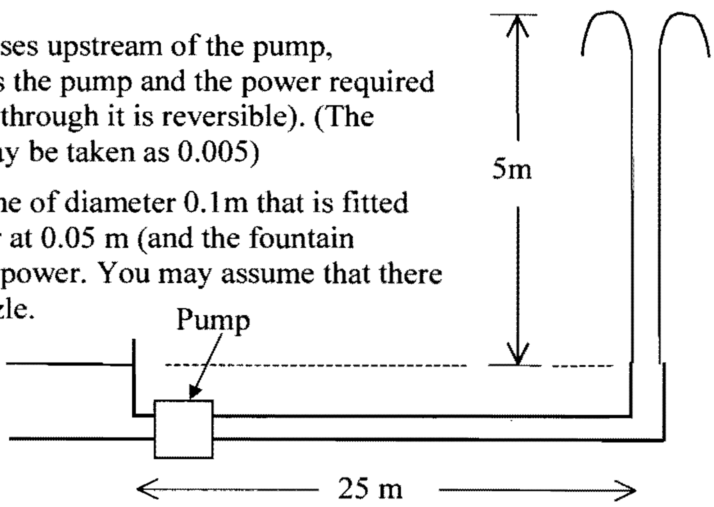
Read lectures 5 and 6.

S Explain what is meant by 'static pressure', 'stagnation pressure' and 'total pressure'. Give a physical interpretation of each of the three terms in the 'total pressure'.

1 Water is supplied from a reservoir to a fountain through a pipe of diameter 0.05 m, as shown.

a) Ignoring all mechanical energy losses upstream of the pump, calculate the total pressure rise across the pump and the power required by the pump (assuming that the flow through it is reversible). (The coefficient of friction for all pipes may be taken as 0.005)

b) If the supply pipe is replaced by one of diameter 0.1m that is fitted with a nozzle to keep the jet diameter at 0.05 m (and the fountain height the same), calculate the pump power. You may assume that there is no loss of total pressure in the nozzle.



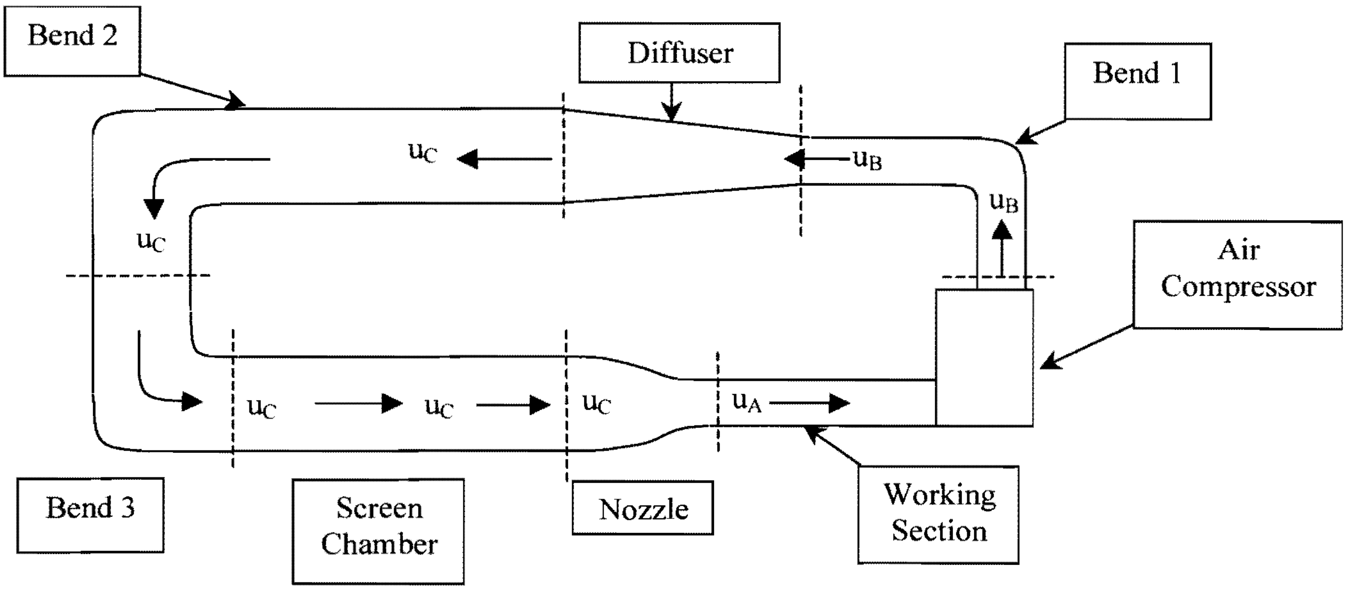
c) For a given volumetric flow rate, how does the rate of dissipation of mechanical energy in the pipe depend on the pipe diameter ?

d) Why is it safe to assume that the exit loss at the nozzle is zero?

2† The figure on the next page shows a low-speed wind tunnel in which air is pumped around a closed circuit by a compressor. The velocity of the air may be taken to be uniform at any cross-section of the flow but varies around the circuit as follows: $u_A = 48\,\mathrm{m/s}$, $u_B = 65\,\mathrm{m/s}$, $u_C = 16\,\mathrm{m/s}$. The density of the air may be taken as constant and equal to $1.2\,\mathrm{kg/m^3}$. Total pressure losses in the components listed below can be expressed in the form

$$\Delta p_{tot} = \frac{1}{2}\rho u^2 K$$

where K is a constant for each item and where u is the velocity at entry to the item, except for the case of the nozzle. The values of K are: bend 0.25; diffuser 0.35; screen chamber 3.8; nozzle 0.15 (based on exit velocity). Determine the total pressure rise across the compressor.



3* The figure below shows a water distribution system from a low reservoir to two high reservoirs. You may assume that losses at the pump entrance and exits are negligible, that the exit loss coefficient at pipes 2 and 3 is 1.0, and that the entrance loss at pipe 1 is negligible.

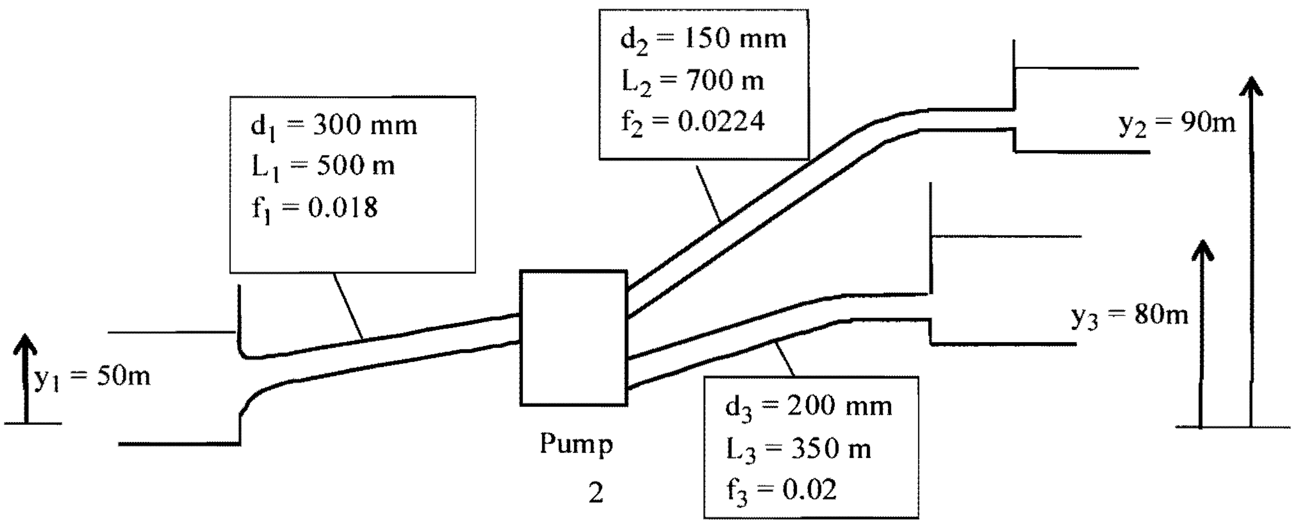
a) If the total pressures delivered by the pump to pipes 2 and 3 are equal, show that

$$V_2^2 = \frac{\left(1.0 + f_3 \frac{L_3}{d_3}\right) V_3^2 + 2g(y_3 - y_2)}{1.0 + f_2 \frac{L_2}{d_2}}$$

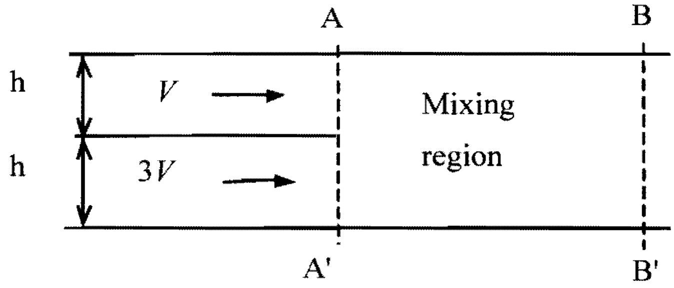
where V_2 and V_3 are the velocities of the flow in pipes 2 and 3 and f_2 and f_3 are the friction factors. (All losses associated with pipe bends may be neglected).

b) Find the total flow rate through the pump when that through pipe 3 is 0.115 m^3/s.

c) Find the rise in total pressure across the pump and the power required, assuming that the pump is reversible.



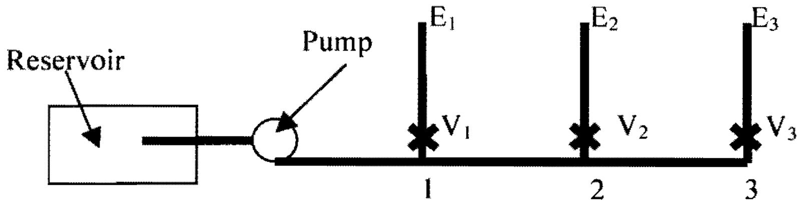
4. Two parallel streams of an incompressible fluid flowing in horizontal rectangular ducts of height h and depth d come together at the location AA' as shown in the figure. They have the same static pressure p_A and speeds V and $3V$ respectively. The two streams mix over a short distance due to the turbulence generated by the unstable shear layer between them.



a) Assuming that viscous shear stress on the solid surfaces can be neglected, calculate the velocity and static pressure at location BB', where the mixing is complete.

b) What is the rate of loss of mechanical energy due to the mixing ?

5* The figure below shows an irrigation system, which lies in a horizontal plane. The reservoir contains water of density ρ. The pipe from the reservoir to the pump is short, and there are negligible losses at the pipe's entry. The increase in total pressure across the pump is $\Delta p_{tot,\text{pump}}$. All pipes have the same internal diameter, d, with friction coefficient, c_f, and all pipes downstream of the pump (i.e. 6 sections in all) have the same length L. You may assume that there is no change in total pressure through the pipe junctions and the bend.



The valves (V_i) are mounted in the three "delivery" pipes, which have open ends at the points E_i, also of inner diameter d, where the water is ejected into open space (i.e. there is no exit loss at the pipe exit). The flow from the reservoir is Q, and the valves are arranged so that the flow in each delivery pipe is the same (i.e. $Q/3$), and so that the pumping power is minimised. When a valve is fully open, its loss coefficient is zero. Otherwise, the total pressure loss across each valve is proportional to Q^2. The substitutions $k = 4c_f L/d$ and $n = \rho/(2A^2) = 8\rho/(\pi^2 d^4)$ are convenient.

(a) Explain why valve V_3 should be fully open and then, by considering the change in total pressure of the flow between the reservoir and the exit at E_3, show that the total pressure rise across the pump can be expressed as $\Delta p_{tot,\text{pump}} = nQ^2(1+15k)/9$.

(b) By considering the flows that exit at E_1 and E_2, obtain expressions for the total pressure drops across valves V_1 and V_2.

(c) With the valve openings unchanged, what is the increase in the flow in the delivery pipes if $\Delta p_{0,\text{pump}}$ increases by 50%?

Boundary Layers and Drag

Read lectures 7 and 8.

S. A 'laminar flow wing' is an aircraft wing over which the boundary layer remains laminar. Why is this so desirable? Why is it so difficult to achieve?

6. The velocity in the laminar boundary layer on a flat plate in an unbounded stream can be modelled by the expression

$$v_x = \frac{V}{2}\left[3\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^3\right] \qquad 0 \le y \le \delta \qquad\qquad v_x = V \;\text{ for }\; y \ge \delta$$

where V is the free stream velocity and δ is the height of the edge of the boundary layer above the plate. This form is assumed to be valid at each value of x along the length of the plate with δ varying with x.

a) Explain why the static pressure is approximately uniform throughout the flow and sketch v_x / V as a function of y/δ.

b) Using the result derived in lectures for mass flow in the boundary layer, find the vertical displacement of the streamline that passes the plate leading edge at a height h in terms of δ.

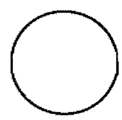
c) Again using results from lectures, calculate the total drag force per unit width on the upper side of the plate. Also calculate the boundary layer thickness at the trailing edge for a 1m plate in a stream of 5 ms^{-1} with fluid properties $\mu = 1.0 \times 10^{-3}$ kg m^{-1}s^{-1}, $\rho = 10^3$ kg m^{-3}.

d) Physically, why does the boundary layer thickness grow in proportion to the square root of x?

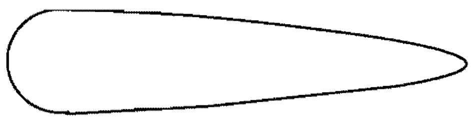
e) Does the analysis in (b) and (c) differ for a turbulent boundary layer? Explain your answer.

7† Sketch the streamlines over the two (two-dimensional) bodies shown when they are inserted into a uniform ideal, *inviscid* flow from left to right. (The nose of the streamlined body is circular with the same radius as the cylinder). Mark on the sketch the regions of highest and lowest static pressure. By considering the likely size of any adverse pressure gradients in these ideal flows, explain why the drag on the streamlined body is much less than that on the cylinder for high Reynolds flow of a viscous fluid over them from left to right.

Which body will have the least drag for low Reynolds Number flow (i.e. $Re \ll 1$) ?

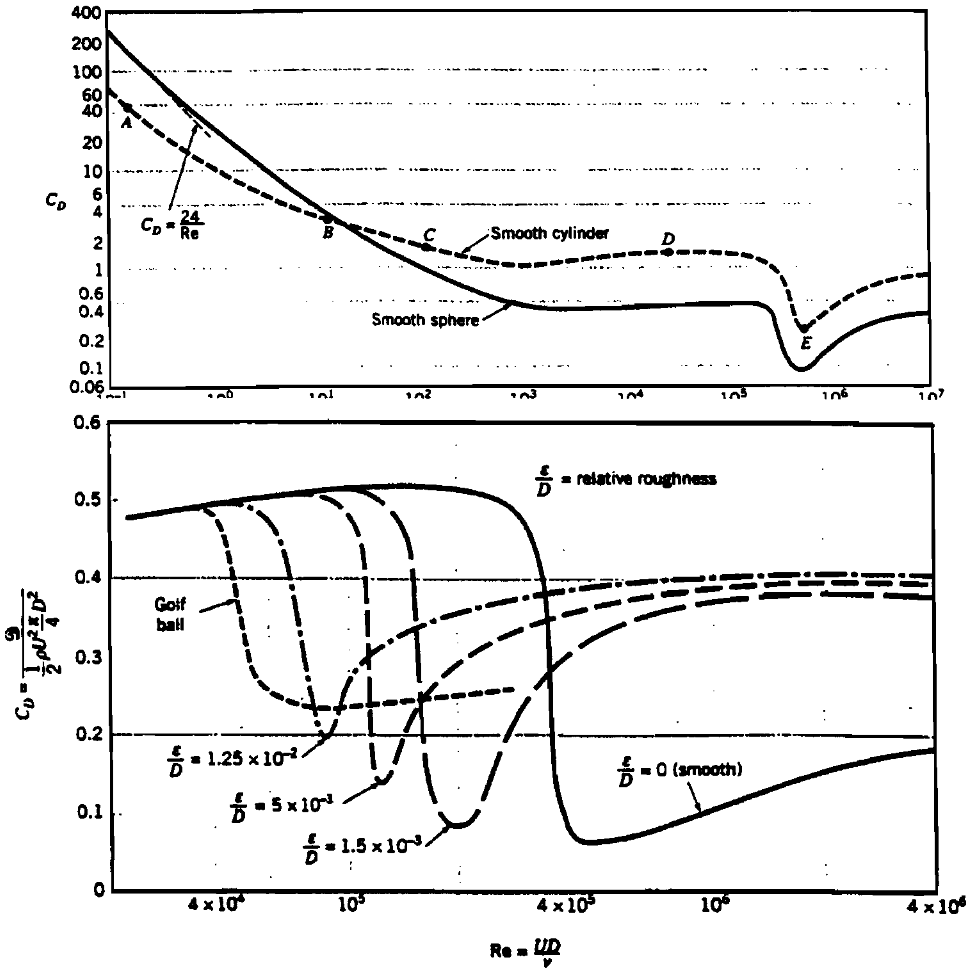


cylinder



streamlined body

8 The measured drag coefficient for a sphere as a function of Reynolds Number is as shown. (a) Using a scaling argument, explain why you would expect the drag coefficient to be proportional to Re^{-1} for low values of Re. (b) Explain why there is a pronounced dip in the value of C_D at high Re.



The following table gives typical speeds and sizes of balls used in various sports. Assuming that the kinematic viscosity ($\nu = \mu/\rho$) of air is 1.5×10^{-5} m^2s^{-1}, calculate typical Reynolds Numbers for the various sports.

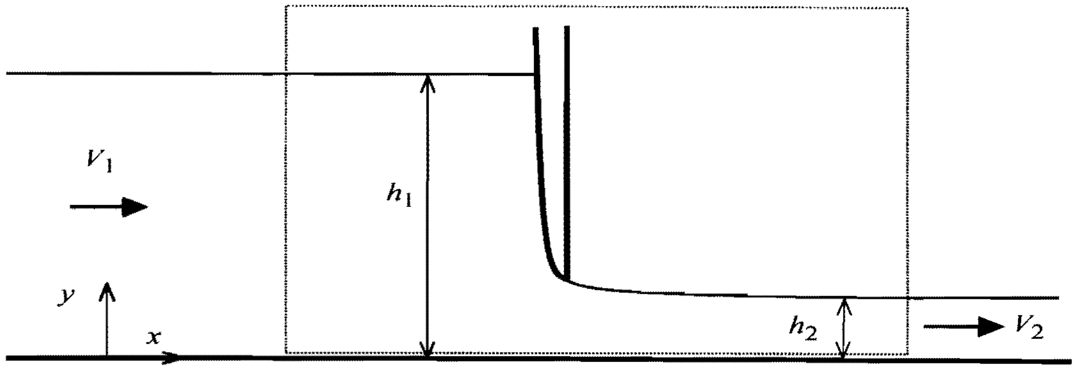
	Golf	Cricket	Football	Tennis	Baseball
U (ms^{-1})	70	40	16	50	45
D (m)	.043	.068	.19	.064	.075

The effect of surface roughness (defined as the ratio of roughness height to diameter) on C_D is also shown for a particular Reynolds Number range. Explain why roughness has this effect.

Discuss why these balls all have some sort of surface treatment. Why is the new ball, which is smoother, usually reserved for fast bowlers in cricket ? How do baseball pitchers and cricket bowlers make a ball swing in flight. Why do cricketers wish to polish one side only of a cricket ball ? Is there any market for a table tennis ball with dimples ?

Similarity and Model Testing

Read lecture 9.



9 (a) Water flows under the sluice gate shown above. If the effects of viscosity are neglected, show using dimensional reasoning that the x-component of force per unit width, F, on the sluice gate must satisfy

$$\frac{F}{\frac{1}{2}\rho V_1^2 h_1} = fn\left(\frac{h_1}{h_2}, Fr\right)$$

where the Froude number Fr is given by $Fr = \dfrac{V_1}{\sqrt{gh_1}}$.

(b) Explain why there is no dependence on the size of the gate or upon V_2 in this expression.

(c) Show that the square of the Froude number is proportional to the ratio of the dynamic pressure to the hydrostatic pressure.

(d) Consider the force on the gate as the sum of a dynamic force and a hydrostatic force. Explain physically why you would expect there to be no dependence on Fr as $Fr \to \infty$.

(e) Use a scaling argument to show that, at low values of Fr,

$$\frac{F}{\frac{1}{2}\rho V_1^2 h_1} \propto Fr^{-2}.$$

(f) By applying the Steady Flow Momentum Equation to the control volume shown, find F and verify the results of (a) to (d). (It may help to look at your notes from 1A for this part).

10. A dam spillway is a channel through which the water behind the dam can overflow. A prototype spillway is 20 m wide and is being designed to carry 125 m^3/s during floods. A 1:15 scale model is constructed to study the behaviour of the flow through the spillway during a flood. If the effects of surface tension and viscosity can be neglected, determine the required model width and flow rate. If the spill is to allow a certain volume of flood water to drain in a 24 hr period, what is the corresponding operating time of the model ?

11.*A ship is 70 m long and is designed to travel at a speed of 7 m/s. A model is made to 1/20 scale and is tested at the Froude number corresponding to that of the full-scale ship. Calculate the test speed.

The engineers running the test assume that

(i) the drag can be considered as the sum of two independent terms, one due to wave drag and the other due to fluid mechanical drag (i.e. viscous drag and form drag).

(ii) the drag coefficient due to fluid mechanical drag is proportional to $Re^{-1/5}$ where Re is the Reynolds number based on length.

The total measured drag on the model is 15 N, and from other tests on the model the fluid mechanical drag is estimated to be half of this value.

Estimate the drag on the ship.

Answers

1. 5.395 bar, 10.5 kW; 1.25 kW; diameter $^{-5}$
2. 2389 Pa
3. 0.144 m^3/s ; 597 kPa; 86 kW
4. $2V$; $p_B = p_A + \rho V^2$; $2\rho d h V^3$
5. $knQ^2/9$; $5knQ^2/9$
6. (b) $3\delta/8$ (c) 7.24 N/m
7. -
8. 2.0×10^5 ; 1.8×10^5 ; 2.0×10^5 ; 2.1×10^5 ; 2.3×10^5
9. (e) $\dfrac{F}{\frac{1}{2}\rho V_1^2 h_1} = 2\left[1 - \dfrac{h_1}{h_2}\right] + \dfrac{g h_1}{V_1^2}\left[1 - \dfrac{h_2^2}{h_1^2}\right]$
10. 1.33 m; .143 m^3/s; 6.2 hr
11. 84 kN

MPJ Michaelmas 2013