Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Example Paper 2

ISSUED ON
25 OCT 2013

Straightforward questions are marked †
Tripos standard questions are marked *

Pole locations

1. Classify the following transfer functions as asymptotically stable, marginally stable, or unstable:

   (a) $1/(s+3)$ (b) $(s-1)/(s+3)$ (c) $1/(s-3)$
   (d) $1/s$ (e) $1/s^2$ (f) $(2s+3)/(s^2+8)$
   (g) $(2s+3)/(s^2-8)$ (h) $10/[(s+5)(s^2-s+1)]$ (i) $10e^{-2s}/[(s+5)(s^2+s+1)]$

   In each case draw the pole–zero diagram.

2. † Figure 1 shows, in the left–hand column, the pole locations of some second–order systems. The right–hand column shows the impulse responses of these systems, but *not in the same order*. Match the diagrams in the left–hand column with the corresponding diagrams in the right–hand column.

3. * The U.S. military specification 'Flying Qualities of Piloted Airplanes' (MIL–F–8785C) lays down the following requirements for the damping factor (ζ) and undamped natural frequency (ω_n) of the so–called 'Dutch Roll' mode of oscillation:

   Flight phase A: $\zeta \geq 0.4$, $\omega_n \geq 1.0$ rad/sec, $\zeta\omega_n \geq 0.35$ rad/sec.
   Flight phase B: $\zeta \geq 0.08$, $\omega_n \geq 0.4$ rad/sec, $\zeta\omega_n \geq 0.15$ rad/sec.

   Assume that, for this mode of oscillation, an aircraft can be modelled as a second–order transfer function, with an atmospheric disturbance as input, and the aircraft's heading as output. (Flight phase A includes reconnaisance and terrain following, while Flight Phase B includes cruising and in-flight refuelling.)

   For each flight phase show, on a diagram of the complex plane, the corresponding regions where the poles of the transfer function may *not* lie in order to satisfy each constraint individually. Describe the behaviour of the aircraft's heading in response to a short–term disturbance, in each of the flight phases (if the full specification is satisfied).

Frequency response

4. † A sinusoidal input is applied to a system with transfer function

$$\frac{1}{(s+1)(s+0.1)}$$

   At which frequency does the output lag behind the input by 90°? If the input amplitude is X, what is the output amplitude at this frequency?

5. Heat propagation in a semi-infinite metal rod is described by the partial differential equation

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2} - \mu\theta$$

where $\theta(x,t)$ is the temperature, $a = \frac{\lambda}{\rho C}$ is the thermal diffusivity of the metal and μ represents the thermal losses.

The transfer function from temperature at the end of the rod to temperature at at distance l along it is given by

$$G(s) = e^{-l\sqrt{(s+\mu)/a}}.$$

Show that

$$\ln|G(j\omega)| \arg G(j\omega) = \frac{l^2\omega}{2a}.$$

Hint: Recall that $\ln x = \ln|x| + j\arg(x)$ *for complex* x.

How would you exploit this result to measure experimentally the thermal diffusivity of a given metal? (This method was discovered by Angstrom.)

6. Draw the Bode diagrams for the following transfer functions on the attached log-lin graph paper:

$$G_1(s) = \frac{(1+sT)}{(1+asT)} \quad \text{for} \quad \begin{matrix} a = 0.1 \\ T = 1 \end{matrix}, \qquad G_3(s) = \frac{1}{(1+2\zeta sT + s^2T^2)} \quad \text{for} \quad \begin{matrix} \zeta = 0.1 \\ T = 10 \end{matrix}$$

$$G_2(s) = \frac{(1+sT)}{(1+asT)} \quad \text{for} \quad \begin{matrix} a = 4 \\ T = 2.5 \end{matrix}, \qquad G_4(s) = \frac{1}{sT(1+2\zeta sT + s^2T^2)} \quad \text{for} \quad \begin{matrix} \zeta = 0.01 \\ T = 0.1 \end{matrix}$$

7. Substitute the parameters from Table 1 in Examples Paper 1 into the transfer function of the Mars Lander, $P(s)$ (found in Q3(d) of Paper 1) to justify the use of the approximation $P(s) = \frac{1}{ms^2} = \frac{1}{200s^2}$ which will be used from now on. For the engine model, assume $H(s) = H_0 = 1121.0$

Sketch the Bode diagram for $L(s) = P(s)H(s)K(s)$ (known as the 'return ratio' of the system) using

(a) $K(s) = k_P$

(b) $K(s) = k_p + k_d s$,

for $k_p = 0.01$ and $k_d = 0.01$

Test: Initialise the lander at 510 m and 700 m, trying different values for k_p and k_d (see 'Simulation Notes' below). What role do each of these parameters appear to perform?

8. In question 6 on Example Paper 1, the transmission path through a telephone network was modelled as a system with impulse response $\beta e^{-\beta t}$. A better model is given by the transfer function

$$\frac{22000s}{(s+300)(s+22000)}$$

Sketch Bode diagrams for the two models, taking $\beta = 22000$. If a sinusoidal signal of frequency ω rad/sec is transmitted through the network, for which values of ω will the received signal be attenuated by less than $1/\sqrt{2}$, according to each of the two models?

Hint: First put the transfer function into the standard form given in lectures (and used in Q6 above).

Check your answer using Octave (or Matlab), by typing

```
G1=tf(22000,[1 22000])
```

followed by

```
bode(G1)
```

and

```
G2=tf([22000 0],conv([1 22000],[1 300]))
bode(G2)
```

In addition,

```
step(G1)
```

and

```
step(G2)
```

will display the step responses of the first and second model, respectively. Are the step responses of the two models consistent with their frequency responses? (Type `help conv`, `help step` and `help bode` to see why the commands given above work, and for further options.)

9. * Figures 2 and 3 give the magnitude and phase plots of the following transfer functions (note that two of the magnitude plots are identical):

$$G_1(s) = \frac{(s+30)}{(s+1)(s+10)(s+100)} \qquad G_2(s) = \frac{s}{(1+0.5s+s^2)}$$

$$G_3(s) = \frac{s}{(1+0.01s)(1+0.5s+s^2)} \qquad G_4(s) = \frac{s(1-0.1s)}{(1+0.1s)(1+0.5s+s^2)}$$

$$G_5(s) = \frac{(1+0.001s+0.01s^2)(1+10s)}{(1+0.01s)^4}$$

Associate the various plots with the correct $G_i(s)$ by considering straight line approximations to each Bode diagram.

Suitable questions on past Tripos papers 2005 Q1, 2007 Q3, 2008 Q1, 2009 Q1.

Simulation Notes for Q7

The controller can be implemented by editing the function defining `throttle` and using the variables `altitude`, `target_altitude` and `speed`. Note that you will always need to add the constant value of F_{eq}/`MAX_THRUST` that defines the equilibrium throttle: remember that all the theory relates to changes about this equilibrium, and that in the simulator H_0 = `MAX_THRUST`. For (i), start by trying $K_p = 0.01$ and for (ii) start with $K_p = K_d = 0.01$.

Answers:

1. Asymptotically stable: (a), (b), (i).
   Marginally stable: (d), (f).
   Unstable: (c), (e), (g), (h).

2. —

3. —

4. $\omega = 1/\sqrt{10}$ rad/sec (or $f = 0.0503$ Hz).
   $\frac{X\sqrt{10}}{1.1}$.

5. —

6. —

7. —

8. First–order model: $\omega < 22000$ rad/sec (or $f < 3501$ Hz).
   Second–order model: $300 < \omega < 22000$ rad/sec (or $47.7 < f < 3501$ Hz).

9. —

Michaelmas 2013

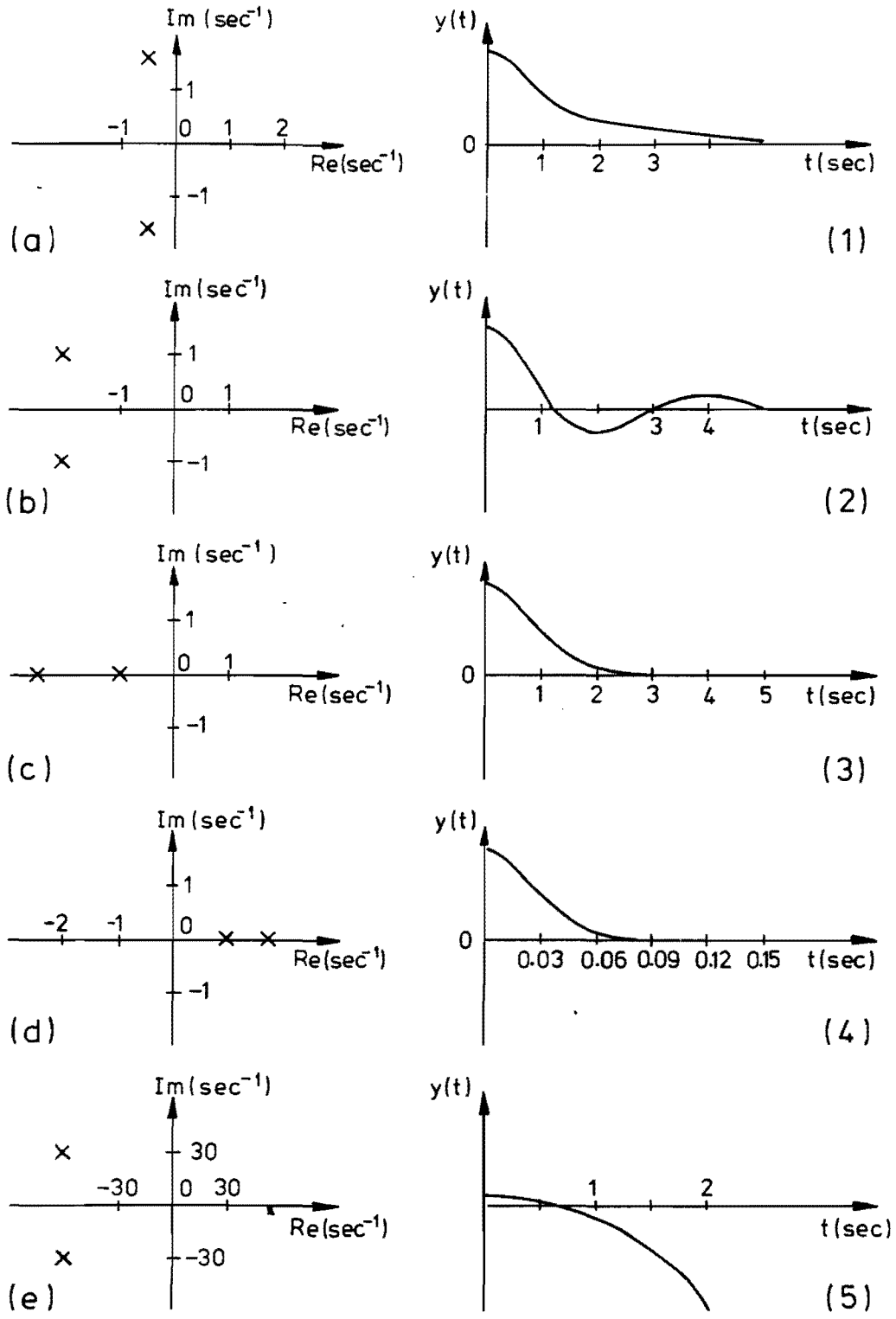


Fig.1

Figure 2

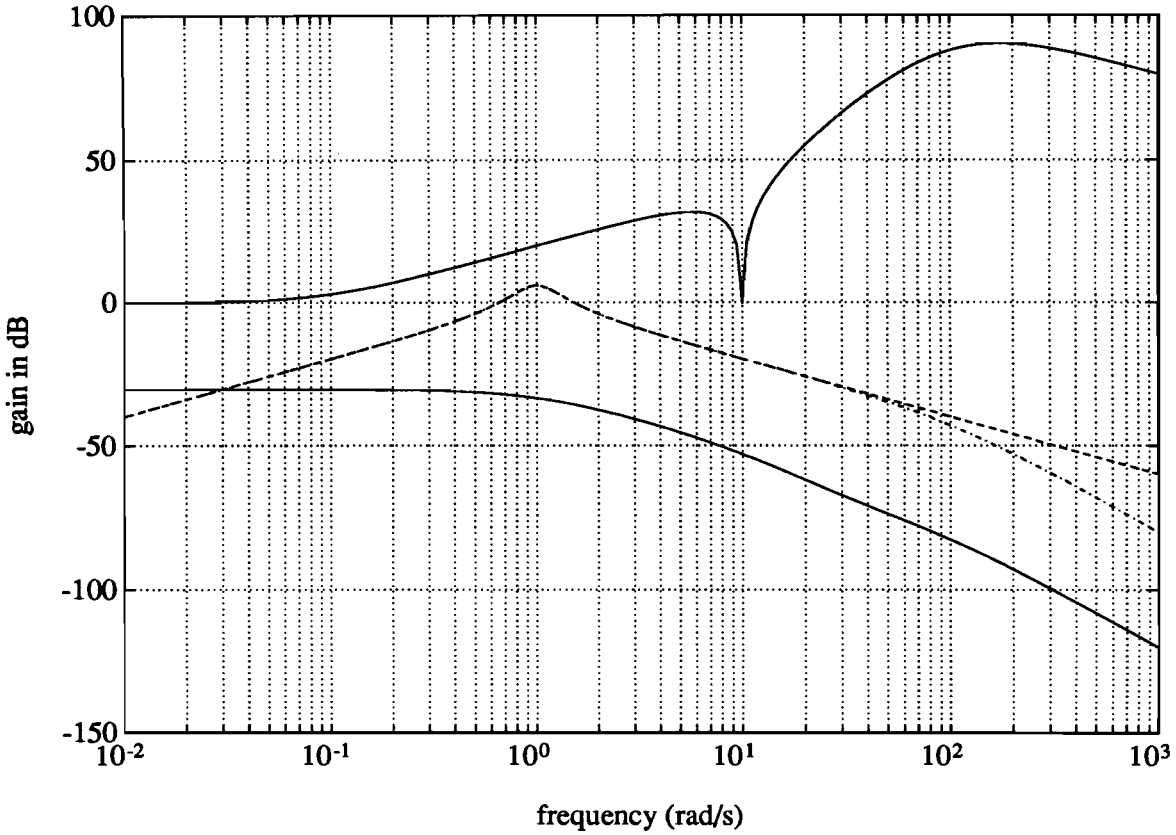
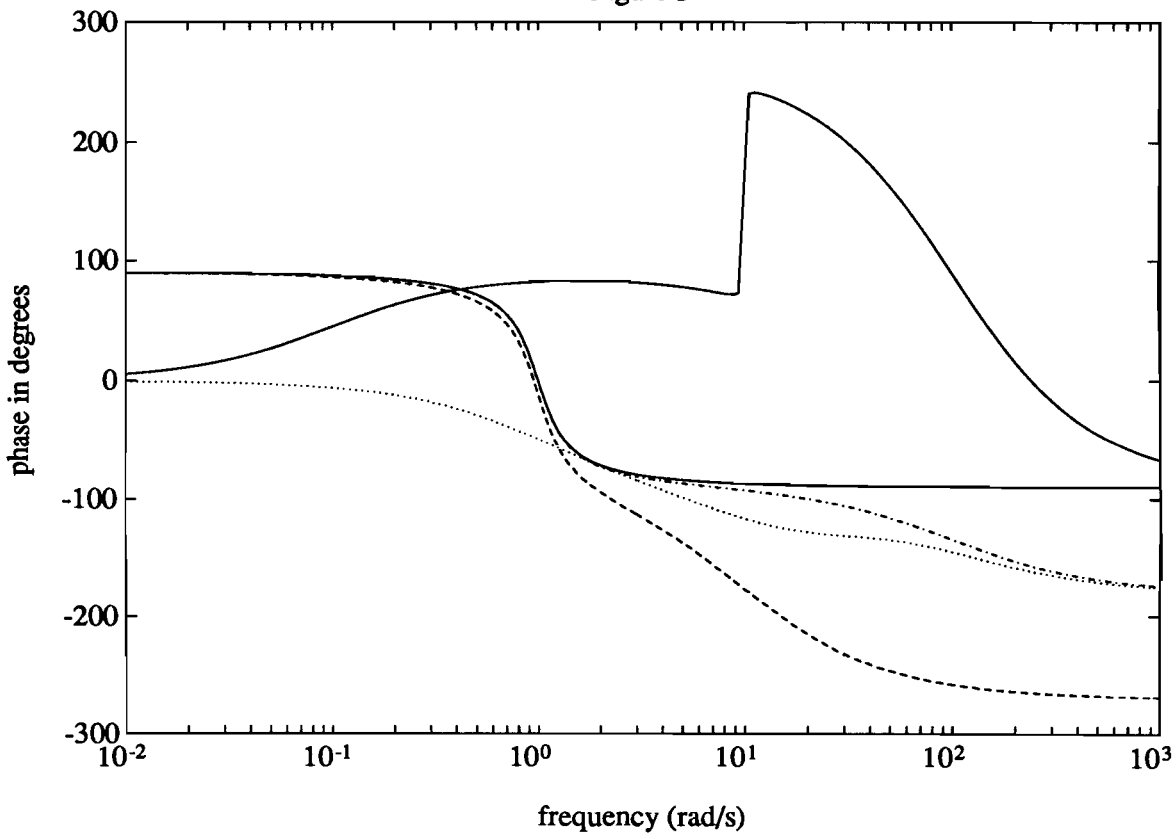
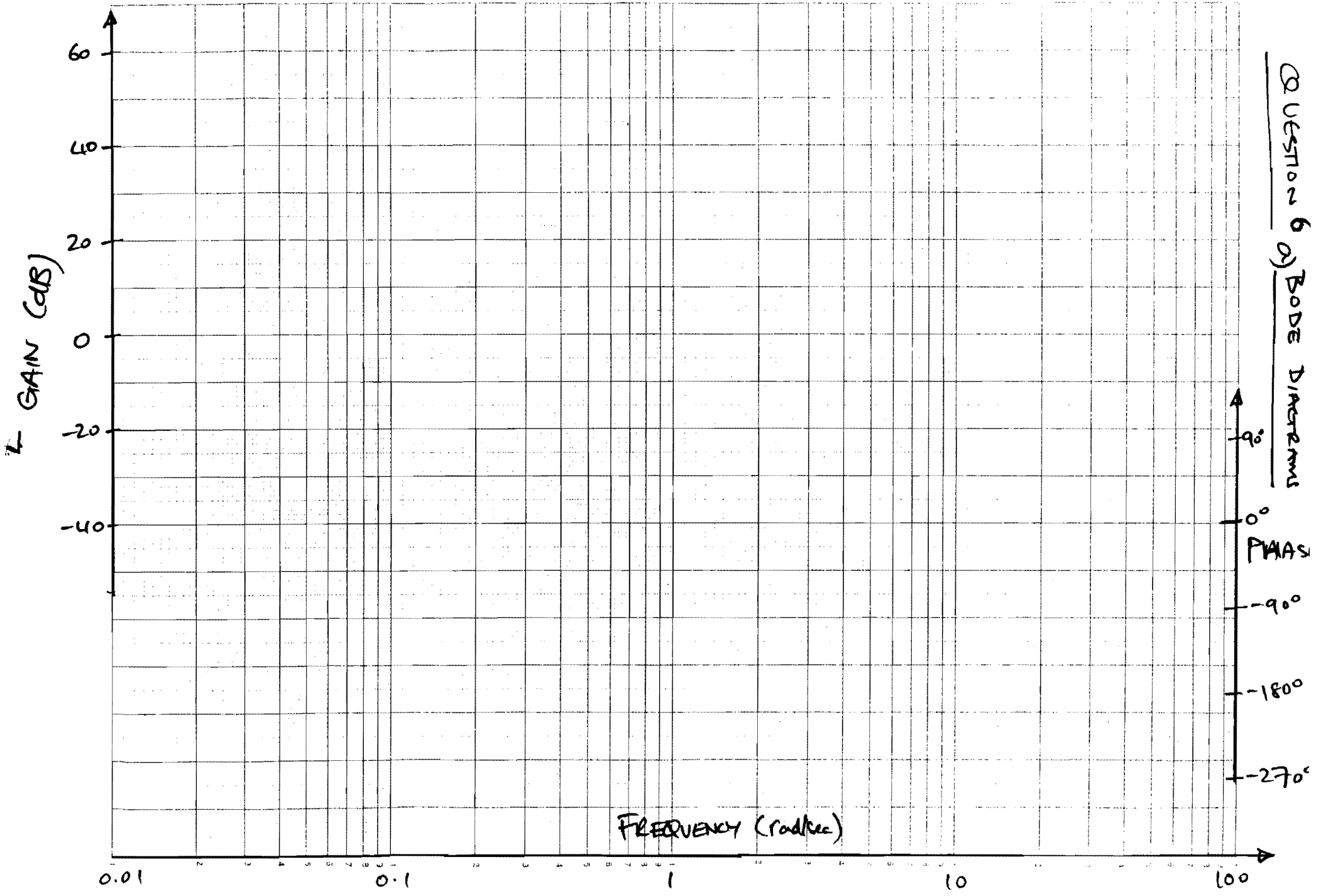
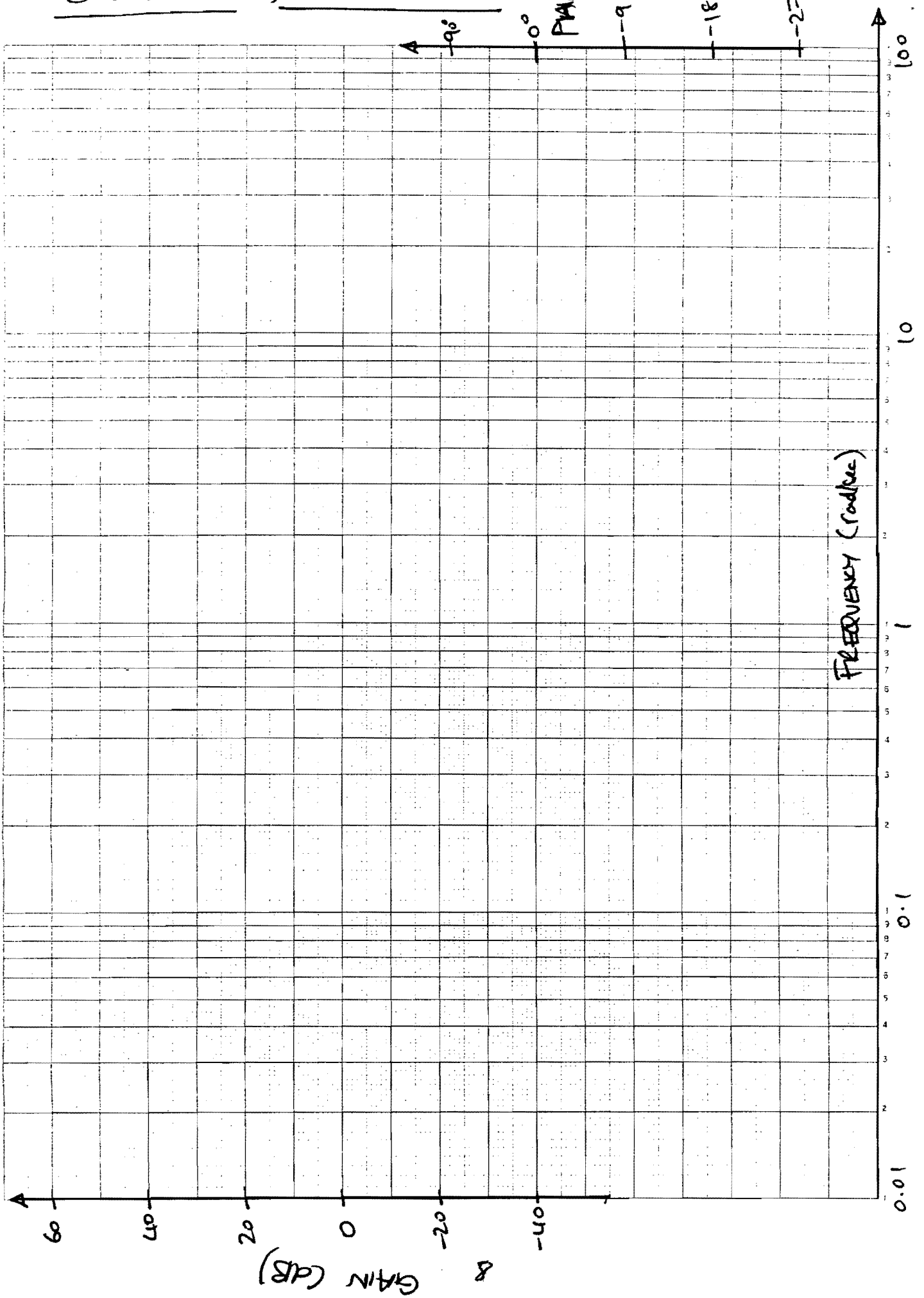


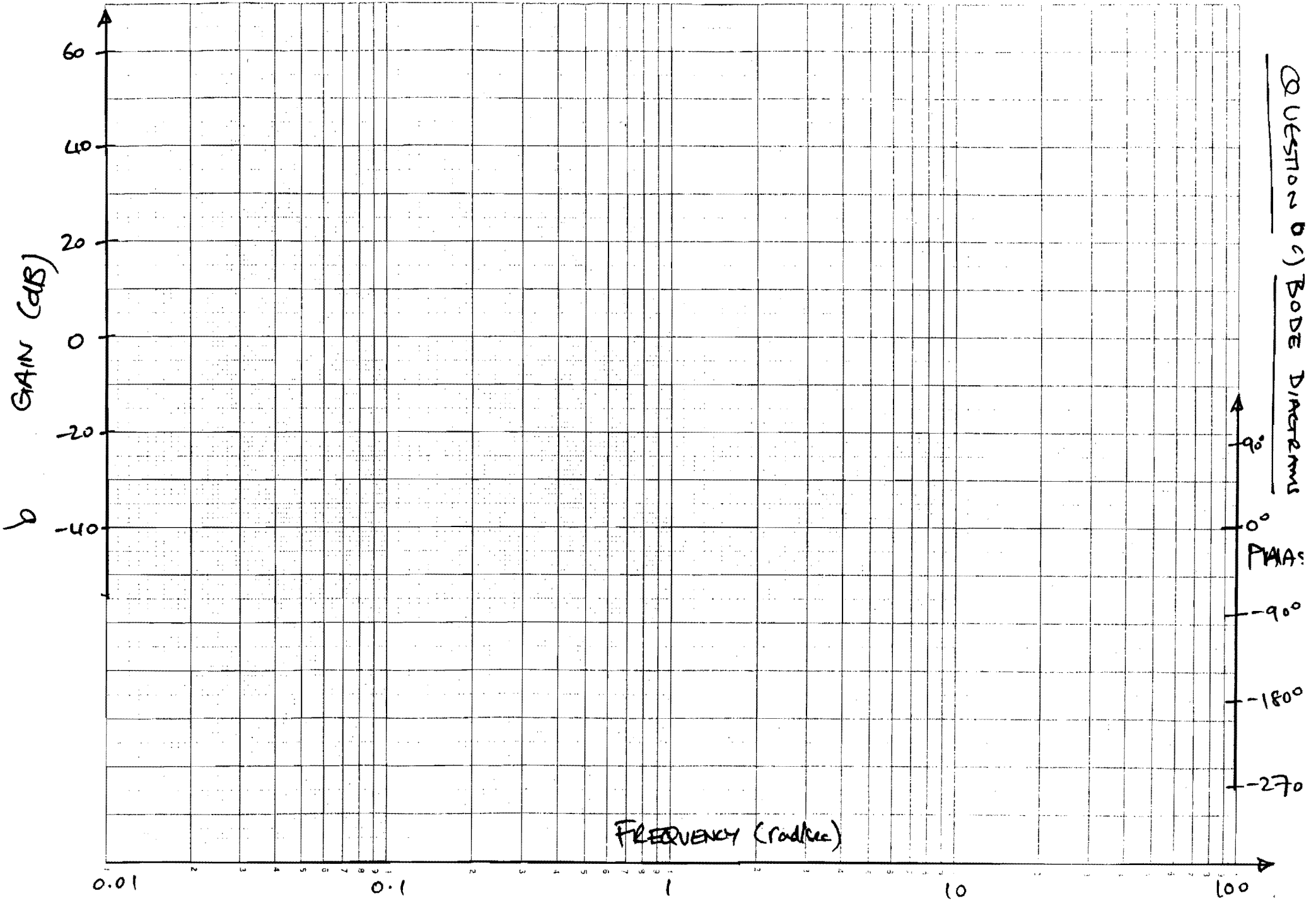
Figure 3

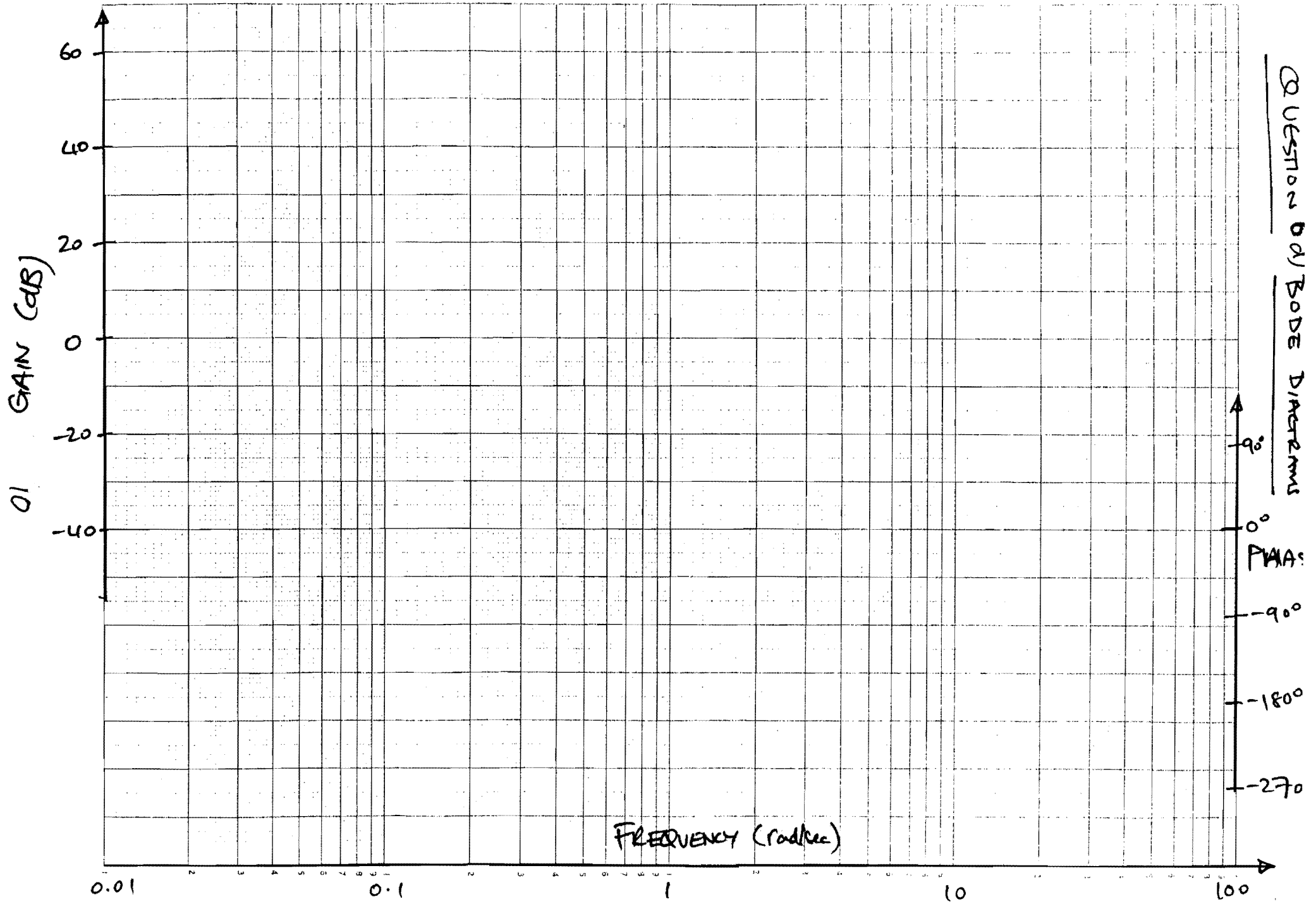



QUESTION 6 a) BODE DIAGRAMS
L GAIN (dB)
60
40
20
0
-20
-40
FREQUENCY (rad/sec)
0.01
0.1
1
10
100
PHASE
90°
0°
-90°
-180°
-270°

QUESTION 6 b) BODE DIAGRAMS




QUESTION 6) BODE DIAGRAMS
GAIN (dB)
60
40
20
0
-20
-40
FREQUENCY (rad/sec)
0.01
0.1
1
10
100
90°
0°
-90°
-180°
-270


QUESTION 01 BODE DIAGRAMS
GAIN (dB)
60
40
20
0
-20
-40
PHASE
90°
0°
-90°
-180°
-270
FREQUENCY (rad/sec)
0.01
0.1
1
10
100