Part IA Paper 1: Mechanical Engineering

MECHANICS

EXAMPLES PAPER 3

Questions marked with a † are of a straightforward nature: those marked * of Tripos standard.

URLs for some web pages related to examples paper questions may be found at
www.eng.cam.ac.uk/~hemh/IAexamples.htm

Velocity diagrams II

1. The crank OA of the mechanism in Fig. 1 rotates anticlockwise at 5000 rpm. Link ABC is a continuous rigid link pinned to the link BE at B. By drawing a velocity diagram, determine the velocity of the ram D in the given position and the angular velocities of the links ABC, EB and CD.

Fig 1 is drawn approximately to scale. The suggested scale for the velocity diagram is 1 mm to represent 1 ms^{-1}. Locate the instantaneous centres for ABC and CD on the space diagram.

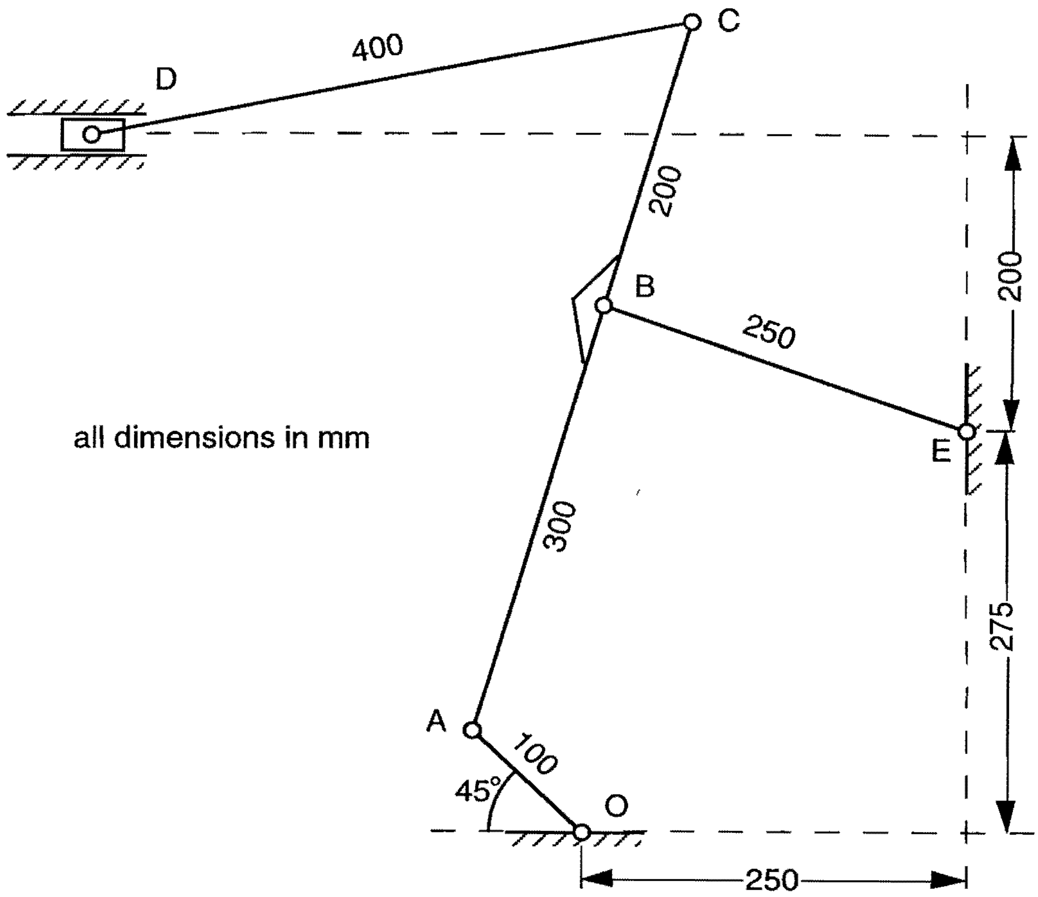

Figure 1

2. Figure 2, which is to scale, shows diagrammatically the space diagram of the operating links of a modern design of dockside crane (called a 'lemniscate' crane). As the link BC rotates clockwise at a constant rate of 0.08 $rads^{-1}$ so the load, which is suspended from E, moves from left to right. Locate on the space diagram the position of E when BC is in position 2 shown in Fig. 2 and also when BC is midway between position 1 and position 2. What do you observe about the motion of E, and hence of the load, as BC rotates?

Using the instantaneous centre method, determine the magnitude of the velocity of E when BC is in position 1 and position 2, and also the magnitude of the velocity of F when BC is in position 1. DE is 30 m long and is horizontal in Fig. 2.

Confirm your answers for position 1 by sketching the velocity diagram. The suggested scale for the velocity diagram is 50 mm to represent 1 ms^{-1}.

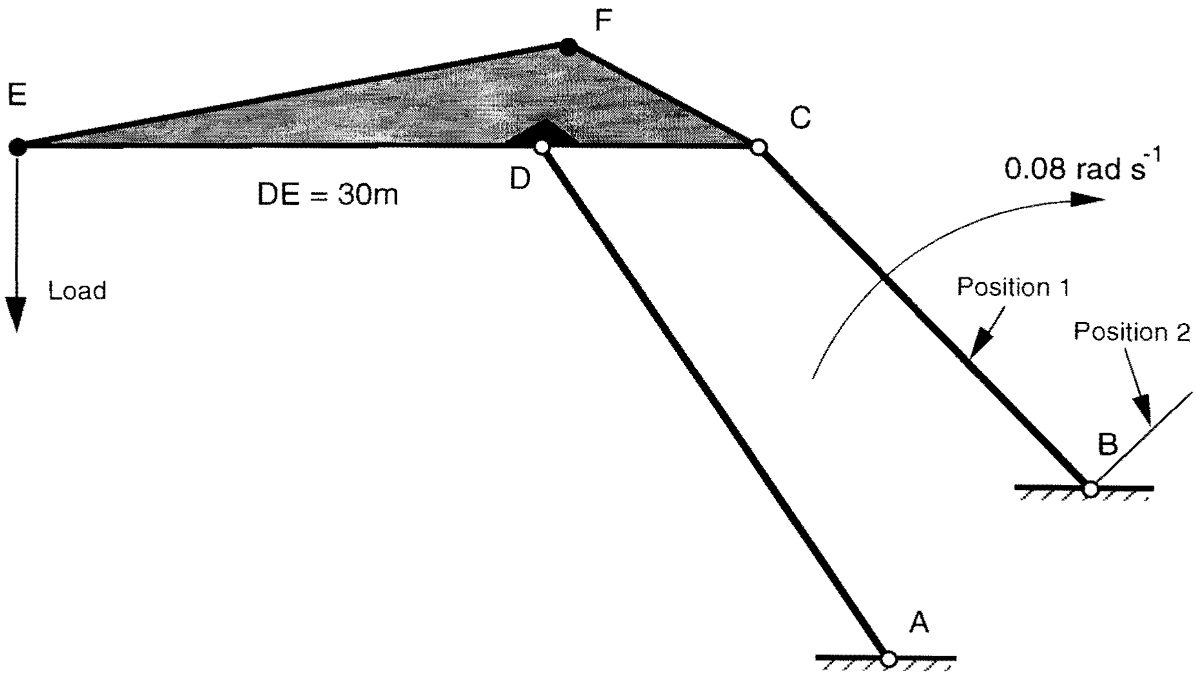

Figure 2

3.* The arm of an excavator is drawn to scale in Fig. 3. The attachment points to the excavator main frame are A and G and these points can be assumed to be fixed in position. With the bucket ram (unlabelled) held at a fixed length and the rams BG, CF moving in their cylinders at 0.06 ms^{-1}, contracting and expanding respectively. By drawing a velocity diagram for the mechanism, find the velocity of the cutting teeth at E. The suggested scale for the velocity diagram is 5 mm to represent 0.01 ms^{-1} .

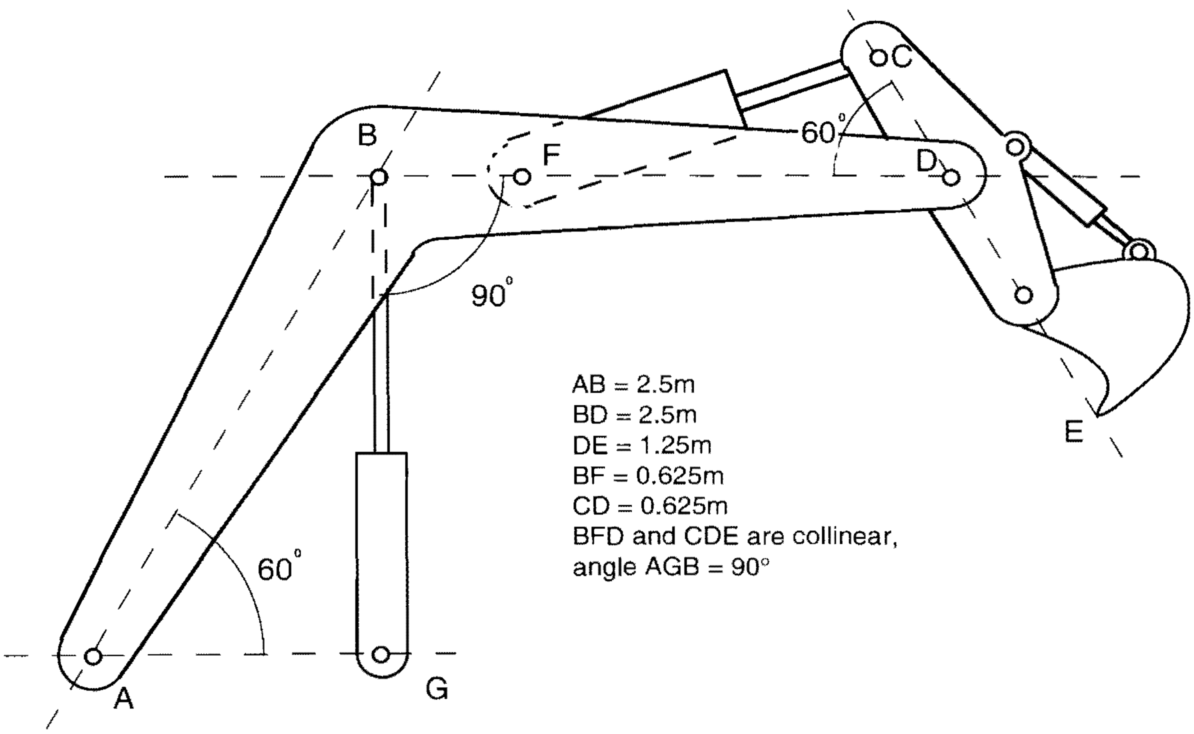

Figure 3

Work and power in mechanical systems

4. (a) The ram D in the mechanism shown in Fig. 1 is *opposed* by a force of 1 kN.

(i) What torque must be applied at O? What assumptions have you made?

(ii)* By how much must the torque be increased if there is a resistance to turning amounting to a frictional torque of 0·5 Nm at each of the pins A, B, C, D and E?

(b) For the excavator shown in Fig 3, a cutting force of 25 kN is applied to the cutting teeth at E. If this force is collinear with the tooth velocity at E, find the power required to supply this cutting force.

5. The lifting jack mechanism shown in Fig. 4 is made of two light rigid bars AB and CD each of length l, with a rotating joint between them at E, and two light rigid bars BF and DF each of length $l/2$.

(a) If the point A remains fixed, sketch the velocity diagram for the case when the point C moves horizontally towards A with a speed of v.

(b) If all the joints are frictionless, evaluate R_x and R_y in terms of the vertically applied load P and the angle α.

(c)* Now if a friction couple Q were to oppose relative motion at joints A to F, find the magnitude of the pair of equal and opposite forces R_x which would just make the tongs operate when $P = 0$.

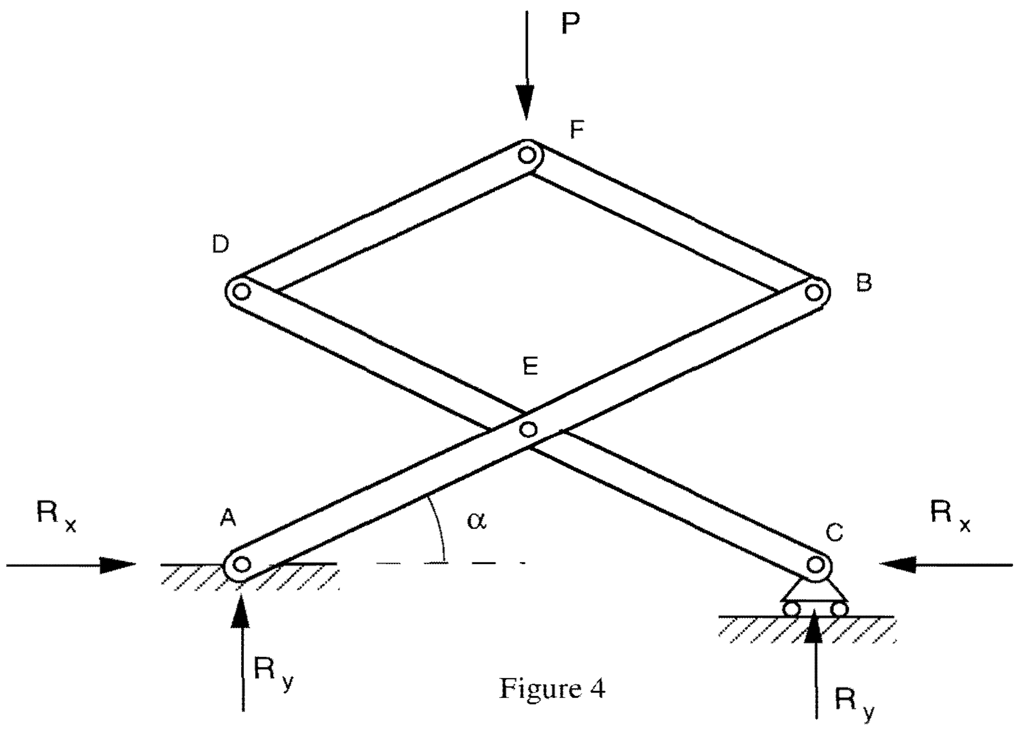

Figure 4

Particle dynamics

6†. A ball of mass m is thrown vertically upwards from the ground with initial velocity v_0 at time $t = 0$. If there is no resistance to motion, show that the height y of the ball above the ground at time t is given by

$$y = v_0 t - \frac{gt^2}{2}$$

If $v_0/g = 0{\cdot}8$ s, find the time t at which the ball first strikes the ground.

7.† A smooth rod is rotating in a horizontal plane about O. A mass of 3·5 kg, which is free to slide along the rod, is attached to a thin wire whose other end is connected to a small winch at O. At the particular instant shown in Fig. 5 the mass is a distance 2 m from O; the magnitudes of the angular velocity and angular acceleration of the rod are 1 rads^{-1} and 3 rads^{-2} respectively and the winch is pulling the mass along the rod so that the magnitudes of its radial velocity and acceleration relative to the rod are 5 ms^{-1} and 4 m s^{-2} towards O respectively. Calculate the tension in the wire and the magnitude and direction of the force exerted by the rod on the mass.

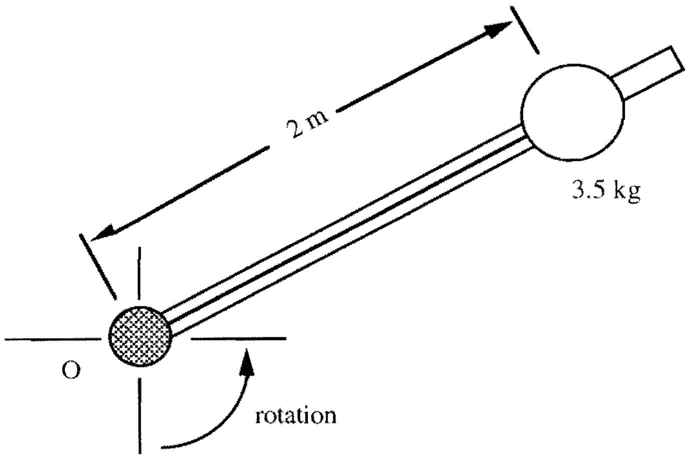

Figure 5

8. An electrostatic cathode-ray tube is operated with its cathode at -2 kV and its anode earthed. The beam of electrons passes between a pair of parallel deflecting plates 10 mm apart and 50 mm long in the axial direction. Estimate the speed with which the electrons enter the space between the deflecting plates.

The beam then strikes a flat screen which is 200 mm from the *centre* of the plates and perpendicular to the axis of the tube. The plates are held initially at $+50$ V and -50 V with respect to earth. Estimate the distance the spot moves on the screen when the leads to the deflection plates are interchanged. Neglect fringing effects. Is gravity important here?

Electron mass $= 9{\cdot}1 \times 10^{-31}$ kg Electron charge $= -1{\cdot}6 \times 10^{-19}$ C

An electron passing through a potential difference of 1 V will increase its energy by $1{\cdot}6 \times 10^{-19}$ J.

Force on an electron = Field strength $\times$ electron charge

Linear momentum, work and energy

9. On landing, an aircraft of mass 5 Mg (i.e. 5000 kg) is travelling horizontally at a speed of 20 ms^{-1}. It is then brought to rest partly by the action of a constant frictional force of 55 kN and partly by the action of a system of wires which exert a restoring force P which varies with the displaced distance x as shown in Fig. 6.

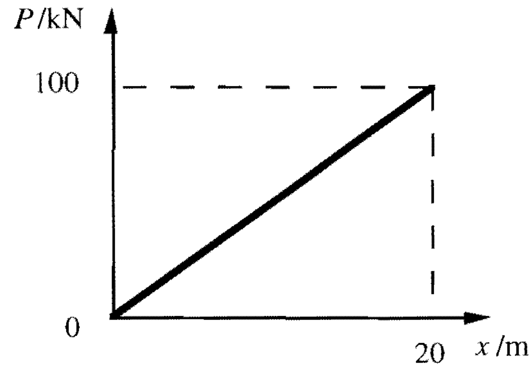

Figure 6

(a) Draw a graph showing the variation with x of the total force acting on the aircraft while it is moving forward. Hence, by using the energy equation, find the value of x at which the aircraft initially comes to rest.

(b) Having initially come to rest, the aircraft begins to move backwards. By considering the *total* force acting on the aircraft for reverse motion find the value of x at which the aircraft finally comes to rest.

10. (a) †A railway wagon of mass 10 Mg moving at 2 ms^{-1} meets a similar stationary wagon. Neglecting rolling and air friction what can you say about the common velocity at the instant when they both move with the same speed? What, if anything, can you say about the velocities after the wagons have separated?

(b) Typical railway wagons carry four buffers, two side by side at each end of the wagon. As wagons meet two buffers on one wagon make contact with two buffers on the other wagon. At maximum compression, how much energy has been transferred to the buffers? While the compression is increasing, the force required to compress a single buffer a distance x is $P_0 + kx$, where P_0 = 5 kN and k = 50 kNm^{-1}; what is the maximum compression of each buffer?

(c) During unloading the buffers do not behave elastically. If the speed of the leading wagon after separation is 1·25 ms^{-1}, calculate the energy recovered from the buffers and hence deduce what fraction of the energy stored in the buffers has been "lost". Where has it gone? Do you think the buffers are functioning effectively?

ANSWERS

1. $10\ \text{ms}^{-1}$ $\quad\omega_{ABC} = 72\ \text{rads}^{-1}$ clockwise

   $\omega_{EB} = 184\ \text{rads}^{-1}$ anticlockwise $\quad\omega_{CD} = 120\ \text{rads}^{-1}$ clockwise

2. E, and hence the load, moves almost horizontally.

   $v_{E1} = 1.62\ \text{m s}^{-1}$; $v_{E2} = 1.71\ \text{m s}^{-1}$; $v_{F1} = 1.76\ \text{m s}^{-1}$

3. $V_e = 277\ \text{mm s}^{-1}$, downwards, $11°$ clockwise from vertical

4. (a) (i) 19 Nm (ii) 1·15 Nm

   (b) Power = 6·9 kW

5. (b) $R_x = 1.5P\cot\alpha$ $\quad R_y = 0{\cdot}5P$

   (c) $\dfrac{10Q}{l\sin\alpha}$

6. 1·6 s

7. 21 N ; 14 N at angle of 90° to it

8. $26{\cdot}5 \times 10^6\ \text{ms}^{-1}$; 50 mm

9. (a) 11.8 m (b) 10.2 m

10. (a) $1\ \text{m s}^{-1}$; add up to $2\ \text{m s}^{-1}$ (b) 10 kJ ; 0·23 m (c) 94%

Suitable practice Tripos questions can be found at
<http://www2.eng.cam.ac.uk/~hemh/IAexamples.htm>.

HEMH/DC

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