Engineering Tripos Part IB SECOND YEAR

Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Example Paper 3

ISSUED ON
- 8 NOV 2013

Straightforward questions are marked †
Tripos standard questions are marked *

Feedback systems

1. A simple speed-control system is shown in figure 1.

   (a) Taking the system constants as
   $K_a = 10$ Amp/Volt $K_b = 50\text{rads}^{-1}/\text{Amp}$ $K_\omega = 0.1$ Volt/rads^{-1}
   $T_a = 0.2$sec $T_b = 0.5$sec
   find the closed-loop transfer function relating controlled and demanded speeds.

   (b) What is the percentage error in steady-state speed response to a step change in demanded speed?

   (c) Sketch the response to a unit step in demanded speed.

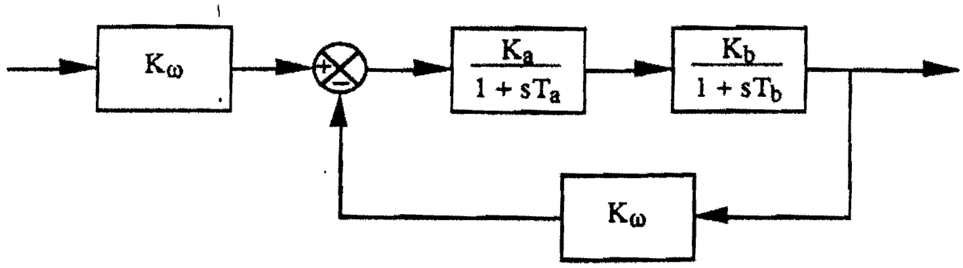

Fig.1

2. An internal combustion engine is directly coupled to an electrical generator. The combined moment of inertia is J and the load torque is B times the speed. The generator voltage V is K times the shaft speed and, owing to magnetic effects, the response lags with a time constant T_1 (i.e. the transfer function will have a factor $1/(1+sT_1)$ due to this). The throttle is adjusted in accordance with the desired voltage V_d and there is proportional-plus-integral control, so that the torque τ developed by the engine, in response to a difference between desired and output voltages is given by

$$\bar{\tau}(s) = K_c\left[1 + \frac{1}{sT_i}\right]\left[\bar{V}_d(s) - \bar{V}(s)\right].$$

Draw a block diagram of the system and establish the closed-loop transfer function relating V to V_d.

3. In an irrigation system, the relationship between the water level at the outflow end of an open water pool, $\bar{y}_i(s)$ and the inflow $\bar{u}_i(s)$ may be approximated as

$$\bar{y}_i(s) = \frac{1}{s(1+Ts)}\bar{u}_i(s).$$

The inflow is to be controlled as

$$\bar{u}_i(s) = K\bar{e}(s),$$

where $e = r_i - y_i$ and r_i is the demanded level

(a) If $T = 0.1$ hours and the damping ratio ζ is to be 0.5 establish the closed-loop transfer function relating y_i to r_i and find the required value of K and the resulting undamped natural frequency ω_n of the closed-loop system.

(b) Use the Mechanics Data Book to find:

(i) the maximum gain for a sinusoidal input and the corresponding frequency

(ii) the maximum overshoot for a step demand of one metre.

4. * Figure 2 shows a block diagram of the shower heater control system used in laboratory experiment 7 ('Process Control'). The voltage θ_d represents a desired water temperature setting, θ represents the outlet water temperature after it has been transduced into a voltage, and θ_i is the inlet water temperature (in °C).

(a) Find the transfer functions $G_1(s)$ and $G_2(s)$, where $\bar{\theta}(s) = G_1(s)\bar{\theta}_d(s) + G_2(s)\bar{\theta}_i(s)$, when the feedback loop is closed.

(b) Assuming that the system eventually settles to a steady state, find the steady-state gains from (i) θ_d to θ, and (ii) θ_i to θ, treating the cases $K_i \neq 0$ and $K_i = 0$ separately.

(c) Suppose that $K_p = 2$ and $K_i = 0.2$ sec^{-1}. Assume that the closed loop system is stable, and that θ_d is constant. If θ_i is oscillating sinusoidally with an amplitude of 2 °C and period of 30 seconds, what is the amplitude of oscillation of θ after any initial transients have died down? (Use results on frequency response.)

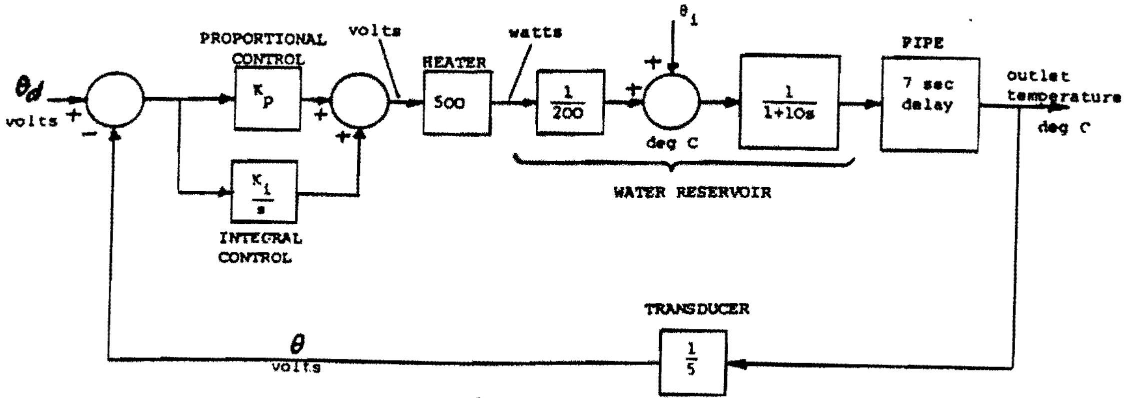

Fig.2

5. * Consider again the Mars Lander with $P(s) = \frac{1}{ms^2}$, engine model $H(s) = H_o$ and an altitude controller which sets the throttle to $k_p(r_d - r) + k_d\frac{d}{dt}(r_d - r)$ as in Figure 3, and with $m = 200$kg, $H_o = 1121$N.

   (a) Assume initially that the disturbance force $f_{\text{dist}} = 0$, and find the closed-loop transfer function from demanded altitude r_d to actual altitude r. Find the closed-loop poles when $k_d = 0$, and predict the stability of the closed-loop system in this case.

   (b) Determine the values of k_p and k_d which give rise to a closed-loop system with damping factor $\zeta = 0.7$ and a decay time constant $1/(\omega_n\zeta) = 2$ seconds.

   (c) Suppose that, with the lander hovering under the control of the altitude controller of (b), a slow combustion instability now occurs in the main lander engine resulting in the actual thrust oscillating about the set thrust. This is modelled as a disturbance of

$$f_{\text{dist}} = 100\cos(0.1t)$$

   in Figure 3. By evaluating the appropriate closed-loop frequency response (and *not* by evaluating the Laplace Transform of $100\cos(0.1t)$), find the steady-state amplitude of the resulting oscillation in altitude and compare it with the steady-state amplitude in the absence of the stabilising feedback system.

   (d) *Test:*

      (i) Initialise the lander at 700 m with $k_p = 0.01$ and $k_d = 0$: to what extent does the simulation agree with your prediction?

      (ii) Now set k_p and k_d as in part (b). Do the simulations agree with the expected step response from the Mechanics Data Book (initialise the simulation at both 510 m and 700 m)?

      (iii) Add in the disturbance of part (c) (see the simulation notes below for how to do this) and compare the resulting altitude oscillations to your predictions, both with and without control.

      (iv) (*Optional*) Can you empirically find values for ζ and ω_n so that the lander does not crash even if you start from 10 km?

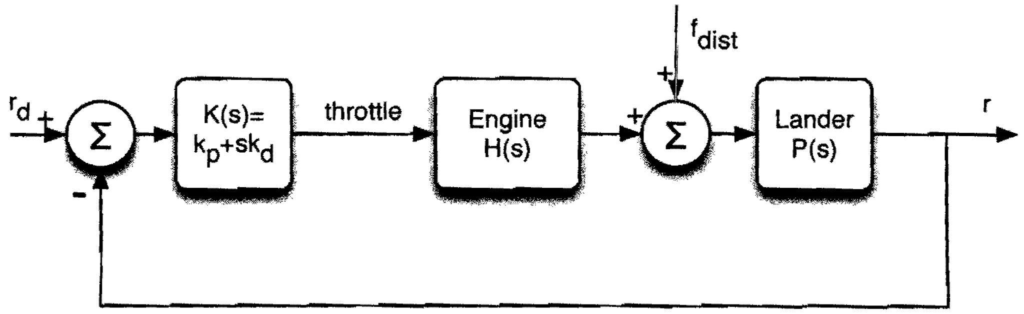

Figure 3

Simulation Notes for Q5

You should have already added the controller in Examples Paper 2, as something like

throttle $= F_{eq}$/MAX_THRUST$+k_p$(target_altitude$-$altitude)$+k_d$speed where `speed` is $(-1)\times$rate of change of altitude (the rate of change of the target altitude being zero).

For the case of vertical, attitude stabilized motion considered here, the disturbance thrust for part (c) can be simulated in the `numerical_dynamics()` function by replacing `thrust_wrt_world()` with

```
thrust_wrt_world() + 100.0*cos(0.1*simulation_time)*position.norm()
```

Don't go using that code in other scenarios though - this is one of the main ways bugs get introduced into flight control systems (causing aircraft to crash)! Can you see why?

Suitable questions on past Tripos papers: 2005 Q.1 and Q.4, 2006 Q.3, 2007 Q.2 and Q.3, 2008 Q.2, 2010 Q.1.

Answers:

1. (a) $\dfrac{500}{s^2 + 7s + 510}$ (b) 2%

2. $$\frac{\bar{V}(s)}{\bar{V}_d(s)} = \frac{KK_c(1 + sT_i)}{sT_i(B + sJ)(1 + sT_1) + KK_c(1 + sT_i)}$$

3. (a) $\dfrac{K}{s^2T + s + K}$, $K = 10h^{-1}$, $\omega_n = 10$
   (b) (i) $2/\sqrt{3}$, $10/\sqrt{2}$ radians/hour. (ii) 0.16m.

4. (a) $G_1(s) = \dfrac{0.5e^{-7s}(sK_p + K_i)}{s(1 + 10s) + 0.5e^{-7s}(sK_p + K_i)}$. $G_2(s) = \dfrac{0.2e^{-7s}s}{s(1 + 10s) + 0.5e^{-7s}(sK_p + K_i)}$.
   (b) (i) 1 if $K_i \neq 0$, but $\dfrac{0.5K_p}{1 + 0.5K_p}$ if $K_i = 0$.
   (ii) 0 if $K_i \neq 0$, but $\dfrac{0.2}{1 + 0.5K_p}$ if $K_i = 0$.
   (c) 0.326 °C

5. (a) $\pm j\sqrt{5.6k_p}$ (b) $k_p = 0.0910$, $k_d = 0.178$ (c) 0.98m (vs 50m).