P2: Structural Mechanics, Examples paper 3

Straightforward questions are marked †.
Tripos standard questions are marked *.

All trusses are two-dimensional. Sign convention: tension and extension +ve.

Displacement diagrams for pin-jointed trusses

† 1. Draw a displacement diagram to find the horizontal and vertical components of the displacement of joint C for the truss shown in Fig. 1 for each of the following sets of bar extensions:

(a) AC +35 mm, BC 0 mm;

(b) AC +35 mm, BC +35 mm;

(c) AC +35 mm, BC –35 mm.

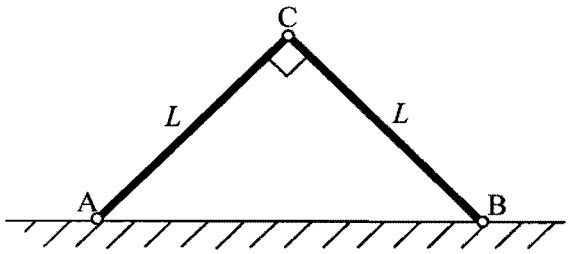


Fig. 1

† 2. The bars in the truss shown in Fig. 2 undergo the following extensions: AB +15 mm, BC –20 mm, AC –10 mm, AD –30 mm, CD –30 mm. Draw a displacement diagram to find the displacements of joints D, C, and B.

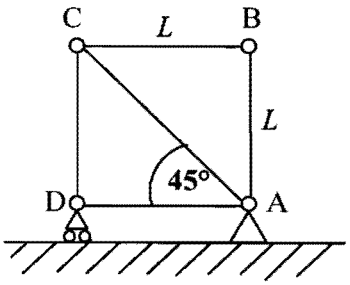
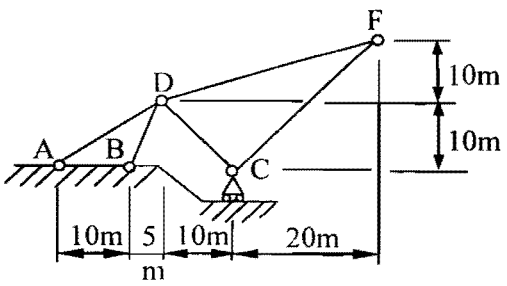


Fig. 2

3. The bars of the structure shown in Fig. 3 undergo the following extensions: AD +17 mm, BD +14 mm, CD –33 mm, DF +25 mm, CF –41 mm. Draw a displacement diagram and find the horizontal movement of C and F (suggested scale: half size).

Fig. 3



* 4. All the bars in Fig. 4 have cross-sectional area A and Modulus of Elasticity E.

(a) Find the axial forces and extensions in all the bars.

(b) Find the vertical deflection of J due to the loading shown.

Suggested scale: $\frac{WL}{AE} = 20$ mm (Hint: start at J and assume that DJ remains vertical. Why is this so?)

(c) By how much does the distance of C from A extend under this loading?

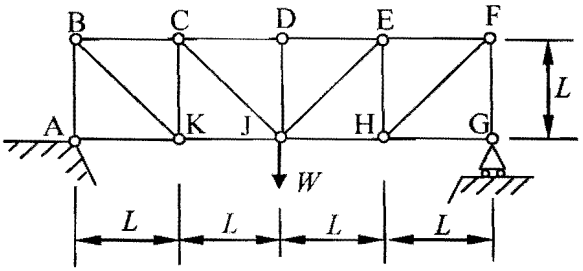


Fig. 4

Virtual work for pin-jointed trusses

† 5. For the truss shown in Fig. 2, use virtual work to find the horizontal component of the displacement of joint C caused by the bar extensions listed in Question 2 above.

† 6. For the plane truss shown in Fig. 4, find the axial forces caused by a vertical unit load $P^* = 1$ acting downwards at C. Hence use virtual work to find the vertical displacement of C due to a vertical load W acting at J as shown in Fig. 4. (Hint: you should already have found the bar extensions due to W, in solving Question 4).

7. For the truss shown in Fig. 4, with a vertical load W applied downwards at joint J, use virtual work to find:

† (a) the vertical support reactions at A and G.

(b) the axial force in member JH.

* 8. For the plane truss shown in Fig. 3, use virtual work to find the movement of C (which is constrained to displace horizontally) when the bars undergo extensions: AD +17 mm, BD +14 mm, CD –33 mm, DF +25 mm, CF –41 mm.

* 9. The members of the truss shown in Fig. 5 have the same cross-sectional area A and Modulus of Elasticity E.

(a) Determine the downwards displacement of joint C under the loading $P_1 = P_2 = W$. (The axial forces due to this loading system have already been obtained in Question 11 of Examples Paper 2).

(b) Use the principle of superposition to find the vertical displacement component of C when $P_1 = 10W$, $P_2 = 4W$.

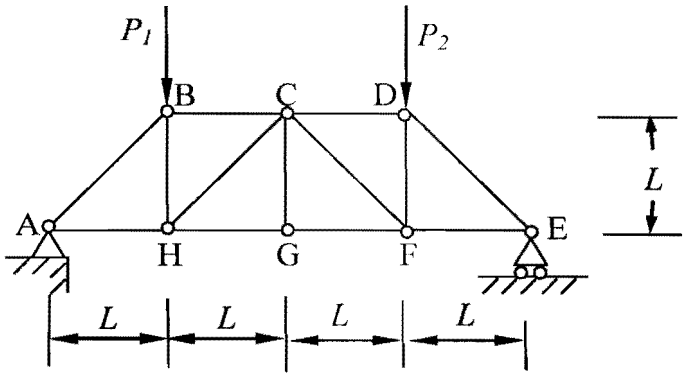


Fig. 5

* 10. Figure 6 shows a truss made from members of equal length L. Joints B, D, and F are constrained to move only in one direction by means of roller supports, shown only schematically in the figure. Thus, joint D can move only along the direction FD, joint F only in a direction perpendicular to FD, and joint B only along the direction FB.

Find the displacement of B when FD extends by an amount e ($e \ll L$).

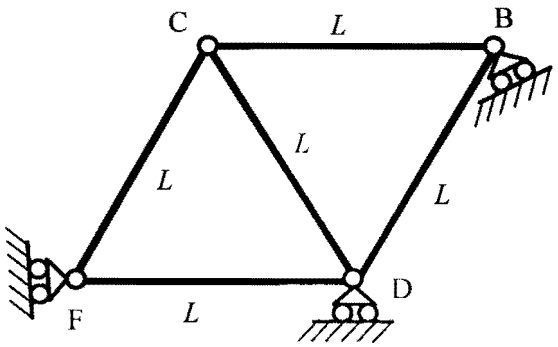


Fig. 6

Suitable Tripos Questions

2004 Q1
2005 Q5
2006 Q3
2007 Q2
2008 Q2
2009 Q2b, Q6c
2010 Q1b, Q5b
2011 Q3b
2012 Q2
2013 Q4

Answers

1. (a) $\delta_{CH} \approx 25\,\text{mm} \rightarrow$; $\delta_{CV} \approx 25\,\text{mm} \uparrow$

   (b) $\delta_{CH} = 0$; $\delta_{CV} \approx 50\,mm \uparrow$

   (c) $\delta_{CH} \approx 50\,mm \rightarrow$; $\delta_{CV} = 0$

2. $\delta_{DH} \approx 30\,mm \rightarrow$; $\delta_{DV} = 0$

   $\delta_{CH} \approx 16\,mm \leftarrow$; $\delta_{CV} \approx 30\,mm \downarrow$

   $\delta_{BH} \approx 36\,mm \leftarrow$; $\delta_{BV} \approx 15\,mm \uparrow$

3. $40\,mm \leftarrow$; $115\,mm \rightarrow$

4. $6.83WL/AE$; $-2.06WL/AE$ Question 6 ↓

Bar	W at J		$P^*=1$ at C
	Bar force $\times W$	Extension $\times WL/AE$	Virtual bar force
DJ	0	0	0
AK	0	0	0
AB	$-1/2$	$-1/2$	$-3/4$
BK	$1/\sqrt{2}$	1	$3/2\sqrt{2}$
BC	$-1/2$	$-1/2$	$-3/4$
CK	$-1/2$	$-1/2$	$-3/4$
KJ	$1/2$	$1/2$	$3/4$
CJ	$1/\sqrt{2}$	1	$-1/2\sqrt{2}$
CD	-1	-1	$-1/2$
HG	0	0	0
FG	$-1/2$	$-1/2$	$-1/4$
FH	$1/\sqrt{2}$	1	$1/2\sqrt{2}$
EF	$-1/2$	$-1/2$	$-1/4$
EH	$-1/2$	$-1/2$	$-1/4$
HJ	$1/2$	$1/2$	$1/4$
EJ	$1/\sqrt{2}$	1	$1/2\sqrt{2}$
DE	-1	-1	$-1/2$

5. $15.9\,mm \leftarrow$

6. $\left(3+\sqrt{2}\right)WL/AE$

7. $V_A = V_G = W/2$; $T_{JH} = W/2$

8. $39.8\,mm \leftarrow$

9. $\left(4+2\sqrt{2}\right)WL/AE$; $7\left(4+2\sqrt{2}\right)WL/AE$

10. $e/\sqrt{3}$

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