ISSUED ON

22 NOV 2013

ENGINEERING **SECOND YEAR**

Part IB Paper 3: MATERIALS

Examples Paper 4: Material deformation and failure

*Straightforward questions are marked †, Tripos standard questions are marked **

Viscoelasticity

1† For the three spring-dashpot networks shown below, **without any calculation** sketch the expected strain response to an applied step of stress. Explain briefly the reasoning behind your three sketches.

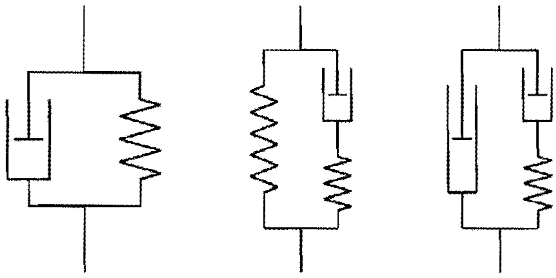

Figure 1

2. (a) (i) Check that the "electrical circuit method" gives the correct answer for the harmonic response of the "standard model" as derived in the lectures by a different method.

(ii) Use the same method to write down the harmonic response of the extended model in Figure 2.

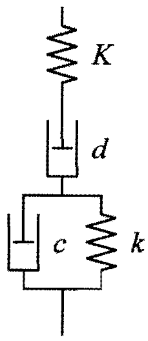

Figure 2

(iii) Calculate the limiting cases when frequency tends to zero, and when frequency is very high. Explain how you could have deduced these limiting cases by looking at the diagram, without calculation.

(b) Young's modulus may be measured in several ways, for example:

(i) deflection in a static 3-point bend test;

(ii) resonant frequency of the first vibration mode of a bending beam, at fairly low frequency;

(iii) measurement of the travel time of an ultrasound pulse at very high frequency.

Discuss what the harmonic responses of the standard and extended models suggest that you might find when comparing measurements made by each of these methods.

What considerations would influence which of these methods you would choose to measure Young's modulus of a particular material?

3. Find a rubber band, and conduct an informal tension – extension test in your fingers by loading and then unloading. What qualitative features can you notice? Sketch the likely form of the tension – extension plot. Which aspects of the behaviour might you be able to reproduce using the simple spring-dashpot models described in the lectures? Which model would you choose as a first candidate to investigate, and why? What aspects of the behaviour are *not* captured by any of these models? Why? No detailed analysis is expected.

4. (a) Figure 3 shows two spring-dashpot models. The response of these models was analysed in the lectures, for the case of an imposed step in stress.

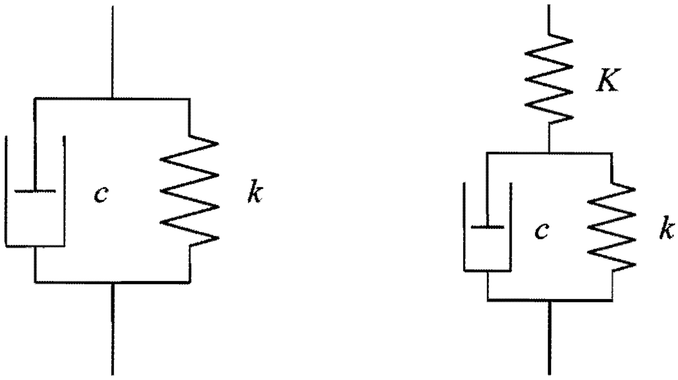

Figure 3

Consider now the response of the same two models for the case of an imposed step in *strain*, with extension given by: $x = \begin{cases} 0 & t < 0 \\ x_0 & t > 0 \end{cases}$

Calculate the resulting tension force as a function of time for both models.

Explain physically why the first model requires infinite force at $t = 0$, whereas the second does not.

(b) When a nylon guitar string is first fitted, sufficient strain is imposed via the tuning peg to achieve a suitable tension so that the string vibrates at the required frequency. It is a common experience that after a while the pitch has dropped, implying that the tension has fallen. This behaviour is known as "relaxation". Do either of the models analysed in part (a) exhibit this behaviour, so that they might be a candidate constitutive model for a drawn nylon monofilament?

(c) Another familiar experience to guitarists is that if you tune a string down to a lower frequency by reducing the tension, the frequency goes back up again somewhat after a few minutes. Does the model predict this as well?

5. The lecture notes contain charts of the creep response of a PVC polymer used for pipes. You wish to analyse the response of such a pipe in a finite-element program. This requires you to enter a constitutive law in the form of parameter values for a spring-dashpot model. The goals are therefore to calibrate the parameters to fit the data, and to test to what extent the model is able to reproduce the behaviour in the charts using a linear model of this general kind. Assume that you are most interested in relatively short loading times, up to a few hours.

Write a Matlab/Octave program to plot a graph of the strain/time response for the step response of the standard model, as derived in the lectures. Make the plot into a similar format to the upper plot in the notes, perhaps using the function SEMILOGX to give the logarithmic time scale. Plot a time range $10-10^4$ s, and use your program to experiment with parameter values for the springs and dashpot to see whether you can match at least some aspects of the empirical data.

You should find that you cannot match the qualitative shape of the curves in at least one important respect. Why is this? What kind of enhancement to the standard model might be needed to get a better fit?

Creep

† 6. (a) Explain briefly, with sketches as appropriate, the *differences* between the following pairs of mechanisms in metal deformation:

(i) room temperature yielding and power-law creep;

(ii) power-law creep and diffusional flow.

(b) Explain the following manufacturing characteristics:

(i) superplastic forming requires a fine grain size material;

(ii) creep resistant alloys are often cast as single crystals.

7. Figure 4 shows a deformation-mechanism map for a nickel alloy. The contours are lines of equal strain-rate for steady-state creep in units of s^{-1}.

(a) What is the maximum allowable stress at 600°C if the maximum allowable strain rate is 10^{-10} s^{-1}? What would be the creep-rate if the stress were increased by a factor of 2, **or** the temperature raised by 100°C?

(b) The strain-rate contours on the map for both diffusional flow and power law creep are based on curve fits to the creep equation:

$$\dot{\varepsilon} = A\,\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

Use the strain-rate contours on the map to estimate the value of the stress exponent n, for diffusional flow and for power law creep.

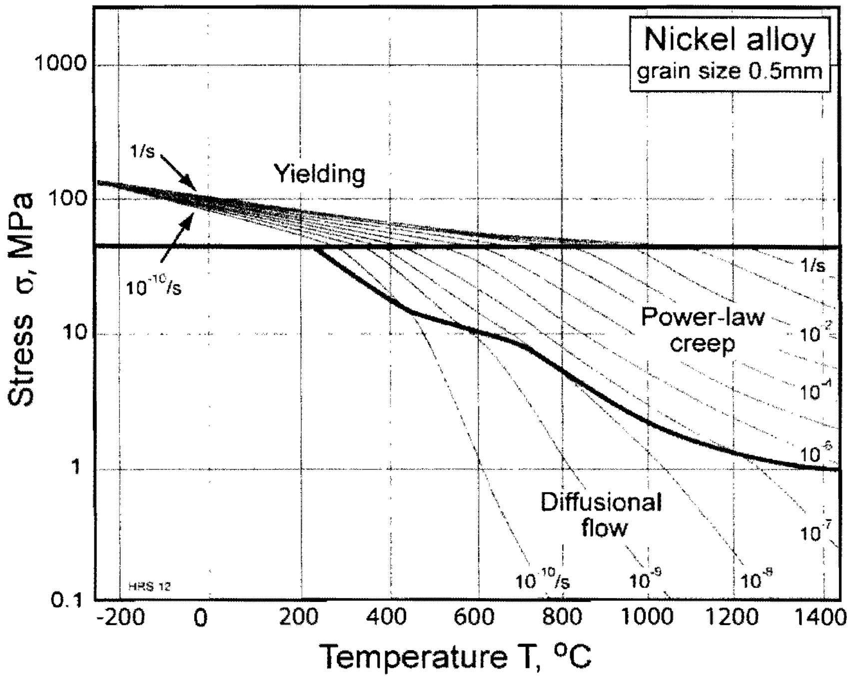

Figure 4

8. The designers of a chemical plant are concerned about creep failure of a critical alloy tie bar. They have carried out creep tests on specimens of the alloy under the nominal service conditions of a stress σ of 25 MPa at 620 °C, and found a steady-state creep rate $\dot{\varepsilon}$ of 3.1×10^{-12} s^{-1}. In service it is expected that for 30% of the running time the stress and temperature may increase to 30 MPa and 650 °C respectively, while for the remaining time the stress and temperature will be at the nominal service values. Calculate the expected *average* creep rate in service.

It may be assumed that the alloy creeps according to the equation:

$$\dot{\varepsilon} = A\,\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

where A and Q are constants, R is the universal gas constant and T is the absolute temperature. For this alloy, $Q = 160$ kJ mol^{-1} and $n = 5$.

* 9. A turbine blade of a nickel-based alloy (ρ=8900 kg.m^{-3}) works at a temperature of 600°C. The root and tip radii r_r and r_t of the blade are 0.2 and 0.25 m respectively from the centre of rotation and the blade rotates at 10,000 rpm as illustrated in Figure 5.

The steady-state (stage II) creep of the blade can be described using the diffusional creep equation:

$$\dot{\varepsilon} = A\,\sigma \exp\left(-\frac{Q}{RT}\right)$$

with $A = 4.3 \times 10^{-11}$ N^{-1} m^2 s^{-1} and $Q = 103$ kJ mol^{-1}.

Assume that the blade has a constant cross sectional area. By considering the stresses on an element as shown in the figure at a position r show that

$$\frac{d\sigma}{dr} = -\rho\omega^2 r$$

and hence use the zero stress boundary condition at the tip of the blade to show that the stress in an element a distance R from the centre of rotation of the blade is given by

$$\sigma(R) = \int_{r=R}^{r_t} \rho\omega^2 r\,dr$$

Use this result to find expressions for the variation of stress and strain rate along the blade.

Given that the rate of change in length of an element of length dr is $\dot{\varepsilon}$ dr, write down an expression for the rate of change in length l of the whole blade with time:

$$\frac{dl}{dt} = \int_{r_r}^{r_t} \dot{\varepsilon}\,dr$$

Hence evaluate the rate of change of length of the blade with time.

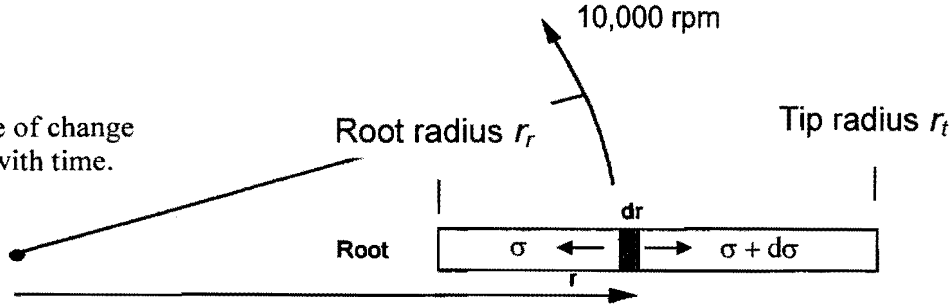

Figure 5

Plasticity and Failure

10. Figure 6 shows the principal stress directions for biaxial loading, with $\sigma_3 = 0$, in:
(i) isotropic materials (e.g. metals, unreinforced concrete), and (ii) uniaxial fibre composites.

Sketch typical failure surfaces (i.e. plots defining the combination of principal stresses leading to failure) for biaxial loading under plane stress conditions for metals, concrete, and uniaxial CFRP. Explain the different shapes in the failure surfaces, with reference to the mechanisms of failure.

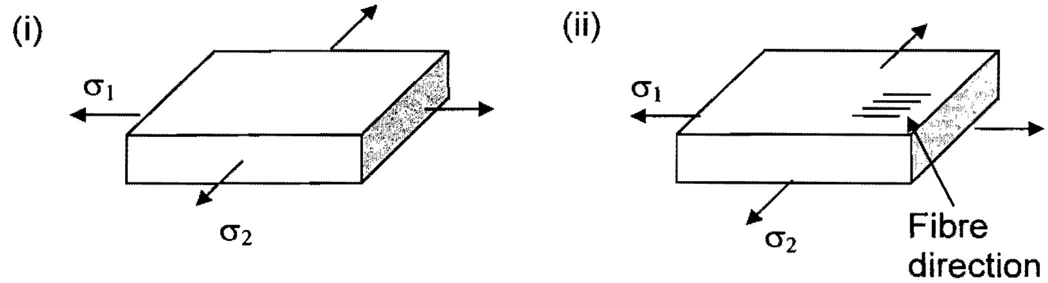

Figure 6

* 11. Die lubricants are used in cold forging of metals to reduce friction between the tool and the work-piece, and to reduce wear of the tool. The frictional shear stress τ between the metal and the tool is given approximately by the Coulomb friction law $\tau = \mu p$ where $p(x)$ is the local pressure between the tool and work-piece and μ is the friction coefficient (a constant, typically $\mu \approx 0.1$)

(a) Consider forging of a long rectangular bar of height 2h and width 2w, see Figure 7. By examining equilibrium of a vertical strip of material of width dx at a general position x as illustrated, show that the pressure p satisfies:

$$\frac{dp}{dx} = -\frac{\mu p}{h} \;, \quad x > 0 \qquad \text{and} \qquad \frac{dp}{dx} = \frac{\mu p}{h} \;, \quad x < 0$$

Hence calculate the pressure distribution $p(x)$ on the bar, assuming the boundary condition that $p = \sigma_y$ for $x = \pm w$.

(b) Discuss qualitatively how the forging load depends upon the friction coefficient μ and aspect ratio w/h, by sketching the dependence of the pressure distribution $p(x)$ upon μ.

(c) Show by a sketch how this analysis forms the basis of a model for the forces and torques used in rolling.

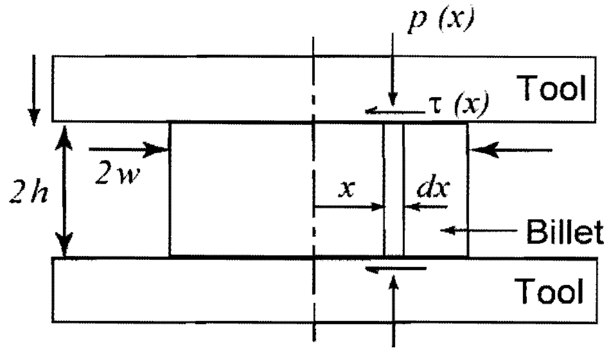

Figure 7

Relevant Tripos Questions from previous years

NB: The course has changed significantly in the past few years. Past questions on deformation process modelling, and some past questions on visco-elastic modelling and creep are relevant. Material on constitutive modelling and failure criteria is relatively new. **Analysis of foam deformation, and failure surfaces for soils, are no longer on the syllabus.**

2008: Q4, 6(a)
2009: Q1, 3(a)
2010: Q3, 6(a)
2011: Q1(a,c), 5
2012: Q2, 5(a)
2013 Q1, 3

NOT 2008: Q5, 2010: Q6(b,c), 2012: Q1

Answers

2. (a) (ii) $\dfrac{X}{F}=\dfrac{1}{K}+\dfrac{1}{i\omega d}+\dfrac{1}{k+i\omega c}$

(iii) $\omega \to 0,\ \dfrac{X}{F}\to\dfrac{1}{i\omega d};\ \omega\to\infty,\ \dfrac{X}{F}\to\dfrac{1}{K}$

4. (a) $f(t)=kx_0+cx_0\delta(t)\ (t\geq 0)$

$$f(t)=x_0\left[\frac{Kk}{K+k}+\frac{K^2}{K+k}e^{-t/T_1}\right],\quad T_1=\frac{c}{K+k}\quad (t\geq 0)$$

7. (a) maximum allowable stress $\approx$ 1.25 MPa
600 °C and 2.5 MPa: strain-rate $\approx 2\times10^{-10}\ \mathrm{s}^{-1}$
700 °C and 1.25 MPa: strain rate $\approx 3\times10^{-10}\ \mathrm{s}^{-1}$

(b) $n\approx 1$ (diffusional flow); $n\approx 4.5$ (power law creep)

8. $6.8\times10^{-12}\ \mathrm{s}^{-1}$

9. $\dot{\varepsilon}(R)=A\dfrac{\rho\omega^2}{2}\exp\left(-\dfrac{Q}{\overline{R}T}\right)\left(r_t^2-R^2\right)$,

$$\frac{dl}{dt}=A\frac{\rho\omega^2}{2}\exp\left(-\frac{Q}{\overline{R}T}\right)\left[rr_t^2-\frac{r^3}{3}\right]_{r_r}^{r_t}$$

Rate of change of length of blade = 8.4×10^{-11} m/s

11. $p(x)=\sigma_y\, e^{\mu(w-x)/h},\quad x>0;\qquad p(x)=\sigma_y\, e^{\mu(w+x)/h},\quad x<0$

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