Part 1A Paper 3: Electrical and Information Engineering,

ISSUED ON
27 NOV 2013

LINEAR CIRCUITS & DEVICES

EXAMPLES PAPER 3

This examples paper is based on material from lectures 8-12. Where possible, the lectures on which the questions are based are indicated, e.g. [L10]. Questions containing material of tripos standard are marked *.

AC Circuit Theory

1. Sketch the following voltage waveform: $V(t) = 2\cos(314t)$. Determine the value of the following quantities:

(a)	The amplitude	(b)	The peak-peak amplitude
(c)	The time-averaged value	(d)	The rms amplitude
(e)	The frequency	(f)	The angular frequency

(Discuss with your supervisor why the rms amplitude is more relevant than the average amplitude in practical cases.)

2. For each of the circuits shown in Fig. 1, find the equivalent complex impedance in both rectangular and polar forms, i.e. X + jY and R $\measuredangle\ \theta$. The reactances of the inductors and capacitors are $X_{L1} = 10\Omega$, $X_{L2} = 15.7\Omega$, $X_{C1} = 40\Omega$, $X_{C2} = 31.9\Omega$, $X_{L3} = 6.28\Omega$.

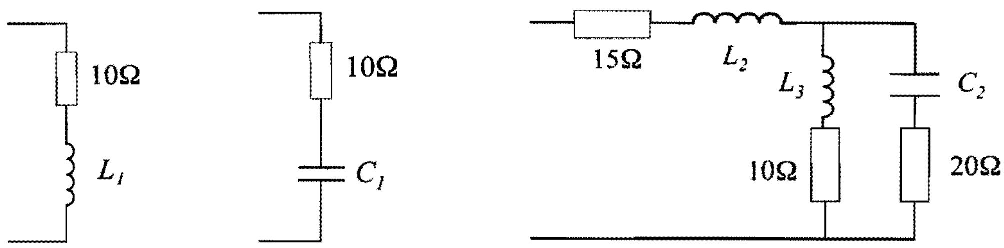

Fig. 1 (a) (b) (c)

3. For each of the bridge circuits shown in Fig. 2, derive conditions for balance, expressing the unknowns R_x, L_x, C_x in terms of the other components. The component in the centre labeled "det" is an ammeter (current meter) that indicates zero at balance. You may assume the general equation for the balance condition of a dc Wheatstone bridge, which may be expressed as $R_x/R_2 = R_4/R_3$.

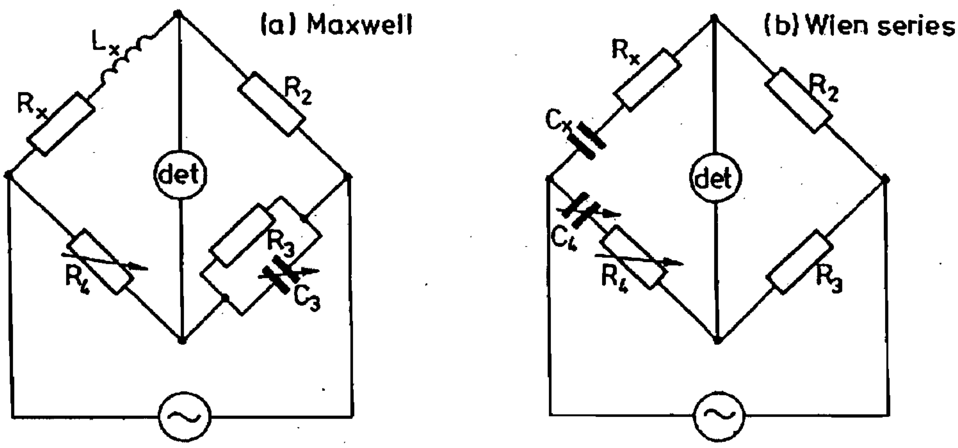

Fig. 2

4. Fig. 3(a) shows an AC source, V_1, whose rms voltage is given as a complex quantity in rectangular form.

   (a) Determine V_1 in polar form

   (b) At the frequency at which the source operates, the impedances Z_1 and Z_2 are shown in Ohms. Find the amplitude and phase of the current I_2, and hence determine the voltage V_2 in rectangular and polar form.

   (c) Determine the phase difference between V_2 and V_1, and the gain of the circuit (i.e. V_2/V_1).

   (d) In order to predict what will happen when further loads are connected across AB, it is desirable to reduce the circuit to its Thévenin equivalent as shown in Fig. 3(b). Determine V_T and R_T in rectangular and polar form.

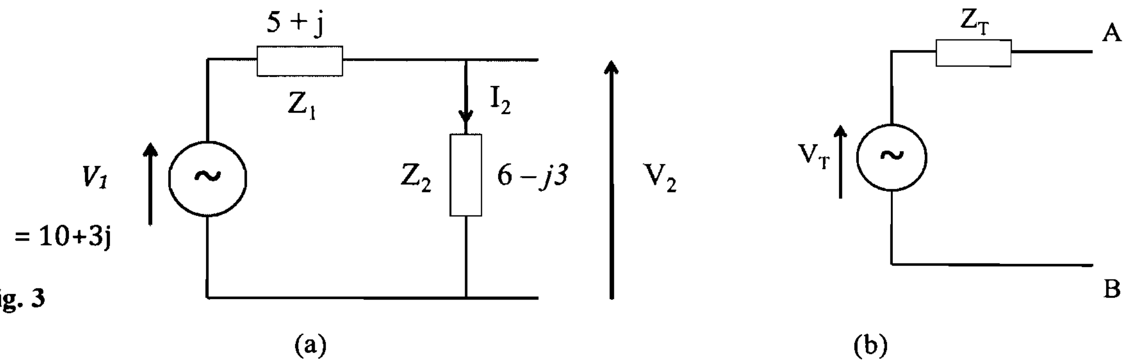

Fig. 3

5. A 50 Hz AC voltage source has a Thévenin equivalent resistance of 100Ω and an rms emf of 20V. It is connected to a load consisting of a 0.159H inductor in parallel with the series combination of a 50Ω resistor and a 63.7μF capacitor. Calculate the rms current that flows through the inductor and its phase with respect to the source.

6. A simple radio receiver circuit is shown in Fig. 4. The antenna, which receives an emf, V_1, of 100 μV has a resistance, $R_{Ant} = 30\Omega$. It is connected to a series resonant circuit that acts as the tuner, the output of which is amplified with a FET (shown) before being sent to the headphone/loudspeaker drive circuitry. The inductor, L and variable capacitor, C are assumed to be ideal.

(a) What value of C is required to make the circuit resonant at 198kHz? What will the value of v_2 then be?

(b) If the inductor is not ideal, but has a winding resistance of 15Ω, what will the value of v_2 at resonance be?

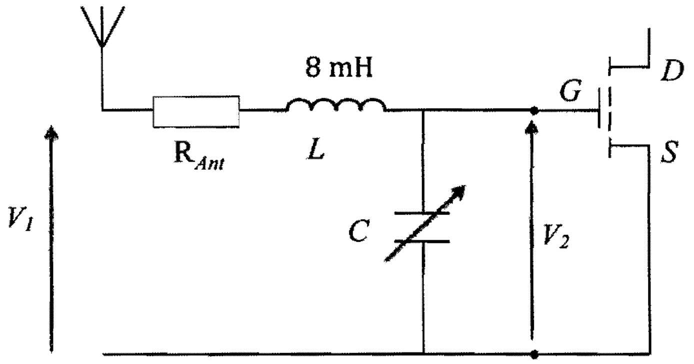

Fig. 4

7. An AC voltage generator has an emf of 100V, and internal resistance of 1MΩ. It is connected to the terminals of a parallel resonant circuit. The capacitor has a value of 100pF and a Q-factor of 1000. The inductor may be represented by an inductance of 10mH in series with a resistance of 10Ω. Calculate:

(a) The resonant frequency

(b) The voltage across the circuit at resonance

(c) The half-power bandwidth

Answers:

1. 2V; 4V; 0V; 1.414V; 50Hz; 314 rad/s
2. $10 + j10\,\Omega$, $14.1\,\Omega \measuredangle 45^\circ$; $10 - j40\,\Omega$, $41.23\,\Omega \measuredangle -75.96^\circ$; $25.9 + j18.6\,\Omega$, $31.9\,\Omega \measuredangle 35.7^\circ$
3. $R_x = R_2R_4/R_3$, $L_x = R_2R_4C_3$; $R_x = R_2R_4/R_3$, $C_x = R_3C_4/R_2$
4. $10.44\text{ V}\measuredangle 16.67^\circ$; $0.832 + 0.424j$ A; $6.25\text{ V}\measuredangle 0.439^\circ$; -16.26^0, 0.6; $6.26 + 0.048j$ V, $3.048 - 0.264j$ Ω

5. $0.179A \measuredangle -63.4^0$
6. 81 pF (81×10^{-12} F), 33.2 mV $\measuredangle$ -90^0; 22.1 mV $\measuredangle$ -90^0
7. 159 kHz; 83.3V; 1.92 kHz

Tripos questions that are appropriate for revision purposes:
Christmas revision 2011 on lectures 1-10
2009 Q2 & Q3
2008 Q4
2007 Q1
2006 Q2

It is recommended that you attempt these questions over the Christmas vacation, as well as any work that your College sets you. In order to optimize the learning process, study the lectures first, and then try the tripos questions. You should attempt these questions without looking at your notes – only do so afterwards to check for yourself and fill in the gaps.

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Michaelmas 2013