Engineering Tripos Part IB SECOND YEAR

Part IB Paper 1: Mechanics

Examples Paper 1

Kinematics

ISSUED ON
17 JAN 2014

Straightforward questions are marked †
*Tripos standard questions are marked *.*

†1. A lamina is in the form of a triangle ABC, Fig. 1. Corner A has velocity and acceleration as shown, parallel to BC and BA respectively. The angular velocity of the lamina at the given instant is 2 rad/s anticlockwise and is increasing at the rate of 1 rad/s^2.

(i) Where is the instantaneous centre I for the lamina ?

(ii) What are the magnitudes of the velocities of B and C ?

(iii) Where is the centre of curvature of the **path** of A ?

(iv) What is the magnitude of the acceleration of B ?

(v) Where is the centre of curvature of the **path** of B ?

(vi) What is the magnitude of the acceleration of C ?

(vii) What is the magnitude of the acceleration of the point on the lamina which is instantaneously coincident with I ?

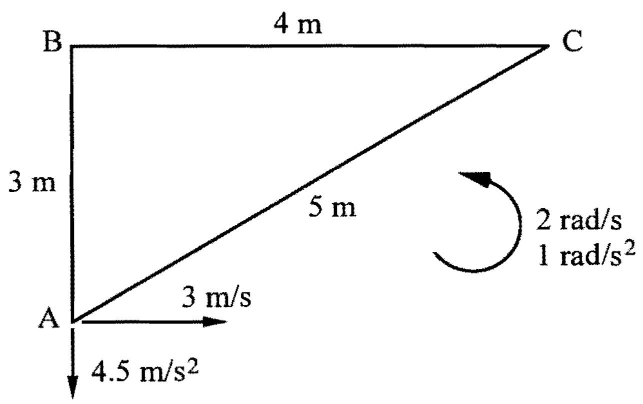

Fig. 1

2. An engine runs at 6000 rpm and has a crank AB 40 mm long and a connecting rod BD 120 mm long. Sketch the velocity diagram for an arbitrary crank angle DAB. What is the acceleration of the piston:

(a) when angle DAB is 180° (bottom dead centre – see lecture notes);

(b) when angle DBA is 90° ?

†3. The rod AB is 2 m long and has the accelerations shown in Fig. 3 at its ends. Sketch the acceleration diagram for the rod and find its angular velocity and acceleration.

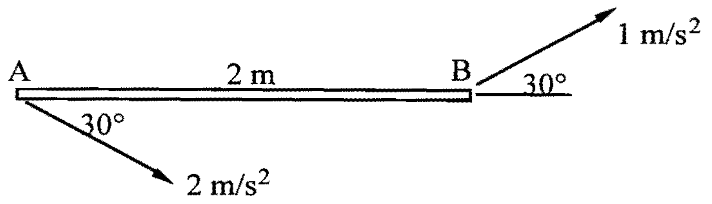

Fig. 3

4. In the mechanism shown in Fig. 2, AB rotates with constant angular velocity ω.

   (a) Draw a velocity diagram and show that the angular velocities of BC and CD are both $-\omega$. Calculate these angular velocities using vectors. (Attach unit vectors to each link and differentiate an expression for vector **AD**.)

   (b) By drawing an acceleration diagram or by differentiating your vector expression from part (a), determine the components of the acceleration of G (the mid-point of BC) parallel and perpendicular to AD ?

   (c) How are the components affected if ω is (a) doubled, (b) reversed in direction ?

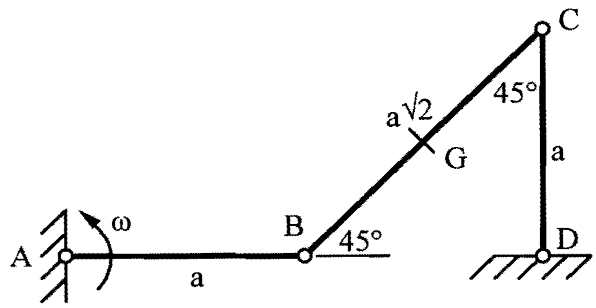

Fig. 2

5. The rigid member ABC rotates in its own plane with constant angular velocity ω (Fig. 4). At the given instant block D is sliding with constant velocity v relative to ABC, and the distance BD is a . Write an equation for the position of point D. What are the components of the acceleration of D along and perpendicular to BC ? Show that the acceleration of a point *on the rod BC* adjacent to D is $a\sqrt{2}\,\omega^2$ towards A.

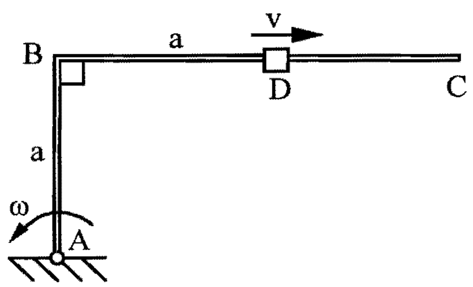

Fig. 4

6. A wire hoop of radius R rotates in its own plane with uniform angular velocity $\omega\mathbf{k}$ about an axis OZ through a point O in its circumference (Fig. 5). Q is the centre of the hoop. A bead P moves round the hoop in the same sense with a uniform speed v *relative* to the hoop.

(a) Show that the *absolute* angular velocity of vector QP is $(\omega + v/R)\mathbf{k}$.

(b) Write an expression for the position vector of P in terms of unit vectors e_1 and e_2 and differentiate it twice to determine the absolute acceleration of P.

(c) Express the absolute acceleration of P in cartesian coordinates ($\mathbf{i}$, $\mathbf{j}$) in terms of the angle θ (= OQP). Take OQ as the $\mathbf{j}$-axis *instantaneously*.

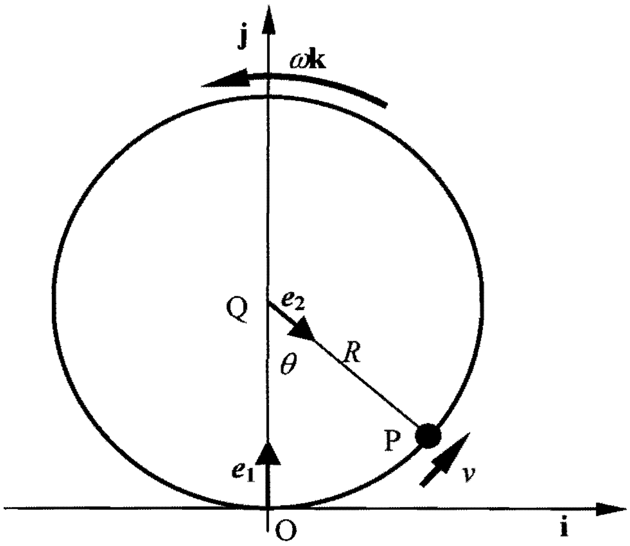

Fig. 5

7*. Fig. 6 shows one of the vanes (or blades) of a centrifugal pump impeller which turns with a constant clockwise angular velocity of 300 rev/min. Fluid enters the impeller through the central hole, is 'flung' out radially by the vanes, and is then collected in a volute (not shown).

The vane surface has a constant radius of curvature R (with centre at C), and the fluid is observed to have an absolute velocity whose **radial component** is 3 m/s at discharge from the vane. The speed of the elements of the fluid measured relative to the vane increases at the rate of 24 m/s^2 just before they leave the vane.

(a) Show that the velocity of the fluid relative to the impeller at discharge from the vane is $3\sqrt{2}$ m/s.

(b) By writing a position vector to the particle via point C and using the same approach as in Question 6, or otherwise, find the magnitude and direction of the absolute acceleration of an element of the fluid just before it leaves the impeller.

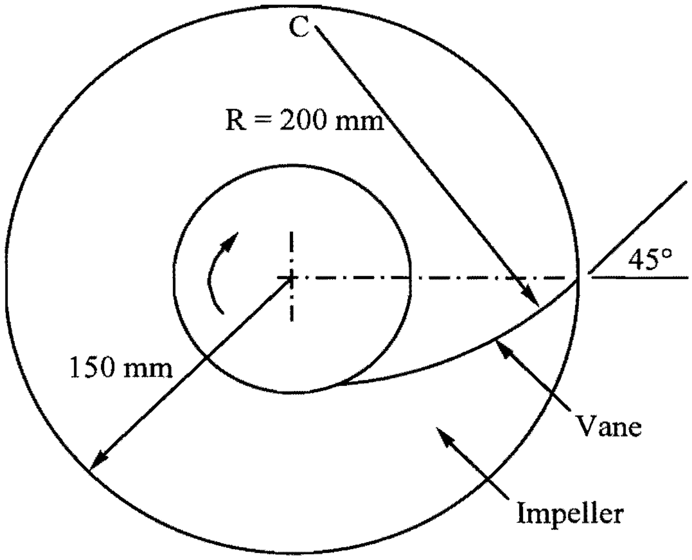

Fig. 6

8. Link AC passes through a fixed swivel B (Fig. 7). AB is 200 mm, BC is 100 mm and the velocity and acceleration of A are as shown.

   (a) Draw the velocity diagram for the rod and find the angular velocity of the rod and the velocity of point C.

   (b) Draw the acceleration diagram for the rod and find the acceleration of point C.

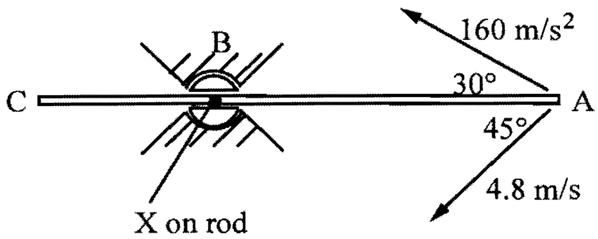

Fig. 7

9. At a certain instant the ends A and B of the hydraulic ram shown in Fig. 8 have zero acceleration. The ram is extending with a speed of 1 m/s and the oil flow is increasing at a rate corresponding to an acceleration of 1 m/s^2.

What are the horizontal and vertical components of the acceleration of the point G on the cylinder, midway between A and B, when AB is 1 m ? Is it possible to specify whether the acceleration of G is up or down ?

Hint: Look at the acceleration of a point in polar co-ordinates, in the data book. What equations can you write down, given that B has zero acceleration relative to A?

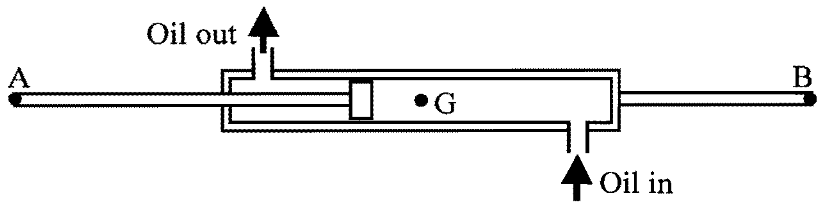

Fig. 8

*10. Fig. 9 shows, to scale, a plane mechanism in which the jack AB operates between a fixed hinge A and the centre point B of the rigid link CD.

If the slider C is moving at the given instant with a constant velocity of 960 mm/s, left to right, determine:

(a) the velocity of sliding of the jack piston relative to its cylinder, and the angular velocities of AB and CD.

(b) the sliding acceleration of the piston relative to the cylinder.

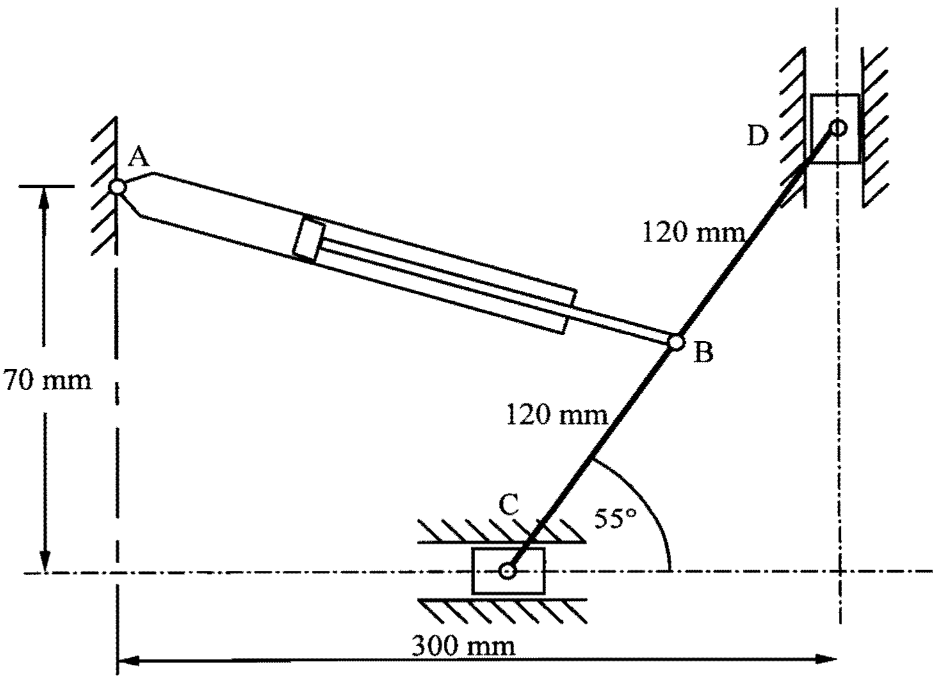

Fig. 9

*11. The wheel, radius r, of a car when turning a corner traces out a path of radius R with a uniform velocity v.

(a) What is the acceleration of point Q, the centre of the wheel?

(b) What is the velocity and acceleration, *relative to* Q, of point P on the tyre which is instantaneously in contact with the ground?

(c) Show that the absolute acceleration of P has components: v^2/r towards the centre of the wheel and v^2/R radially *outwards* from the centre of the curve.

(d) Sketch the paths traced-out by Q and by P as the wheel rolls along its curved path. Account for the outward direction of the radial acceleration of P on geometric grounds.

For further practice try the following IB Mechanics Tripos questions:

2000 Qs 3 & 6; 2001 Qs 1 & 2 2002 Q2; 2003 Q6; 2004 Q2a,b; 2005 Q6; 2006 Q5, 6; 2007 Q3b; 2008 Q2a,b

(Ignore any parts of the above questions which ask about forces, torques, &c., until these have been covered later in the course.)

ANSWERS

1. (i) midway between A and B (ii) 3 m/s; 8.54 m/s
   (iii) 2 m from A on BA extended (iv) 16.8 m/s^2
   (v) 0.546 m from B on BA (vi) 22.7 m/s^2
   (vii) 10.6 m/s^2
2. (a) 10530 ms^{-2} upwards (b) 616 ms^{-2} downwards
3. 0.658 rad/s clockwise or anticlockwise; 0.75 rad/s^2 anticlockwise
4. 1.5 aω^2 ; 0.5 aω^2 (a) they are quadrupled (b) unaffected
5. – aω^2 ; 2vω – aω^2
6. (b) $-R\omega^2\mathbf{e_1} - R(\omega + v/R)^2\,\mathbf{e_2}$
   (c) $-(\omega^2R + v^2/R + 2\omega v)\sin\theta\,\mathbf{i} + \{(\omega^2R + v^2/R + 2\omega v)\cos\theta - \omega^2R\}\,\mathbf{j}$
7. 108 ms^{-2}, 86.7° anticlockwise from inward radius
8. 16.97 rad/s clockwise, 3.79 m/s at 26.6° to AB; 143 ms^{-2} at 68.6° to AB
9. 0.5 ms^{-2}, 1.0 ms^{-2}, no
10. (a) 0.36 m/s, ω_{AB} = 1.93 rad/s anti-clockwise; ω_{CD} = 4.90 rad/s anticlockwise
    (b) 1.94 ms^{-2}.

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