Engineering Tripos   **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 2 – Elastic Deformation and Properties

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

*You will need to look up data in the Materials Databook, and use the
Cambridge Engineering Selector (CES) software.*

Elastic Stress and Strain

† **1.**   Draw diagrams to define: (i) tensile strain ε, (ii) shear strain γ, (iii) Poisson's ratio ν, in the elastic straining of a solid.

† **2.**   (a) Figure 1 shows a typical element of material with Young's modulus E and Poisson's ratio ν, which is subjected to normal stresses σ_1, σ_2 and σ_3. Derive simultaneous equations for the strain in each direction ε_1, ε_2 and ε_3, as a function of these stresses.

(b) Define the dilatation, and derive an expression for the dilatation for a general state of strain ε_1, ε_2 and ε_3 (assuming that the strains are small).

(c) Define the bulk modulus, K, and write down the expression relating K to E and ν (from the Databook), explaining how it is derived. Compare K and E for metals ($\nu \approx 0.3$) and for rubber $\nu \approx 0.5$? What does the value for rubber mean physically?
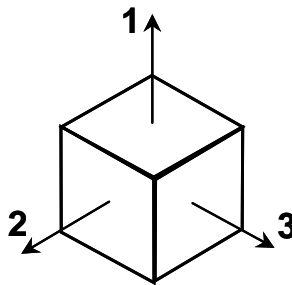
Figure 1

3. A butyl rubber mounting is designed to cushion a sensitive device against shock loading. It consists of a cube of the rubber of side-length $L = 40\text{mm}$ located in a slot in a rigid plate, as shown in Figure 2. The slot and cube have the same cross-sectional dimensions, so that the strain in one direction is constrained to be zero. The slot surfaces are lubricated so that friction on the cube is negligible. The compressive load is $F = 50$ N.

Find the deflection δ of the block in the direction of loading, and hence the stiffness (F/δ) in this constrained condition. By what factor is the stiffness increased, compared to loading the block of rubber without transverse constraint? (Use a mid-range value of E for butyl rubber from the table in the Materials Databook, and assume $\nu = 0.5$.)

[Hint: use the expression derived in lectures for the "effective modulus" (σ_1/ε_1) in terms of Young's modulus E and Poisson's ratio ν, for the case of constraint in one transverse direction.]
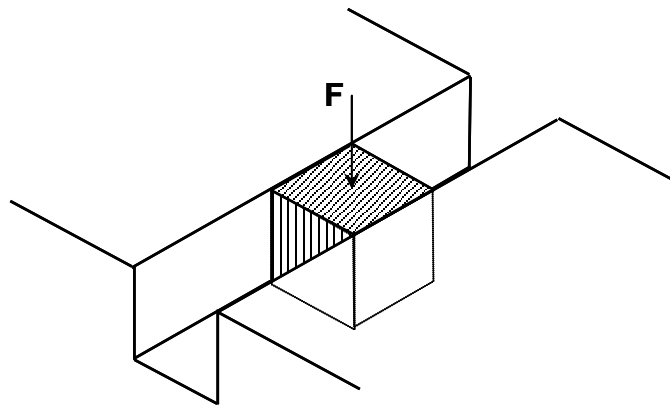

Figure 2

* **4.** A cube of linear elastic material (as in Figure 1) is again subjected to a compressive stress σ_1 in the 1-direction, but is now constrained ($\varepsilon = 0$) in both the 2 and 3 directions.

(a) Derive an expression for the "effective modulus" (σ_1/ε_1) in this case.
[Hint: first find σ_2 and σ_3 in terms of σ_1.]

(b) Sketch the variation of effective modulus with ν and comment on the limiting values when $\nu = 0$ (foam) and $\nu \approx 0.5$ (rubber).

(c) A variant is proposed in the design of the rubber mounting (Figure 2). The block is to be replaced with an axially loaded cylinder of rubber completely filling a cylindrical hole in the steel plate. Use the result in (a) to explain whether this is a good idea.

(d) Explain why the rubber soles of running shoes are designed with some combination of air or gel pockets, partially foamed rubber, and a tread.

* **5.**   Square porcelain tiles are to be manufactured with a uniform layer of glaze, which is thin compared to the thickness of the tile, as shown in section in Figure 3. They are fired at a high temperature T_1 and then cooled slowly to room temperature T_o (= 20°C). The relevant coefficients of thermal expansion and Young's moduli for the glaze and the porcelain are defined in the figure.

(a) Derive an expression for the thermal strain in the glaze, assuming first that it is unconstrained by the porcelain. Use superposition to derive an expression for the stress in the glaze after cooling. What properties determine whether the stress is tensile or compressive?
[Hint: note that the induced thermal stress is biaxial, with the same value in two perpendicular directions].

(b) Two tile types are to be manufactured from porcelain. The first type is decorative, using tension in the surface to form a mosaic of cracks. For the second type, compression in the surface is desirable (to resist surface cracking and give improved strength). Use the data below to select a suitable glaze (A, B or C) for each of these applications.

Firing temperature = 700°C
Young's modulus of glaze = 90 GPa
Poisson's ratio of glaze = 0.2
Tensile failure stress of glaze = 10 MPa

Coefficients of thermal expansion:

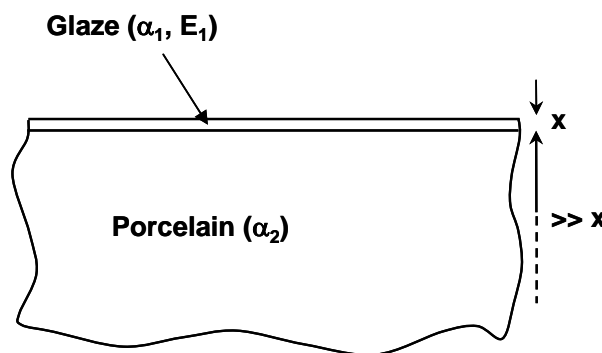|	$\alpha \times 10^{-6}$ (K^{-1})
Porcelain	2.2
Glaze A	1.5
Glaze B	2.3
Glaze C	2.5

Figure 3

6.   (a) Iron has a BCC crystal structure (known as the α phase, or ferrite) at room temperature (20°C). Its theoretical density at this temperature was found in Examples Paper 1 (Exercise E6a). Given that the linear thermal coefficient of expansion of iron = 12×10^{-6} K^{-1}, determine the density of BCC iron at 910°C.

(b) At 910°C BCC iron can transform to an FCC form (the γ phase, or austenite). What is its density at this temperature after the transformation? (Hint: consider the atomic packing fraction). Assuming that there is no change in phase or in expansion coefficient from 910 to 1300°C, sketch how the length of a sample of iron would change with temperature from 20°C to 1300°C. (Note: dimensional changes of this type have been used extensively as a simple method for quantifying phase changes in iron alloys).

Elasticity of Solids

7.   (a) Use the modulus-density property chart in the Materials Databook to identify the ranges of Young's modulus for ceramics, metals and polymers. Explain briefly how the nature of the bonding in these material classes accounts for the relative magnitude of the elastic properties.

(b) Below the glass-transition temperature T_g, most polymers have a modulus of order 1-2 GPa. Above T_g, the different classes of polymers show wide variations in elastic response: some are barely affected, in others the modulus falls to ≈1 MPa or less. Sketch the variation of Young's modulus with temperature for the different classes of polymer, and explain the following in terms of bonding and microstructure:

      (i)   the elastic response of a typical amorphous polymer;
      (ii)   the effect of crystallinity on the response;
      (iii)   the effect of cross-linking.

Explain why some polymers are much easier to recycle than others.

Manipulating Elastic Properties: Foams and Composites

8. Assuming the relationship $E_{foam} = E_{solid} \left(\dfrac{\rho_{foam}}{\rho_{solid}} \right)^2$, estimate the range of modulus-density combinations which might be achievable by foaming the following metals to relative densities in the range 0.1 – 0.3: Ni alloys, Mg alloys. [Use mid-range data from the Materials Databook]. Locate approximate "property bubbles" for these new metallic foams on the E- ρ chart in the Materials Databook. With which existing materials would these foams appear to compete, in terms of modulus and density? Think of a property for which the metallic foams will differ significantly from the competition.

9. Medical prosthetic implants such as hip replacements have traditionally been made of metals such as stainless steel or titanium. These are not ideal as they are much stiffer than the bone, giving relatively poor transfer of load into the bone. New composite materials are being developed to provide a much closer stiffness match between the implant material and the bone. One possibility uses high density polyethylene (HDPE) containing particulate hydroxyapatite (HA, the natural mineral in bone, which can be produced artificially). Data for some experimental composites are provided below.

(a) Plot the upper and lower bounds for the Young's modulus of HDPE-HA composites (formulae in the Materials Databook) together with the experimental data.

(b) Which of the bounds is closer to the data for the particulate composite? Use the experimental data to identify the volume fraction of hydroxyapatite particulate that is required to match the stiffness of bone. Is this practical?

[Young's modulus (typical values): Bone 7 GPa, Hydroxyapatite 80 GPa, HDPE 0.65 GPa]

Volume fraction of HA	Young's modulus (GPa)
0	0.65
0.1	0.98
0.2	1.6
0.3	2.73
0.4	4.29
0.45	5.54

* **10.** (a) Derive an expression for the density ρ_c of a composite containing a volume fraction V_f of reinforcement, in terms of the densities of the matrix ρ_m and of the reinforcement ρ_f. Does the result depend on the form of the reinforcement (fibres or particulate)?

(b) A composite material consists of parallel fibres of Young's modulus E_f in a matrix of Young's modulus E_m.   The volume fraction of fibres is V_f. Derive an expression for E_c (Young's modulus of the composite along the direction of the fibres) in terms of E_f, E_m and V_f.

(c) Use the material property data in the Table below to find ρ_c and E_c for the following composites: (i) carbon-fibre/epoxy resin ($V_f = 0.5$), (ii) glass-fibre/polyester resin ($V_f = 0.5$).

Material	Density (Mg m^{-3})	Young's modulus (GPa)
Carbon fibre	1.90	390
Glass fibre	2.55	72
Epoxy resin	1.15	3
Polyester resin	1.15	3

(d) A magnesium company has developed an experimental metal-matrix composite (MMC), by casting magnesium containing 20% (by volume) of particulate SiC. Use the lower bound estimate for Young's modulus (Materials Databook) to estimate E_c for this MMC. Mid-range data for E and ρ of Mg alloys and SiC should be taken from the Materials Databook.

(e) Find the *specific stiffness* E/ρ for the three composites in (c,d). Compare the composites with steels, for which $E/\rho \approx 28$ GPa Mg^{-1} m^3.

Answers

2. (a) $\Delta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

   (c) $K = E/3(1-2\nu)$

3. $(F/\delta) = 80$ N/mm; 33% stiffer.

4. $\dfrac{\sigma_1}{\varepsilon_1} = \dfrac{E(1-\nu)}{(1-\nu-2\nu^2)}$

5. (a) Thermal strain $= (\alpha_1 - \alpha_2)(T_1 - T_o)$; Thermal stress $= \dfrac{E}{(1-\nu)}(\alpha_1 - \alpha_2)(T_1 - T_o)$

   (b) Glaze C, Glaze A.

6. (a) BCC iron at 910°C: $\rho = 7629$ kg m^{-3}

   (b) FCC iron at 910°C: $\rho = 8300$ kg m^{-3}

8. Ni alloy foam: 2.05 – 18.5 GPa

   Mg alloy foam: 0.45 – 4.0 GPa

9. Approx. 50% HA

10. (a) $\rho_c = \rho_f V_f + (1-V_f)\rho_m$

    (b) $E_c = E_f V_f + (1-V_f)E_m$

    (c) Carbon-fibre epoxy: $\rho_c = 1.53$ Mg m^{-3}, $E_c = 197$ GPa

       Glass-fibre polyester: $\rho_c = 1.85$ Mg m^{-3}, $E_c = 37.5$ GPa

    (d) $\rho_c = 2.1$ Mg m^{-3}, $E_c = 54$ GPa

Suggested Tripos Questions

2008   Q7(c,d)
2009   Q7, 11(a,b)
2010   Q8
2011   Q7
2012   Q8(b), 10(c), 11(a)
2013   Q11

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