ISSUE DATE
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Part IA Paper 1: Mechanical Engineering

THERMOFLUID MECHANICS

Examples Paper 2

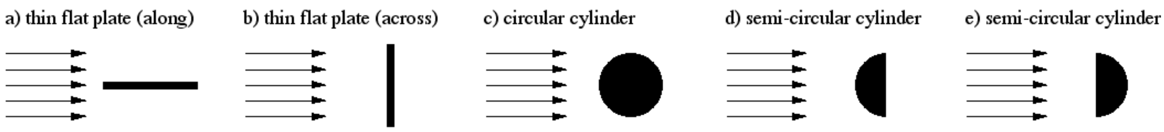
Starter questions are marked S
Straightforward questions are marked †
*Tripos-standard questions are marked **

Introduction and Terminology of Fluid Dynamics

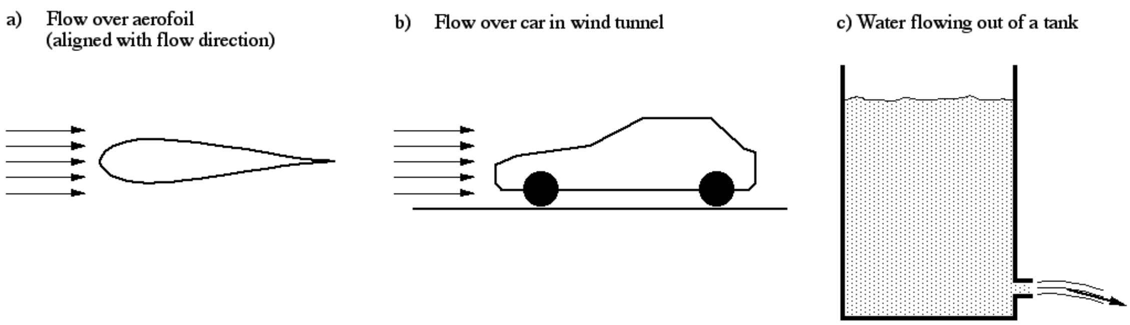
†1. Each of the following statements is either true or false. Identify the appropriate category for each one and give a practical example or illustrate your answer.

(a) The velocity of the fluid at a stationary solid wall is zero (in viscous flow)
(b) The velocity of the fluid at a stationary solid wall is zero (in inviscid flow)
(c) Streamlines can terminate at solid boundaries.
(d) Streamlines always represent particle trajectories.
(e) The velocities at all points along a streamline must be equal.
(f) In steady flow, the streamlines have a fixed shape.
(g) Streamlines can cross one another. (Exclude cases where the velocity is zero.)
(h) In a steady flow, individual fluid particles experience no acceleration.
(i) When flow separates a vacuum is formed.

†2. Sketch the flow patterns you would expect around the shapes given below. In each case, mark the location of the stagnation point, any regions of separated flow and separation points (if you think there are any) and indicate whether the flow is likely to be unsteady or steady.



†3. Are the following flows mainly steady or unsteady?



†4. As you will learn later in the course the relationship between density ρ, Temperature T and pressure p of a gas is:

$$\rho = \frac{p}{R\,T} \text{ (where } R \text{ is a constant).}$$

For liquids we can use the bulk modulus K to relating pressure changes to density changes (at constant temperature):

$$\frac{\rho_1}{\rho_2} = 1 + \frac{p_1 - p_2}{K}$$

For water an approximate value is: $K \approx 2.2\times10^9$ N/m^2.
If a 5% variation in density is to be considered negligible what pressure increase Δp from atmospheric pressure (10^5 Pa) can we allow for
(a) air,
(b) water,
at constant temperature?

†5. The new Airbus A380 is to have a typical take-off mass of around 500 tons. The wing area at take-off is likely to be around 600 m^2. Estimate the average pressure difference Δp needed between the upper and lower surface of the wing to generate sufficient lift at take-off. Would it still be OK to treat the flow around the wings as incompressible?

Control volumes and conservation equations

Notes:

- **Be clear about what is (and is not) included in the control volume**
- **Indicate the direction of the coordinate system and be consistent with signs**
- **Be clear about whether the forces are acting on the fluid or on the surrounding structure.**

Mass conservation

S6. A water tank receives a continuous flow rate of 0.2 kg/s and delivers 0.1 kg/s to an outlet. What is the rate at which the tank water mass grows? If the initial water mass is 100 kg, when will it have 1000 kg?

S7. Water with a density of 1000 kg/m^3 flows through a channel of rectangular cross section of 1 m by 2 m at a volumetric flow rate of 5400 m^3/h. Determine the velocity in m/s and the mass flow rate in kg/s.

†8. A mixing tank has two inlets and one outlet, all of the same area, 0.01 m^2. Liquids of density 825 kg/m^3 and 985 kg/m^3 flow into the tank through separate inlets. They have uniform inlet velocities of 2.4 m/s and 1.6 m/s respectively. Mixing is assumed to be complete, and the conditions at the tank outlet are uniform. Assuming steady state conditions, find
(a) the mass flows into and out of the tank
(b) the mean density and velocity of the mixture flowing out of the tank
Note: The mean velocity can be calculated based on the assumption of incompressible flow.

9. A cylindrical tank with base area A and maximum height H receives a steady flow rate $\dot{m}_1$ from a tank, and drains a mass flow rate $\dot{m}_2$. The inlet pipe has a radius R and uniform velocity V_o. The outlet pipe also has the same radius R and a velocity profile $v(r) = V_o(1-(r/R)^2)$. At time t=0, the volume of liquid in the tank is at

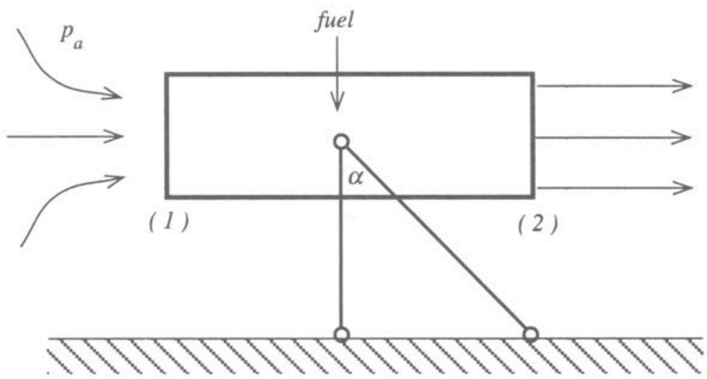
level h_o from the base. The density of the fluid is constant and equal to ρ. In each case, determine the expression as a function of R, V_o, A, H, h_o, and ρ, as necessary.
(a) Determine the mass flow rates through the inlet and outlet.
(b) Determine the rate of mass accumulation in the tank.

Steady Flow Momentum Equation

S10. Ping pong balls of 10 g of mass hit a wall with a velocity of 1 m/s, and bounce back in the opposite direction with the same absolute velocity.
(a) Determine the overall change in momentum for each ball.
(b) If the average number of ping pong ball hits per unit time is 10 per second, determine the momentum flux into and out of the wall, and the average force exerted by the balls on the wall.

11. A pipe of 2.10^{-4} m^2 in area delivers a flow of 1 kg/s of water (ρ=1000 kg/m^3) into a mixing tank at a 30 degree angle with the normal to the entrance. Determine:
(a) The velocity of the flow in the pipe
(b) The normal and tangential velocity components into the tank
(b) The momentum flow in the normal and tangential directions.

*12. A gas turbine is being tested under steady flow conditions in the rig shown in the figure above. The engine thrust is balanced by the tension force F in the diagonal strut, which makes an angle $\alpha = 45^o$ with the vertical. The mass flow rate of air from the ambient atmosphere into the engine is $\dot{m}_a$= 100 kg/s at a density ρ_1 = 1.2 kg/m^3. The pressure at the inlet is p_1=9.167.10^4 Pa. The mass flow rate of fuel into the engine is $\dot{m}_f$ = 2 kg/s. The exhaust gas density is ρ_2 = 0.4 kg/m^3. The intake and exhaust flow areas are both equal, $A_1 = A_2$ = 1 m^2. The outlet pressure is equal to atmospheric pressure p_a = 10^5 Pa. Calculate
(a) the inlet and outlet velocities V_1 and V_2
(b) the tension F in the diagonal strut.



*13. A submerged submarine is towed horizontally at a steady speed U in deep, still water. Far behind at a fixed distance from the submarine an axially-symmetrical wake is formed in which the water velocity may be assumed to vary from U on the axis to zero at radius R. as follows:

$$u(r) = U\left(1 - \left(\frac{r}{R}\right)^2\right)$$

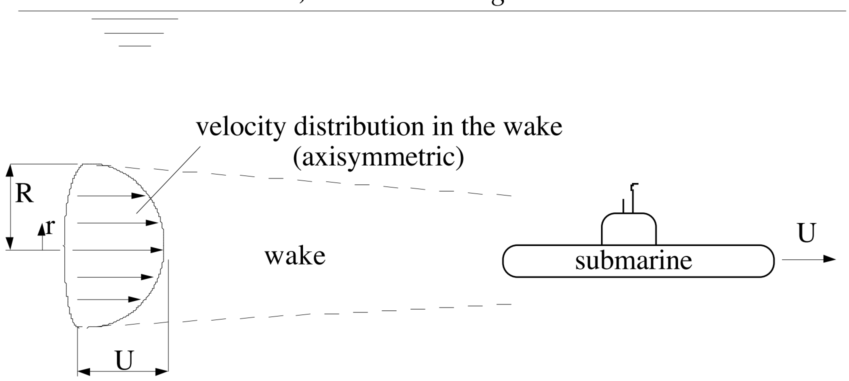
The variation of the water pressure with depth may be assumed to be unaffected by the presence of the submarine. The density of the water is ρ.

(a) In order to use the continuity and steady flow momentum equations we employ a moving control volume, in which the submarine appears stationary. Why do we need to do this? What is the velocity distribution in the wake relative to the moving control volume?

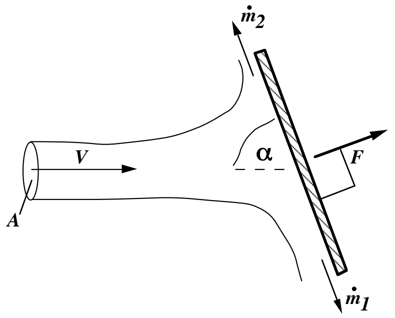
(b) Show that the drag on the submarine is given by:

$$F_D = \pi\rho U^2 R^2 / 6$$

Hint: Before applying momentum equation, make sure all mass flows are accounted for in the control volume used. There are two obvious control volumes to take, which should give the same answer.



*14. The following example was used in the lecture:



In the lectures the force F was calculated by using a reference co-ordinate system aligned with the plate direction. Show that you can obtain the same result using a co-ordinate system aligned with the incoming jet (jet direction = x-direction) and determine the mass fluxes $\dot{m}_1$ and $\dot{m}_2$ as well as the force on the plate.
You may assume that the flow speeds at 1 and 2 are equal to that of the jet *V* and that the fluid pressures in the incoming jet as well as at 1 and 2 are all atmospheric.

Forces on fluid particles

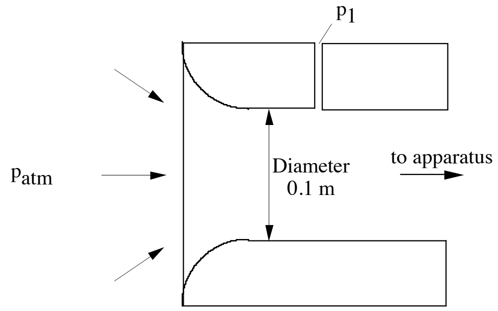
15. (a) Show that a fluid particle moving in a pressure gradient experiences a resultant pressure force which is proportional to its volume and the magnitude of the pressure gradient (you need only consider gradients in one co-ordinate direction, eg *dp/dx*).

(b) Then show that under hydrostatic conditions this is equivalent to Archimedes' principle.

Bernoulli's Equation

16. Consider two identical tanks filled with water to the same height *H*. One of the tanks has a hole of area *A*; the second tank has a tube of height *h* and area *A* connected to the hole. The flow can be considered inviscid. If both holes are open at the same time, which one will drain faster? Explain why.

†17. Many experimental rigs (e.g. the turbocharger experiment) use a 'Bell-mouth' to measure the mass flow of air into an apparatus.

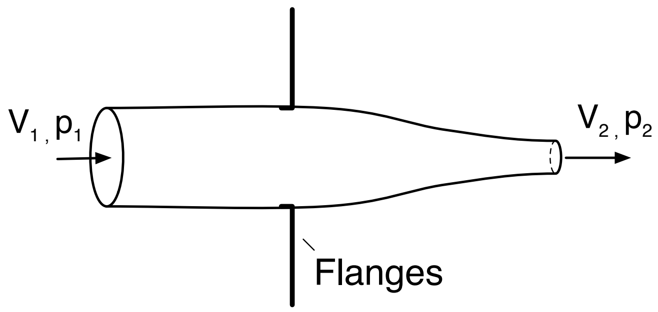


The air density outside the bell-mouth is ρ=1.22 kg/m^3. Consider a volume flux of 100 litres/s through a bell-mouth with 10 cm diameter.

(a) What pressure difference p_{atm}-p_1 would be measured (assuming air to be incompressible). Is the assumption of incompressibility justified?

(b) If a vertical water manometer is used to measured the pressure difference indicated, what is the height of the water column difference?

*18. The figure below shows air flowing through a nozzle. The inlet pressure is p_1 = 105 kPa and the pressure in the exhausting jet is p_2 =101.3 kPa (which is equal to the ambient pressure at the nozzle exit). The nozzle has an inlet diameter of 60 mm, an exit diameter of 10 mm, and the nozzle is connected to the supply by flanges. Assume the air has a constant density of 1.22 kg/m^3, and there are no viscous forces. Neglect the weight of the nozzle.
(a) Determine the air speed at the exit of the nozzle
(b) Determine the necessary force to hold the nozzle stationary.



Answers

1. a) yes, b) no, except at stagnation points, c) yes, d) not in unsteady flow, e) no, f) yes, g) no, h) no, i), no

2 & 3. Discuss in supervisions

4. 5000 Pa, 105 Mpa

5. 8175 Pa (borderline case)

6. 9000 seconds

7. 0.75 m/s, 1500 kg/s

8. (a) 19.80 , 15.76 and 35.56 kg/s (b) 889 kg/m^3 and 4.0 m/s

9. (a) $\dot{m}_1 = \rho V_o \pi R^2$, $\dot{m}_2 = \rho V_o \pi R^2 / 2$ (b) $\rho V_o \pi R^2 / 2$

10.	(a) 0.02 kg.m/s	(b) 0.1 N each way for a 0.2 N force	
11.	(a) 5 m/s	(b) 4.3 and 2.5 m/s	(c) 4.3 and 2.5 N
12.	(a) 83.3 m/s 255 m/s	(b) 36.8 kN	
14.	$\frac{1}{2}\rho AV(1+\cos\alpha)$	$\frac{1}{2}\rho AV(1-\cos\alpha)$	$\rho AV^2 \sin\alpha$
16.	The second tank		
17.	(a) 99 Pa	(b) 1.0 cm	
18.	(a) 77.9 m/s	(b) –9.89N	

Recommended Tripos Questions (Part 1 Section A)

2012	1, 2, 3
2010	1, 2
2009	1, 5 (except c)
2004	1
2003	1, 2