ISSUED ON
31 JAN 2014

Engineering Tripos Part IB SECOND YEAR

Part IB Paper 1: Mechanics

Examples Paper 2

Rigid Body Accelerations and Forces

Straightforward questions are marked †
*More challenging questions are marked *.*

†1. (i) Derive, from first principles, the *polar* moment of inertia, J of the thin disc shown in Fig. 1(a). The mass of the disc is m and its radius R. Recall that

$$J = \int r^2 \, \mathrm{d}m \ . \tag{1}$$

(ii) Derive the perpendicular axis theorem:

$$J = I_{xx} + I_{yy}$$

by substituting $r^2 = x^2 + y^2$ into equation (1). Use this theorem to calculate the *diametral* moment of inertia $I = I_{xx} = I_{yy}$ of the disc. Confirm your result using the Mechanics Data Book.

(iii) Derive, from first principles, the moment of inertia I_{yy} of the thin rod of mass m and length ℓ shown in Fig. 1(b). Confirm the result using the Mechanics Data Book.

(iv) The rod and disc are welded together as shown in Fig. 1(c). Determine the location of the centre of mass G of the combined system (calculate the coordinate $\bar{x}$). Use the parallel axis theorem to calculate the moment of inertia I_{yy} of the combined system about G.

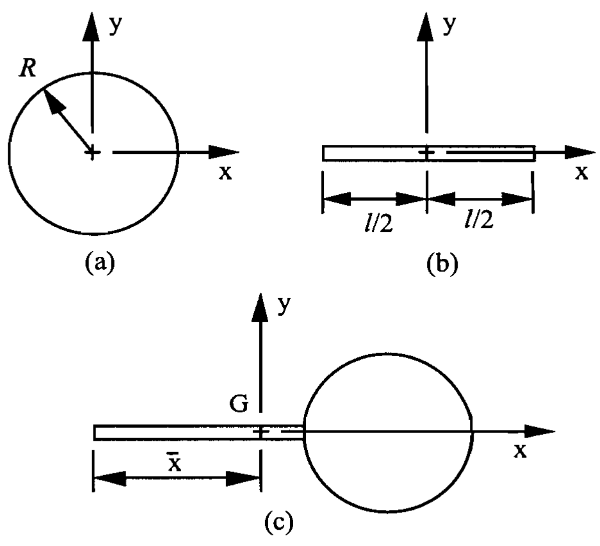

Fig. 1

†2. In Fig. 2, the uniform rod AB of length $2l$ and mass m has an extra concentrated mass m attached at A. The rod is moving freely under the action of gravity and a horizontal force $P = mg$ applied at B. At the given instant the rod is inclined at 30° to the horizontal and has zero angular velocity.

(i) What is the moment of inertia of the system about an axis through its centre of mass G perpendicular to the rod ?

(ii) What are the vertical and horizontal components of the acceleration of G ?

(iii) What is the angular acceleration of the rod ?

(iv) What are the vertical and horizontal components of the acceleration of B ?

(v) Which of these answers would be different if the angular velocity of the rod were not zero at the given instant ?

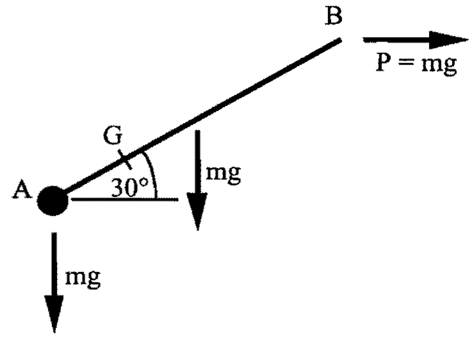

Fig. 2

3. A thin uniform circular wire ring of mass m and radius a is supported vertically by two pegs at the same level $a\sqrt{2}$ apart, with its centre below the pegs. One peg is removed. Assuming no slip at the other peg, find the initial angular acceleration of the ring and so determine the normal and tangential components of the reaction at the remaining peg. Hence show that there will be no initial slip if the coefficient of friction exceeds 0.5.

4. A uniform ladder is held vertically against a wall standing with its foot in the angle between the wall and the floor. The top of the ladder is moved a short distance away from the wall and then released.

(i) If the floor is rough so that the foot of the ladder does not slip, calculated the magnitude and direction of the horizontal force acting on the foot of the ladder just before the ladder strikes the ground.

(ii) Now assume that the floor is smooth. Through what angle does the ladder turn before its foot starts to come away from the wall ?

Try this with a 300mm ruler on your desk.

*5. A uniform plank of length $2l$ is held with its centre of gravity projecting a distance a beyond the edge P of a horizontal shelf, as shown in Fig. 3. The plank is released from its horizontal position and initially rotates about edge P without slipping. During this motion, its inclination to the horizontal is θ.

(a) Calculate the angular velocity and acceleration $\dot{\theta}$ and $\ddot{\theta}$ during the initial rotation.

(b) Show that the plank will rotate about the P until θ reaches the value given by

$$\tan\theta = \frac{l^2}{9a^2 + l^2}\tan\phi,$$

where $\tan\phi$ is the coefficient of friction between the plank and the shelf.

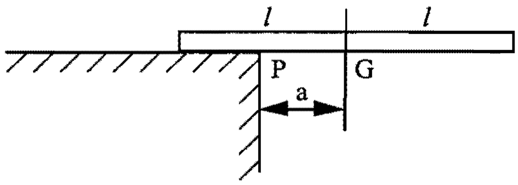

Fig. 3

6. A uniform cylinder of radius r and mass m rests on a rough slope with its axis horizontal, as shown in Fig. 4. One end of a light string is wrapped around and attached to the curved surface of the cylinder, and the other end is fixed to a light spring of stiffness k, mounted such that the straight part of the string is always parallel to the slope.

†(i) The cylinder is initially held with the string just taut and the spring unstretched, and then gradually allowed to roll down to its equilibrium position. How much will the spring extend by, if the cylinder rolls though an angle α?

(ii) Assuming that no slipping takes place, write down an expression for the total energy of the mass-spring system as a function of the angle through which the cylinder rotates, and hence find the angle through which the cylinder has rotated when it reaches equilibrium. Use this to find the tension in the string at equilibrium, and check your answer by statics.

(iii) The cylinder is now rolled a small amount away from its equilibrium position, again without slipping, and released. Find the frequency of the resulting oscillations. (You are advised to use Section 4.1 of the Data Book.)

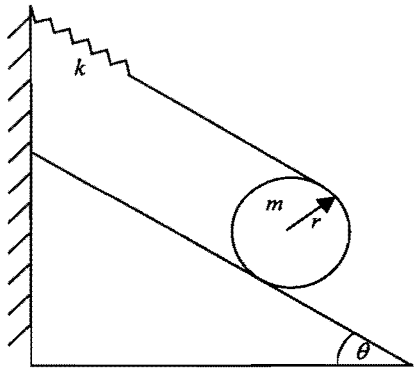

Fig. 4

*7. In the mechanism shown in *plan* in Fig. 5, AB is a uniform rigid rod hinged to a fixed point A and to the mid-point of a similar second rod CD of distributed mass m. AB = CD = l. The motion of CD is constrained by a light tie-rod CE also of length l.

The mechanism is initially at rest in the position indicated with CD parallel to AB and the angle DCE equal to 60°. A torque T is applied to AB to give it an angular acceleration α.

(i) Find the initial angular acceleration of CD and the initial acceleration of point B.

(ii) Determine the value of T and the corresponding force in CE.

Assume that AB and CD are parallel and that the distance between them is negligible. Note that gravity does not act in the plane of the mechanism.

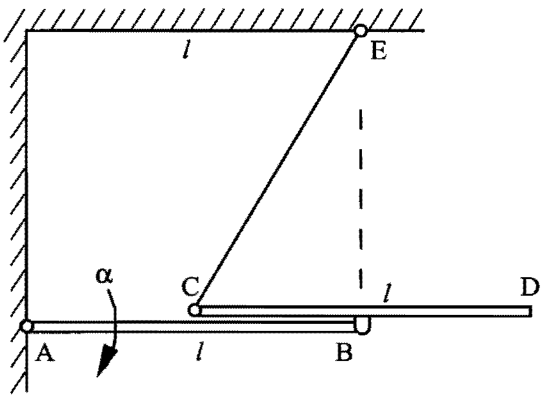

Fig. 5

*8. The mechanism shown in plan in Fig. 6 consists of two *light* links AB (of length a) and CD (of length $a\sqrt{2}$), and a uniform *heavy* link BC of mass m and length a. The joints are frictionless. For the position shown with AB rotating at constant angular velocity ω, find:

(i) the angular velocity of CD;

(ii) the angular acceleration of BC, and the acceleration of the centre of mass of BC;

(iii) the instantaneous value of the driving torque T necessary to maintain the motion.

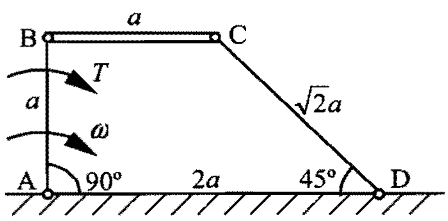

Fig. 6

For further practice try the following IB Mechanics Tripos questions:

1996 Q5; 1998 Q5, 6; 2000 Q4; 2001 Q3, 4; 2002 Q6; 2003 Qs 1, 2; 2004 Qs 2, 4; 2006 Q 2a,b; 2007 Q1; 2008 Q3.

ANSWERS

1 (i) $mR^2/2$ (ii) $mR^2/4$ (iii) $ml^2/12$

(iv) $3l/4 + R/2$, $M\left\{\frac{5l^2}{24}+\frac{lR}{2}+\frac{3R^2}{4}\right\}$

2 (i) $5Ml^2/6$ (ii) g downwards, $g/2$ to the right (iii) $0.9g/l$

(iv) $(1+\frac{27\sqrt{3}}{40})g$; $\frac{47g}{40}$ (v) only the acceleration components of B.

3 $\ddot{\theta}=\frac{g}{2a\sqrt{2}}$, $N=\frac{mg}{\sqrt{2}}$, $T=\frac{mg}{2\sqrt{2}}$

4 (i) $\frac{3}{2}mg$ towards wall; (ii) $\cos^{-1}(2/3)$

5 $\dot{\theta}^2=\frac{6ag\sin\theta}{l^2+3a^2}$ $\ddot{\theta}=\frac{3ag\cos\theta}{l^2+3a^2}$

6 (i) $2r\alpha$ (ii) $\frac{mg\sin\theta}{4rk}$, $\frac{mg\sin\theta}{2}$ (iii) $\omega_n=\sqrt{\frac{8k}{3m}}$

7 (i) 2α clockwise, $l\alpha$ perpendicular to the rod, away from E (ii) $\frac{5}{3}ml^2\alpha$, $\frac{2}{3\sqrt{3}}ml\alpha$

8 (i) ω clockwise (ii) $2\omega^2$, $\frac{1}{2}a\omega^2$ to the left and $2a\omega^2$ downwards (iii) $-\frac{5}{3}ma^2\omega^2$

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