ISSUED ON
12 FEB 2014

Engineering **FIRST YEAR**

Part IA Paper 2: Structures and Materials

MATERIALS

Examples Paper 3 – Stiffness-limited Design; Plastic Deformation and Properties

Straightforward questions are marked with a †

Tripos standard questions are marked with a *

You will need to look up data in the Materials Databook, and use the Cambridge Engineering Selector (CES) software.

Material Selection: Stiffness-limited Design

Questions 1-3 introduce material selection in design. All can be solved using the Materials Databook or the *Cambridge Engineering Selector (CES)* software (though it is usually easier in software). Try both methods, at least for some of the questions.

CES may be downloaded from the CUED Website (PCs running Windows only; instructions provided in lecture notes). CES is also available on PWF. Complete the *CES Tutorial* (or follow the online audio-video demonstrations) before doing the CES problems.

1. (a) A component is made from brass (a copper alloy). Identify three alternative classes of alloy which offer higher Young's modulus for this component.

(b) Find materials with $E > 40$ GPa and $\rho < 2$ Mg/m^3, and identify the cheapest.

(c) Find metals and composites that are both stiffer and lighter than: (i) steels; (ii) Ti alloys; (iii) Al alloys.

(d) Compare the specific stiffness, E/ρ, of steels, Ti alloys, Al alloys, Mg alloys, GFRP, CFRP and wood (parallel to the grain).

(e) Comment on the usefulness of the approaches in (c) and (d) for seeking improved performance in lightweight, stiffness-limited design.

(CES tips: in (d), either plot E-ρ and use a line of slope unity, as in the Databook, or use the "Advanced" feature in a graph stage to plot a bar chart of E/ρ).

2. Polymer ropes and lines for use on water are often designed to float, to aid in their retrieval and to avoid applying a downwards load to an object or person attached to them in the water. Excessive stretch is undesirable, so a lower limit of 0.5 GPa is also imposed on Young's modulus.

(a) Identify suitable polymers (in the Databook, refer to the tables of data rather than the charts, which do not show individual polymers).

(b) Look at the environmental resistance of these polymers, and comment on any possible weaknesses for this application.

(c) Use the information for material applications to see if the polymers identified are used in practice.

(CES tips: (a) requires a limit or graph stage; (b) requires a limit stage, or browsing on the results of (a); (c) requires browsing, or use of the Search facility).

3. A sailing enthusiast is seeking materials for lightweight panels to use in a sea-going yacht. The panels are of rectangular cross-section and will be loaded in bending, as shown in Figure 1. The span L and width b of the panels are fixed, but the thickness d may vary (up to a maximum specified value). The required stiffness is specified as a maximum allowable deflection δ under a given central load W, for a simply supported span. The designer is interested in two scenarios: (i) minimum mass; (ii) minimum material cost. In either case, the environmental resistance to sea water must be above average.

NB: The central deflection of a simply supported span under a point load is given by:

$$\delta = \frac{W L^3}{48 E I} \quad \text{where } I = \frac{b d^3}{12} \quad \text{(from Structures Databook).}$$

(a) Show that the stiffness and geometric constraints lead to the relationship $E d^3 = \text{constant}$, where E is Young's modulus. Explain why the design specification leads to a minimum allowable value for E.

(b) Write down an expression for the first objective to be minimised (mass) and use the stiffness constraint to eliminate the free variable (thickness). Hence define the material performance index to *maximise* for a minimum mass design.

(c) On a Young's modulus – density property chart, how are materials with the best values of the performance index identified? Use this method to identify a short-list of candidate materials (excluding ceramics and glasses - why is this?) Comment on the influence on your candidate materials of the other factors in the design specification:

(i) environmental resistance to salt water;

(ii) maximum allowable thickness (for which assume $E > 5$ GPa is required).

(d) Modify your material performance index to describe the alternative objective of minimum material cost. Use a suitable material property chart (in CES) to identify a short-list of materials (excluding ceramics and glasses as before, and noting materials which may be excluded on grounds of inadequate resistance to sea water or excessive thickness).

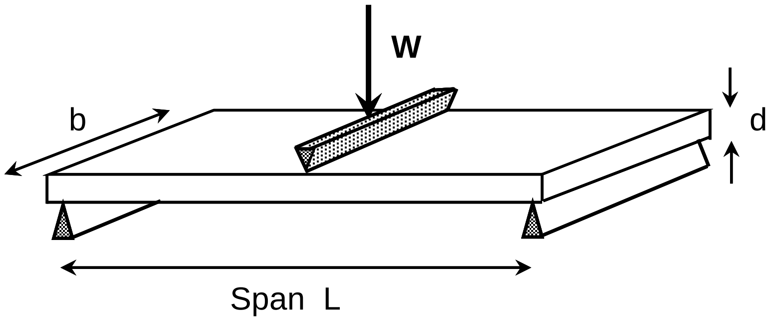

Figure 1

Plastic Deformation

† **4.** (a) Give the meaning of the yield stress σ_y, the tensile strength σ_{ts}, the hardness H, and the tensile ductility (or elongation) ε_f of a material. Explain why the ductility is not strictly a material property.

(b) The 0.1% proof stress is often quoted for metals which do not have a distinct yield point. Explain the meaning of a proof stress.

5. (a) Write down expressions for the nominal strain and the true strain for the following situations:

(i) in a tensile test where the length of the specimen increases from ℓ_0 to ℓ;

(ii) in a compression test where the height of the specimen decreases from h_0 to h.

(b) In what circumstances can the true strain be approximated by the nominal strain?

6. (a) In a two-stage elongation of a specimen, the length is first increased from ℓ_0 to ℓ_1 and then from ℓ_1 to ℓ_2. Write down the true strains ε_1 and ε_2 respectively for each of the two elongation processes considered separately. Show that the **true** strain for the overall elongation ℓ_0 to ℓ_2 is given by the sum of the true strains for each of the two separate processes. Is the same true for the **nominal** strains?

(b) In a “tandem” rolling mill, wide metal strip passes continuously between sets of rolls, each of which apply a reduction in thickness, as shown in Figure 2. The strip elongates without getting wider, and therefore accelerates. For an incoming thickness h_{in} = 25mm moving at speed v_{in} = 5cms^{-1}, and an exit thickness h_{out} = 3.2mm, calculate:

(i) the overall true strain applied to the material;

(ii) the exit speed v_{out}.

State any assumptions made.

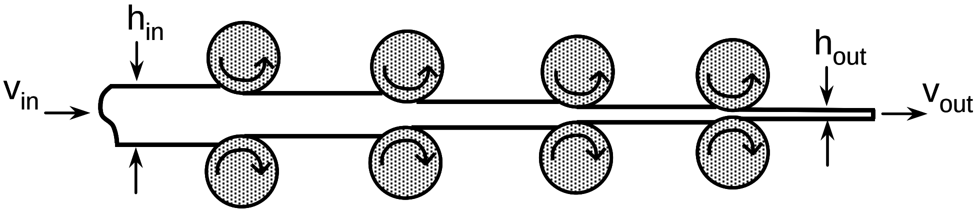

Figure 2

7. When a nuclear reactor is shut down quickly, the temperature at the surface of a thick stainless steel component falls from 600°C to 300°C in less than a second. Due to the relatively low thermal conductivity of the steel, the bulk of the component remains at the higher temperature for several seconds.

(a) Explain why the stress and strain in the underlying material is effectively zero, and hence state the sum of the thermal, elastic and plastic strains in the surface layer.

(b) Find the thermal, elastic and plastic strains in the surface layer (parallel to the surface), after rapid cooling from 600°C to 300°C. (The coefficient of thermal expansion of stainless steel is 1.2×10^{-5} K^{-1}, and Poisson's ratio $\nu = 0.33$; for other properties use mid-range values from the Materials Databook.)
[Hint: the elastic strain in a surface layer carrying an equal biaxial stress was found in Examples Paper 2, Q5.]

8. (a) By considering the area under a uniaxial load-extension response of a linear-elastic material, and converting to nominal stress and strain, derive an expression for the maximum elastic energy per unit volume which may be stored in the material loaded in tension, in terms of the yield stress σ_y and Young's modulus E.

(b) On a sketch of a typical stress-strain curve for a ductile metal, distinguish between the maximum elastic energy stored per unit volume and the total energy dissipated per unit volume in plastic deformation.

(c) Give examples of engineering applications in which a key material characteristic is:
(i) maximum stored elastic energy per unit volume;
(ii) maximum plastic energy dissipated per unit volume.

Microstructure of Materials: Physical basis of Strength

9. (a) Explain the distinction between the ideal strength of a single crystal, the shear stress τ_y required to move a dislocation in a single crystal, the shear yield stress k of a polycrystalline material, and the tensile yield stress σ_y. Note any approximate relationships between these quantities.

(b) Explain briefly what is meant by a dislocation, and how they enable plastic deformation under an applied shear stress that is much less than the ideal strength of the material. Distinguish between edge, screw and mixed dislocations. Show, with diagrams, how the motion of an edge dislocation leads to an increment of plastic deformation in a crystal.

(c) Explain briefly the three main microstructural mechanisms by which metals are hardened. Identify which mechanisms account for the following increases in yield stress:
(i) Pure annealed aluminium: 25 MPa; cold rolled Al-Mn-Mg alloy: 200 MPa
(ii) Pure annealed copper: 35 MPa; cast 60-40 Brass (60% Cu, 40% Zn): 105 MPa
(iii) Pure annealed iron: 140 MPa; quenched & tempered medium carbon steel: 550 MPa.

(d) For each of the hardening mechanisms discussed in part (c), give other examples of important engineering alloys which exploit these mechanisms.

10. (a) An iron cube of side length 1cm has a dislocation density of 10^8 mm^{-2}. Assuming a square array of parallel dislocations, and an atomic spacing of 0.2 nm, estimate:

(i) The total length of dislocation in the sample;
(ii) The distance between the dislocations;
(iii) The number of atoms between the dislocations.

(b) An aluminium alloy used for making cans is cold rolled into a strip of thickness 0.3mm and width 1m. It is coiled round a drum of diameter 15cm, and the outer diameter of the coil is 1m. In the cold rolled condition, the dislocation density is approximately 10^{15} m^{-2}. Estimate:

(i) The mass of aluminium on the coil;
(ii) The total length of strip on the coil;
(iii) The total length of dislocation in the coiled strip.

11. (a) Derive an expression for the shear stress τ_y needed to bow a dislocation line into a semicircle between small hard particles a distance L apart.

(b) A polycrystalline aluminium alloy contains a dispersion of hard particles of diameter 10^{-8} m and average centre-to-centre spacing of 6×10^{-8} m measured in the slip planes (the shear modulus G of aluminium is 26 GPa and $b = 0.286$ nm). Estimate their contribution to the yield stress of the alloy.

(c) The alloy is used for the compressor blades of a small turbine, which operates at a temperature of 150˚C. At this temperature, atomic diffusion causes the particles to *coarsen* slowly (i.e. increase in average size and spacing). After 1000 hours they have grown to a diameter of 3×10^{-8} m and an average spacing of 18×10^{-8} m. Estimate the decrease in yield stress.

12. Figure 2.2 in the Materials Databook shows the typical nominal stress-strain response of some common polymers. Account for the difference in behaviour of PMMA and polypropylene.

Answers

3. (b) $E^{1/3}/\rho$

6. (b) (i) – 2.06; (ii) 39.1 $\mathrm{cms^{-1}}$.

7. $\varepsilon_{thermal} = -3.6 \times 10^{-3}$; $\varepsilon_{elastic} = 1.96 \times 10^{-3}$; $\varepsilon_{plastic} = 1.64 \times 10^{-3}$

8. (a) $\frac{1}{2}\sigma_y^2 / E$

10. (a) (i) 10^8 m; (ii) 100nm; (iii) 500 atoms.
    (b) (i) approx. 2.1 tonnes; (ii) approx. 2.6 km; (iii) approx. 0.77×10^{15} m.

11. (a) $\tau_y = \dfrac{Gb}{L}$; (b) 450 MPa; (c) 300 MPa.

Suggested Tripos Questions

2006	Q9, 10
2007	Q7, 8
2008	Q8, 12
2009	Q8
2010	Q12
2011	Q10
2012	Q8(a), 10(a), 12
2013	Q8

(NB. Most material selection questions combine material from this and later Examples Papers)

H.R. Shercliff
Lent Term 2014