Version GP/9

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 3 June 2014   2 to 4

Paper 4

THERMOFLUID MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on each cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 (a) A refrigerator uses the refrigerant R-134a as the working fluid. Dry saturated fluid leaves the evaporator at a temperature of -20 °C and enters the adiabatic compressor. The saturation temperature in the condenser is 30 °C but the fluid leaves the compressor superheated by 20 °C. The fluid leaves the condenser wet saturated and passes through a throttle before returning to the evaporator.

(i) Sketch temperature-entropy, *T*-*s*, and pressure-enthalpy, *p*-*h*, diagrams for the refrigerator. State any assumptions made. [4]

(ii) Calculate the specific work input required for the compressor and evaluate its isentropic efficiency. [5]

(iii) Define and evaluate the coefficient of performance (COP) for the refrigerator. [3]

(b) A heat engine, based on a Rankine cycle, uses the refrigerant R-134a as the working fluid. Wet saturated fluid from the condenser at a temperature of 10°C enters the feed pump where its pressure is raised to 26.33 bar. Dry saturated fluid leaves the evaporator and enters the turbine which has an isentropic efficiency of 0.85.

(i) Sketch a *T*-*s* diagram for the heat engine. [3]

(ii) Calculate the specific work output from the turbine. [6]

(iii) Explain how superheating could be used to improve the cycle. [4]

2 (a) The specific steady flow availability function b is defined as

$$b = h - T_0 s$$

where h is specific enthalpy, s is specific entropy and T_0 is the dead state temperature.

(i) Using expressions for the First and Second Laws of Thermodynamics applied to a steady flow process from state 1 to state 2, show that [4]

$$b_2 - b_1 = -w_x \quad + \quad \int_1^2 \left(1 - \frac{T_0}{T}\right) dq \quad - \quad T_0 \Delta s_{irrev}$$

where the symbols have their usual meanings.

(ii) Provide a physical interpretation for each of the terms on the right hand side of this equation. [4]

(b) Two separate streams of air each have a pressure of 20 bar. One stream has a mass flow rate of 20 kg s^{-1} and a temperature of 900 K. The other stream has a mass flow rate of 80 kg s^{-1} and a temperature of 1800 K. The two streams enter a constant pressure co-flow heat exchanger in which heat is transferred until they reach a common temperature T_m. The dead state temperature T_0 is 300 K. The air should be treated as a perfect gas with ratio of specific heats $\gamma = 1.4$ and specific heat capacity at constant pressure c_p = 1.01 kJ kg^{-1} K^{-1}.

(i) Stating any assumptions made, evaluate T_m. [2]

(ii) Calculate the power potential that has been lost in the heat exchanger. What is the cause of this loss? [5]

(iii) The combined flow of air leaving the heat exchanger (at pressure 20 bar and temperature T_m) is passed through an isentropic turbine that has an exit pressure of 1 bar. Calculate the shaft power output from the turbine. [2]

(c) Instead of using a heat exchanger, two separate isentropic turbines, each with an exit pressure of 1 bar, are used to extract work from the two streams of air at the initial conditions given in Part (b). Calculate the total shaft power output from the two turbines. Comment on the comparison of this answer with the results of Part (b). [5]

(d) Identify a process that could be used to combine the two streams in Part (b) with no loss in power potential. [3]

3 (a) Show that for radial heat conduction through an annular layer of thermal conductivity λ, with inner and outer radii of r_1 and r_2 respectively, the thermal resistance per unit length is

$$\frac{\ln(r_2/r_1)}{2\pi\lambda}.$$ [3]

(b) Air flows along a cylindrical pipe of internal diameter d = 0.2 m at a mass flow rate of $\dot{m} = 1$ kg s^{-1}. The temperature T_1 of the air at inlet to the pipe is 50 °C and the pressure may be assumed to be constant at 1 atm. The pipe is made of a plastic with thermal conductivity $\lambda = 0.2$ W m^{-1} K^{-1} and a wall thickness of 10 mm. The pipe is encased in a 50 mm thick annular layer of insulation material with $\lambda = 0.05$ W m^{-1} K^{-1}. The external surface heat transfer coefficient is 200 W m^{-2} K^{-1}. The temperature of the environment external to the pipe is T_∞ and is lower than T_1. The air in the pipe may be treated as a perfect gas with specific heat capacity at constant pressure $c_p = 1.01$ kJ kg^{-1} K^{-1}.

(i) Explain why there are two correlations for the convective heat transfer in circular pipes in the Thermofluids Data Book. [5]

(ii) Evaluating any required fluid properties at temperature T_1, choose the appropriate correlation from the Thermofluids Data Book and evaluate the heat transfer coefficient for the internal surface of the pipe. [7]

(iii) Evaluate the total thermal resistance per unit length R_{tot} between the flow in the pipe and the external environment. [5]

(iv) Find an expression for the variation of air temperature with distance along the pipe x in terms of T_1, T_∞, c_p, R_{tot} and $\dot{m}$. Hence find the length of pipe required to reduce the temperature difference between the air in the pipe and the external environment to half of the inlet value. [5]

SECTION B

*Answer not more than **two** questions from this section.*

4 An incompressible Newtonian fluid flows along a horizontal duct of annular cross section. The flow is axisymmetric and, in cylindrical polar coordinates (r,θ,z), the velocity and shear stress distributions are $u_z(r)$ and $\tau_{rz}(r)$. The inner and outer radii are R_i and R_o respectively.

(a) By considering the force balance on a suitable annular fluid element, show that,

$$\frac{1}{r}\frac{d}{dr}\left(r\tau_{rz}\right) = -\left|\frac{dp}{dz}\right|$$

where dp/dz is the pressure gradient in the duct. [10]

(b) Hence show that the velocity distribution is given by

$$u_z = \frac{1}{\mu}\left|\frac{dp}{dz}\right|\left(A - \frac{r^2}{4} + B\ln r\right)$$

where μ is the dynamic viscosity of the fluid and A and B are constants of integration. [5]

(c) Find an expression for B in terms of R_i and R_o. [5]

(d) Hence find an expression for the viscous force per unit length exerted by the fluid on the inner surface of the duct, $r = R_i$, in terms of R_i, R_o and $\left|dp/dz\right|$. [5]

5 Consider the incompressible flow through a pipe expansion as shown in Fig. 1. The upstream radius is R_1 and the downstream radius is R_2 . The flow is steady-on-average, the turbulence is restricted to the region immediately downstream of the expansion, and the flow well upstream and downstream of the expansion may be considered as uniform, with speeds V_1 and V_2 respectively. To a reasonable approximation, the pressure on the back face of the expansion may be taken as equal to the upstream pressure, p_1 , as indicated in Fig. 1.

(a) Using the control volume shown in Fig. 1, show that the pressure rise across the expansion is

$$p_2 - p_1 = \rho V_2 (V_1 - V_2) \,.$$

You may neglect viscous stresses acting on the control volume. [9]

(b) Show that the loss of mechanical energy Δe per unit volume of fluid flowing through the expansion is

$$\Delta e = \frac{1}{2}\rho (V_1 - V_2)^2 \,. \qquad [8]$$

(c) A sphere is mounted on the centreline of the pipe just downstream of the expansion. It is held in place by wires which are attached to the pipe wall. The net drag force on the sphere and wires is F_d . The turbulence in the wake of the sphere enhances the rate of dissipation of mechanical energy. Taking this into account, derive a new expression for the mechanical energy loss per unit volume of fluid passing through the expansion. [8]

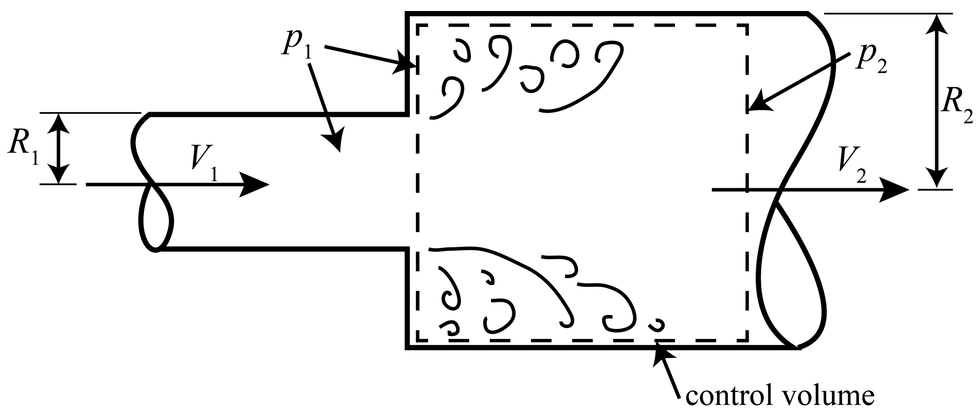

Fig. 1

6 An incompressible fluid rotates at an angular velocity Ω above a large, flat, stationary, horizontal plate. A thin boundary layer of thickness δ is established adjacent to the plate. Outside the boundary layer the fluid is in a state of rigid body rotation with an azimuthal velocity of $u_\theta = \Omega r$ in (r,θ,z) coordinates. Within the boundary layer the azimuthal velocity drops from $u_\theta = \Omega r$ at the top of the boundary layer, $z=\delta$, to $u_\theta = 0$ at the surface of the plate, $z=0$.

(a) Outside the boundary layer the viscous forces can be neglected and the radial pressure gradient is given by

$$\frac{dp}{dr} = \rho \frac{u_\theta^2}{r} \ .$$

Show, with the aid of a sketch, that this equation follows directly from Newton's second law of motion. Derive an expression for the radial pressure distribution. [8]

(b) Use dimensional analysis to show that the boundary layer thickness δ is of the order of

$$\delta \sim \sqrt{\nu/\Omega}$$

where ν is the kinematic viscosity of the fluid. [5]

(c) The radial pressure distribution calculated in Part (a) is imposed on the fluid within the boundary layer. It is observed that, within the boundary layer, the fluid spirals radially inward with a radial velocity comparable to the azimuthal velocity, $|u_r| \sim u_\theta$. Explain, with the aid of a sketch, why this radial inflow is driven by the imposed pressure gradient. [6]

(d) Outside the boundary layer it is observed that the fluid has a small but finite axial velocity, u_z, which is independent of r and directed away from the surface of the plate. Using a cylindrical control volume of height $\delta = \sqrt{\nu/\Omega}$ and radius r, show that

$$u_z \sim \delta\Omega \ .$$ [6]

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