EGT1
ENGINEERING TRIPOS PART IB

Wednesday 3 June 20152 to 4

## Paper 5

## ELECTRICAL ENGINEERING

Answer not more than four questions.

Answer not more than two questions from any one section and not more than one question from each of the other two sections.

All questions carry the same number of marks.

The approximate number of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book

## 10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

1 (a) The circuit symbol for an Op-Amp is shown in Fig. 1.
(i) List six aspects of non-ideal Op-Amp behaviour.
(ii) Explain briefly why the Op-Amp circuit of Fig. 1 is not a good design, and sketch a corrected version of the circuit.
(b) The INA116 instrumentation amplifier integrates three Op-Amps into a single circuit, as shown in Fig 2. It performs an amplifier function with a gain set by $\mathrm{R}_{\mathrm{G}}$. In use, the circuit can be drawn as shown in Fig. 3.
(i) Find the respective values of $\mathrm{R}_{\mathrm{G}}$ for gains of 20 and 1000 .
(ii) How might a gain of 1 be achieved?
(iii) Give an example of an application where the use of the INA116 may be justified and state your reasons.
(c) Show that non-ideal Op-Amp behaviour typically results in a fixed GainBandwidth product. The bandwidth of the INA116 is found to vary according to the table below. Find the Gain-Bandwidth products of Op-Amps, $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and $\mathrm{A}_{3}$.

| Gain | Bandwidth $(\mathrm{kHz})$ |
| :---: | :---: |
| 1000 | 7 |
| 100 | 70 |
| 10 | 500 |
| 1 | 800 |



Fig. 1


Fig. 2


Fig. 3
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2 (a) Figure 4 shows the input stage to an Op-Amp. State the class of amplifier it belongs to and give your reasoning.
(b) Approximately what voltage is required between the base and emitter of $\mathrm{Q}_{1}$ in Fig. 4 to ensure correct operation as a long tailed pair? The operating point of the transistors is to be $I_{\mathrm{C}}=1 \mathrm{~mA}, V_{\mathrm{CE}}=2 \mathrm{~V}$, where $h_{F E}=200$. Hence find values for $R_{1}$, $R_{2}$ and $R_{3}$.
(c) Draw a small signal equivalent circuit for the circuit shown in Fig. 4 and find an expression for the voltage gain ( $h_{o e}$ and $h_{r e}$ may be neglected).
(d) Find the input resistance, if $h_{i e}$ is $1 \mathrm{k} \Omega$ and $h_{f e}$ is 200 and comment on your result with regard to common-mode and differential-mode signals.
(e) How might the input impedance be increased?


Fig. 4

## SECTION B

3 (a) Give three reasons why a balanced three phase system is used in electrical power.
(b) Making reference to the power factor, explain why a load need not be specified as star or delta connected and justify the use of line quantities in an equation for the total power in a load.
(c) A scenic rural town is supplied at 50 Hz and $33 \mathrm{kV}, 10 \mathrm{~km}$ from the nearest generating capacity. The 33 kV lines each have a total resistance of $3 \Omega$ and an inductance of 36 mH . The town draws 12 MW at a power factor of 0.8 lagging.
(i) Find the star connected capacitance required at the town to correct the power factor to unity.
(ii) With this capacitance connected and assuming that the town remains at 33 kV , what is the line voltage at the generator end?
(d) To improve the scenery, the 33 kV lines are to be replaced by a three phase underground cable with phase resistance of $0.3 \Omega$ per km , negligible inductance and line to line capacitance of $1 \mu \mathrm{~F}$ per km . Assume that the line to line capacitance appears at the ends of the lines and is equally split between each end. The town remains at 33 kV and the power factor correcting capacitors remain in place.
(i) Find the new line current and voltage at the generator end.
(ii) What issues would arise if the underground cable were extended so as to connect the town to an alternative generator at a distance of 50 km ?

4 (a) The Cambridge area 33 kV network has a connection to the Combined Heat and Power (CHP) plant at the Wissington Sugar Factory. The turbine is rated at 70 MW , and the generator is rated at $95 \mathrm{MVA}, 11 \mathrm{kV}$ and a power factor of 0.85 . Sketch a load chart for this generator, marking on your sketch the limits in the overexcited generator region, if the synchronous reactance per phase is $1.45 \Omega$.
(b) The CHP plant is connected to the power grid as shown in Fig. 5. The system is controlled so that the 11 kV and 33 kV infinite buses operate at their nominal voltage. The cables used throughout are of the same steel cored aluminium construction, with a reactive impedance of $0.122 \Omega$ per km . Draw a new diagram of the section shown in Fig. 5 using the per unit system. Mark on your diagram the per unit values for each component noting that the infinite buses are held at 1 pu and are operated at unity power factor.
(c) In winter the CHP plant can only deliver a total of 54 MW to the infinite buses because of the thermal limit on Line 2. Find the currents in Line 2 and Line 1.
(d) Reactance is to be added to Line 2 so that Line 1 and Line 2 carry the same current.
(i) Find the value of the reactance to be added to each phase of Line 2.
(ii) Calculate the new total maximum power that may be delivered.
(iii) Find the generator excitation voltage and load angle when the new maximum power is being delivered.


Fig. 5

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5 (a) Explain the significance of the term slip and its importance with regard to induction motors.
(b) Making reference to a transformer or otherwise, explain why the per-phase equivalent circuit of an induction motor contains a variable resistor $\frac{R_{2}^{\prime}}{s}$ and show how this term is derived.
(c) A three phase star connected $415 \mathrm{~V}, 50 \mathrm{~Hz}$, induction motor has four poles and is designed to run at 1425 rpm . Its equivalent circuit parameters are: stator resistance of $3 \Omega$; referred rotor resistance of $3 \Omega$; Stator leakage reactance of $1.5 \Omega$; referred rotor leakage reactance of $1.5 \Omega$. The magnetising reactance and iron loss resistance may be ignored.
(i) Find the torque and power factor at starting direct on line.
(ii) Find the rated torque at the rated speed of 1425 rpm .
(iii) Carefully sketch the torque speed curve for rated torque to zero torque, marking on your sketch the value of the rated torque and the value of slip at which it occurs.
(d) In an application, the steady load torque on the motor is known to be $80 \%$ of the rated torque. Using your graph or otherwise, estimate the speed at which the motor will run.

## SECTION C

6 (a) The wave propagation of an electric field may be described by:

$$
\operatorname{Re}\left[E_{0} e^{j(\omega t-\beta z)}\right]
$$

(i) Show that this represents a travelling wave and sketch a graph to show the direction in which it is propagating.
(ii) Using Maxwell's Equations, show that the peak magnetic field $H_{0}$ is related to the peak electric field $E_{0}$ by the intrinsic impedance. Hence show that peak values, $E_{0}$ and $H_{0}$, occur at the same values of $z$. Using the Poynting Vector, show that the power per unit area is given by $\frac{E_{0} H_{0}}{2}$.
(b) A ground based weather radar system uses short pulses of electromagnetic waves at 2.9 GHz to detect water in the path of the wave and has a range of $100-500 \mathrm{~km}$ from the radar station.
(i) A single antenna is used for both detection and emission of the wave. Estimate the longest pulse that can be used such that the transmitted pulse and received pulse do not interfere at the antenna for ranges over 100 km .
(ii) A shower of rain behaves like a solid body with a relative permittivity of 1.5 for a 2.9 GHz wave. Estimate the gain of the antenna required for a reflected power density of $1 \mathrm{Wm}^{-2}$ at 500 km if the power output from the radar is 850 kW .
(c) The scattering of the wave by the rain shower results in the reflected wave having an on-axis gain of only 500 . If the antenna has an effective area of $4 \mathrm{~m}^{2}$ and has a matched impedance of $75 \Omega$, calculate the peak current measured at the receiver.
(d) By considering the wavelength, make a sketch to explain why using an electromagnetic wave at 90 GHz would interact with individual raindrops. State one further consideration regarding the choice of frequency.



7 (a) The inductance and capacitance per unit length of a lossless coaxial cable transmission line with an inner conductor of radius $r_{1}$ and an outer conductor of inner radius $r_{2}$ are given by

$$
C=\frac{2 \pi \varepsilon_{0} \varepsilon_{r}}{\ln \left(r_{2} / r_{1}\right)} \text { and } L=\frac{\mu_{0} \mu_{r} \ln \left(r_{2} / r_{1}\right)}{2 \pi}
$$

(i) Show that the velocity of a wave in this coaxial cable depends only on the material used for the dielectric inbetween the inner and outer conductors. Hence compare the mechanism of wave propagation in free space with that in a transmission line. Explain why the phase constant $\beta$ depends on the signal frequency.
(ii) Derive an expression for the characteristic impedance of this coaxial cable. How does it compare with the intrinsic impedance of free space?
(iii) The wave velocity in a $50 \Omega$ coaxial cable is given as $66 \%$ of the speed of light. Find the relative permittivity of the dielectric used and suggest suitable values for $r_{1}$ and $r_{2}$ if the cable is to be used at an oscilloscope input. (Assume that the dielectric has the permeability of air.)
(b) (i) An amplifier operates from dc to 50 MHz and is connected to an oscilloscope using a length of $50 \Omega$ coaxial cable, as shown in Fig. 6. Suggest why the input impedance of the oscilloscope should be set to $50 \Omega$. Find the source emf $E$ when the measured voltage at the oscilloscope is a 20 ns pulse of 1 V .
(ii) The instructions for an alternative dc to 50 MHz amplifier require that the oscilloscope input impedance must be set to $1 \mathrm{M} \Omega$ and the amplifier must be connected to the oscilloscope using the 1 m of $50 \Omega$ coaxial cable as supplied. Find the new source emf $E$ when the measured voltage is a 20 ns pulse of 1 V .
(iii) Calculate the power produced by each amplifier at the source when the measured voltage is 1 V dc.
(c) In the alternative system of Part (b) (ii), a voltage standing wave can be produced in the coaxial cable. Show that a voltage node is produced at the amplifier end of the cable when the measured voltage is a 1 V rms sinewave at 50 MHz , assuming the cable of Part (a) is used.


Fig. 6

## END OF PAPER

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## ANSWERS:

Q1. $2.63 \mathrm{k}, 50.05 \Omega, \sim 7 \mathrm{MHz}, \sim 7 \mathrm{MHz}, \sim 800 \mathrm{kHz}$
Q2. $\sim 1.2 \mathrm{k}, \sim 1.2 \mathrm{k}, \sim 0.9 \mathrm{k}, \sim 1.99 \mathrm{k}$, (For $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ )
Q3 $26 \mu \mathrm{~F}, 34.3 \mathrm{kV}, 227 \mathrm{~A}, 34.2 \mathrm{kV}$
Q4. $629 \mathrm{~A}, 315 \mathrm{~A}, 0.538 \mathrm{j} \Omega, 70 \mathrm{MW}, 1.4$ p.u., $38^{\mathrm{O}}$

Q5. $73 \mathrm{Nm}, 0.85,16.6 \mathrm{Nm}, 0.05,1440 \mathrm{rpm}$.
Q6. $667 \mu \mathrm{~s}, 3.6 \times 10^{8}, 2.9 \mu \mathrm{~A}$
Q7. 2.3, $2 \mathrm{~V}, 1 \mathrm{~V}, 40 \mathrm{~mW}, 1 \mu \mathrm{~W}$,

