1B - Paper 2 - 2015

① a) Point Load → $\delta_{B,v} = \dfrac{W\ell^3}{3EI}$ $I = \frac{1}{12}\left(102(42)^3 - 98(38)^3\right)$

$= 181{,}600 \text{ mm}^4$

$$\delta_{B,v} = \frac{(1000)(1000)^3}{3(210\times10^3)(181{,}600)} = 8.74 \text{ mm}$$

Torque → $\phi = \dfrac{\oint \frac{ds}{t}}{4A_e^2}\dfrac{T}{G} = \dfrac{(280/2)}{4(40\times100)^2}\dfrac{2\times10^6}{81\times10^3} = 5.4\times10^{-5}$ rad/mm

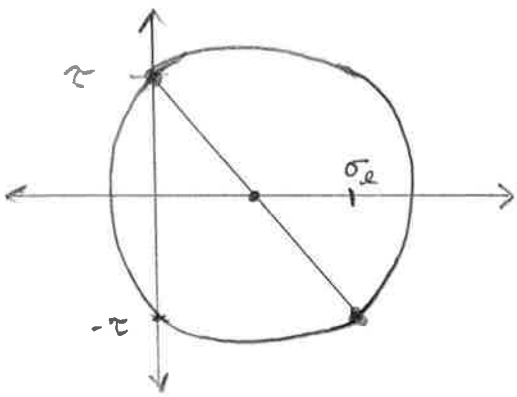
$$\text{Twist} = (5.4\times10^{-5})500 = 0.027 \text{ rad}$$

$$\delta_{B,\text{total}} = 8.74 + 0.027(50 \text{ mm}) = \underline{\underline{10.1 \text{ mm}}}$$

b) Point C:

$$\sigma_\ell = \frac{My}{I} = \frac{(1000\text{ N})(1000\text{ mm})(21)}{181{,}600} = 115.6 \text{ MPa}$$

$$\tau = \frac{T}{2A_e t} = \frac{2\times10^6 \text{ N}\cdot\text{mm}}{2(40\times100)2} = 125 \text{ MPa}$$



$$\tau_{max} = \sqrt{\left(\frac{\sigma_\ell}{2}\right)^2 + (\lambda\tau)^2} = \frac{Y}{2}$$

$$(\lambda\tau)^2 = \frac{Y^2}{4} - \frac{\sigma_\ell^2}{4}$$

$$\lambda = \frac{1}{\tau}\sqrt{\frac{Y^2}{4} - \frac{\sigma_\ell^2}{4}}$$

$$\therefore \lambda = \frac{1}{125}\sqrt{\frac{355^2}{4} - \frac{115.6^2}{4}} = \underline{\underline{1.34}}$$

① (b) cont.

Point D: $\sigma_x = 0$, $\tau_{torque} = 125$ MPa

$$\tau_{point\ load} = \frac{S A\bar{y}}{It}$$

$$\left\{ \begin{aligned} A\bar{y} &= (20\times2)10(2) \\ &\quad + 100(2)(20) \\ &= 4800 \text{ mm}^3 \end{aligned} \right.$$

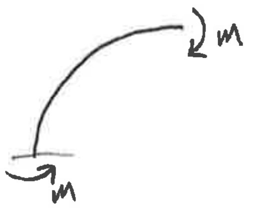
$$\tau_{pl} = \frac{1000\,(4800)}{181{,}600\,(4)} = 6.61 \text{ MPa}$$

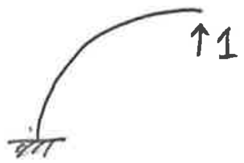
$$\tau_{total} = 125\lambda + 6.61 = \frac{Y}{2}$$

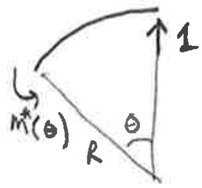
$$\lambda = \frac{1}{125}\left[\frac{355}{2} - 6.61\right] = \underline{\underline{1.37}}$$ does not govern

Torque can be increased by 1.34x → $\underline{\underline{0.7 \text{ kNm}}}$

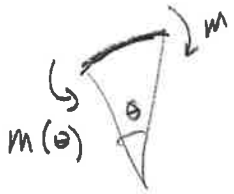
② a) <u>Real:</u> <u>Virtual:</u>







$$m^*(\theta) = -R\sin\theta\,(1)$$



$$m(\theta) = M$$

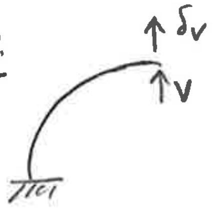
$$\therefore\ \kappa(\theta) = \frac{M}{EI}$$

<u>V.W.</u> $\rightarrow$ $\quad \Sigma F^*\delta = \int_0^{\pi/2} \kappa\ m^*\ ds$

$$\delta_{V,B} = \int_0^{\pi/2} \frac{M}{EI}(-R\sin\theta)R\,d\theta = \frac{+MR^2}{EI}\left(\cos\theta\right]_0^{\pi/2} = \underline{\underline{\frac{-MR^2}{EI}}}$$

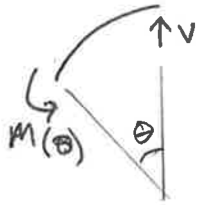
b) Find Reaction @ B

<u>Real:</u>



<u>Virtual:</u> same as above.

$$m^*(\theta) = -R\sin\theta$$



$$m(\theta) = -VR\sin\theta$$

$$\therefore\ \kappa(\theta) = \frac{-VR\sin\theta}{EI}$$

<u>V.W.</u> $\rightarrow$ $\quad \delta_{V,B} = \int_0^{\pi/2} \frac{-VR\sin\theta}{EI}(-R\sin\theta)R\,d\theta = \frac{+VR^3}{EI}\int_0^{\pi/2}\sin^2\theta\,d\theta$

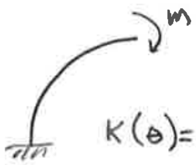
$$\delta_{V,B} = \frac{\pi}{4}\frac{VR^3}{EI}$$

<u>Find V:</u> $\quad \frac{MR^2}{EI} = \frac{\pi}{4}\frac{VR^3}{EI} \longrightarrow \underline{\underline{V = \frac{4M}{R\pi}}}$

② b) (cont.)

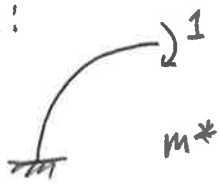
Find rotation @ B:

Real:



$K(\theta) = \frac{M}{EI}$

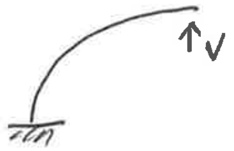
Virtual:



$M^* = 1$

Real

+



$K(\theta) = \frac{-VR\sin\theta}{EI}$

"

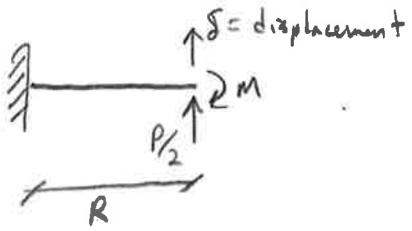
$$1\,(\theta_B) = \int_0^{\pi/2} \left[\frac{M}{EI} - \frac{VR\sin\theta}{EI}\right] (1)\, R\, d\theta$$

$$= \frac{MR}{EI}\frac{\pi}{2} - \frac{VR^2}{EI}\Big[\cos\theta\Big]_0^{\pi/2} = \frac{MR}{EI}\frac{\pi}{2} - \frac{4M}{R\pi}\frac{R^2}{EI}$$

$$\theta_B = \frac{MR}{EI}\left[\frac{\pi}{2} - \frac{4}{\pi}\right]$$

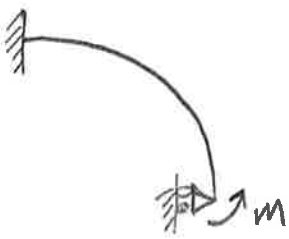
c) consider two structures:

(i)



+

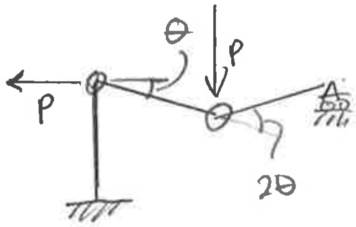
(ii)



→ Find M by setting rotation @ right end to be equal.

→ Find δ using databook cases for (i)

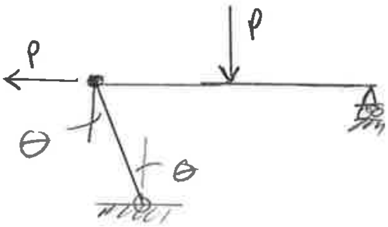
③ (a) Beam:



$$M_p(3\cancel{\theta}) = P\left(\cancel{\theta}\frac{L}{2}\right)$$

$$P = \frac{6M_p}{L}$$

Sway:

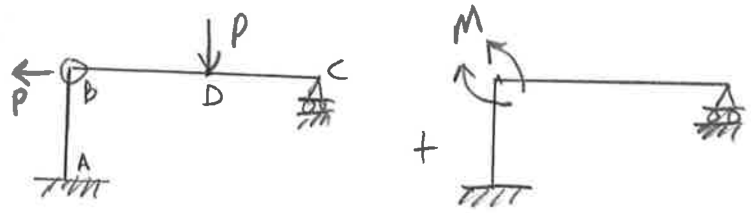


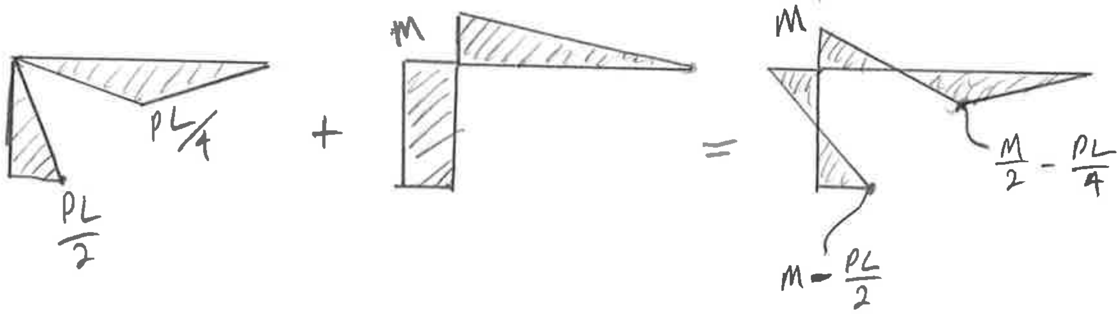
$$2M_p(\theta) + M_p(\theta) = P\left(\frac{\theta L}{2}\right)$$

$$3M_p\theta = \frac{P\theta L}{2}$$

$$P = \frac{6M_p}{L}$$

③ b)





(i) $\underline{AB}$: $M_{max} \rightarrow M_{p_{AB}} = M$

$M_{min} \rightarrow -M_{p_{AB}} = M - \frac{PL}{2} \rightarrow -2M_{p_{AB}} = -\frac{PL}{2} \rightarrow M_{p_{AB}} = \frac{PL}{4}$

$\underline{BC}$: $M_{max} \rightarrow M_{p_{BC}} = M$

$M_{min} \rightarrow -M_{p_{AB}} = \frac{M}{2} - \frac{PL}{4} \rightarrow -\frac{3}{2}M_{p_{BC}} = -\frac{PL}{4} \rightarrow M_{p_{BC}} = \frac{PL}{6}$

$$\Rightarrow \underline{\underline{M_p = PL/4}}$$

(ii) $\underline{BC}$: same as above $\rightarrow \underline{\underline{M_{p_{BC}} = \frac{PL}{6}}}$

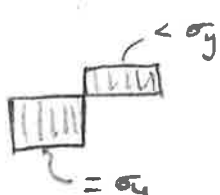
AB: $M_{max} \rightarrow M = M_{p_{BC}}$

$M_{min} \rightarrow -M_{p_{AB}} = M - \frac{PL}{2} = \frac{PL}{6} - \frac{PL}{2} \rightarrow \underline{\underline{M_{p_{AB}} = \frac{PL}{3}}}$

$$\text{Efficiency} = \frac{PL}{6}(L) + \frac{PL}{3}\left(\frac{L}{2}\right) = \frac{PL^2}{3} < \underbrace{\frac{PL}{4}\left(\frac{3}{2}L\right) = \frac{3PL^2}{8}}_{\text{part b) i)}}$$

iii) Segment BC: Select x-section with $M_p > \frac{PL}{6}$ (no axial load)

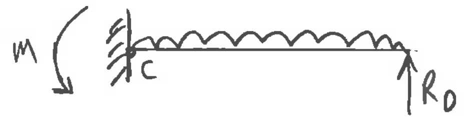
Segment AB: Axial Load + $\left(M_p = \frac{PL}{3}\right)$



Increase x-section to account for axial stress

→ Also check buckling

④ (a) Point C will not rotate or move (symmetry)

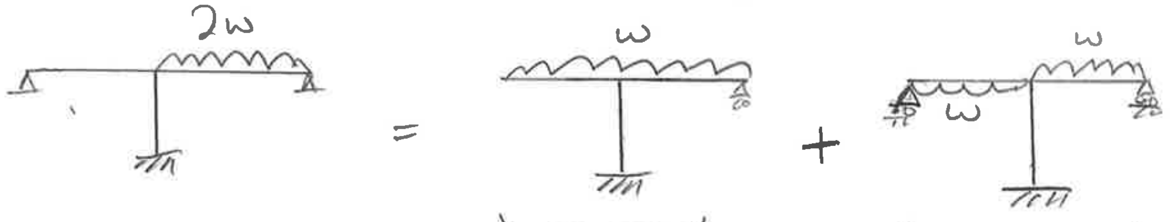


$M = \frac{wL^2}{8}$ (Data Book)

$\Sigma M_C \rightarrow R_D(L) - wL\left(\frac{L}{2}\right) + \frac{wL^2}{8} = 0$

$$\underline{\underline{R_D = \frac{3wL}{8}}}$$

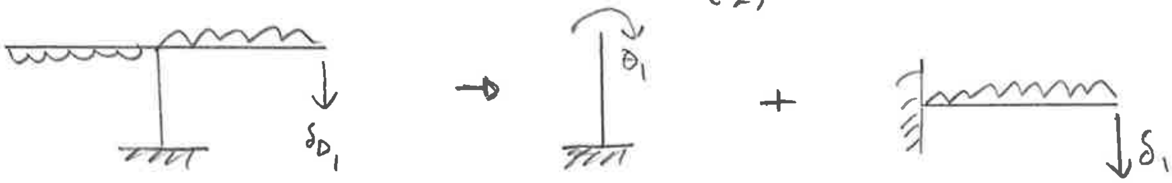
(b)



Remove rollers!

$M = wL\left(\frac{L}{2}\right)2 = wL^2$

(i)

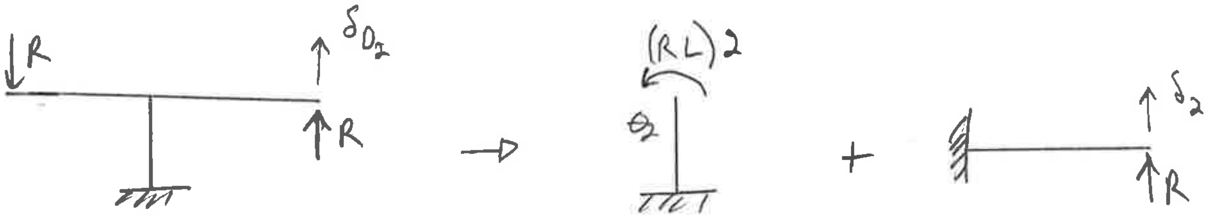


$\theta_1 = \frac{(wL^2)L}{EI}$

$\delta_1 = \frac{wL^4}{8EI}$

$\delta_{D_1} = \frac{wL^3}{EI}L + \frac{wL^4}{8EI} = \frac{9wL^4}{8EI}$

(ii)

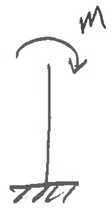


$\delta_{D_2} = \frac{(2RL)L}{EI}(L) + \frac{RL^3}{3EI} = \frac{7RL^3}{3EI}$

$\delta_{D_1} = \delta_{D_2} \rightarrow \frac{9wL^4}{8EI} = \frac{7RL^3}{3EI} \rightarrow \underline{\underline{R = \frac{27wL}{56}}}$

④ (b)(i) Total reaction @ D: $\frac{3wL}{8} + \frac{27wL}{56} = \underline{\underline{\frac{6wL}{7}}}$

(ii) Only need to consider anti-symmetric case:



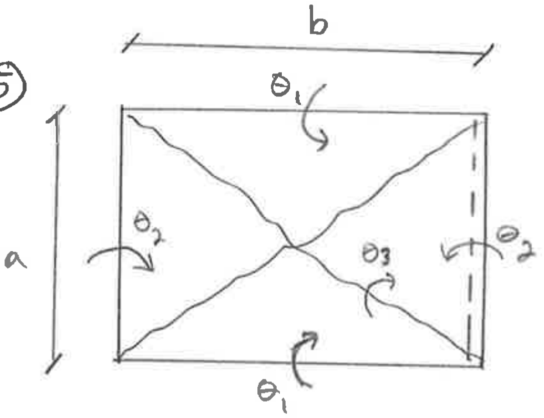
$$\frac{27}{56}wL$$

$$M = wL\left(\frac{L}{2}\right)2 - RL(2)$$

$$= wL^2 - \frac{54}{56}wL^2 = \frac{wL^2}{28}$$

$$\delta_D = \frac{ML^2}{2EI} = \frac{wL^2}{28}\left(\frac{L^2}{2EI}\right) = \underline{\underline{\frac{wL^4}{56EI}}}$$

(5)



$$\theta_1 = \frac{\delta}{a/2} \qquad \theta_2 = \frac{\delta}{b/2}$$

Projection method:

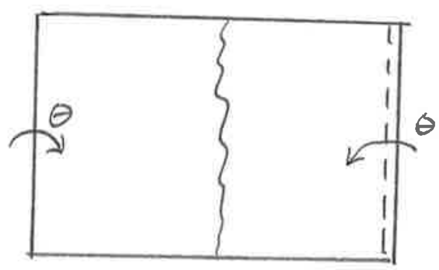
$$\theta_3 = \frac{b}{2}\theta_1 + \frac{a}{2}\theta_2 = \frac{b}{2}\frac{2\delta}{a} + \frac{a}{2}\frac{2\delta}{b} = \frac{b}{a}\delta + \frac{a}{b}\delta$$

$$E.D. = ma\left(\frac{2\delta}{b}\right) + 4m\left(\frac{b}{a}\delta + \frac{a}{b}\delta\right) = 6m\frac{a}{b}\delta + 4m\frac{b}{a}\delta$$

$$W.D. = \underbrace{p\left(\frac{a}{2}\frac{b}{2}\frac{\delta}{2}\right)\frac{1}{3}}_{\text{pyramid}} + \underbrace{p\left(\frac{a}{2}\frac{b}{2}\frac{\delta}{2}\right)}_{\text{rectangular prism}} = \frac{pab\delta}{6}$$

$$\Rightarrow \frac{pab\cancel{\delta}}{6} = 6m\frac{a}{b}\cancel{\delta} + 4m\frac{b}{a}\cancel{\delta} \rightarrow p = \frac{36m}{b^2} + \frac{24m}{a^2}$$

(b) Choose mechanism (e.g.)



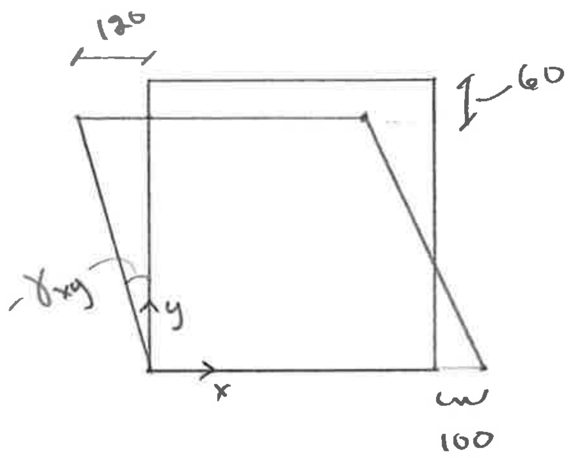
$$\theta = \frac{\delta}{b/2}$$

$$E.D. = ma(3\theta) = ma\,3\left(\frac{2\delta}{b}\right) = 6\frac{ma}{b}\delta$$

$$W.D. = p\left(\underbrace{\frac{\delta}{2}\frac{b}{2}\frac{a}{2}\frac{1}{2}}_{\text{triangular prism}} + \underbrace{\frac{\delta}{2}\frac{b}{2}\frac{a}{2}}_{\text{rectangular prism}}\right) = p\delta ba\left(\frac{1}{16} + \frac{1}{8}\right) = p\delta ba\left(\frac{3}{16}\right)$$

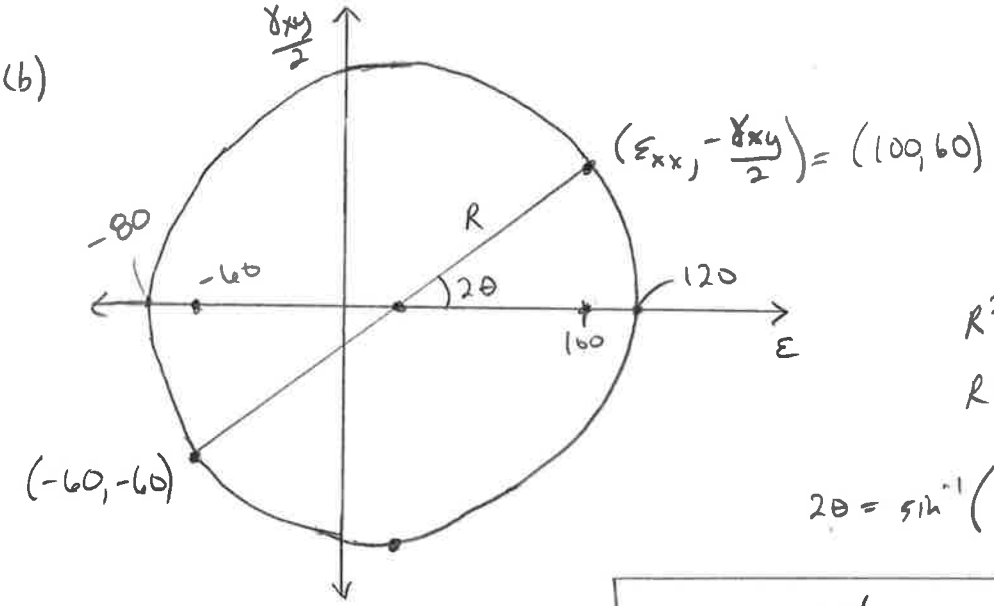
$$\Rightarrow \frac{6m\cancel{a}}{b}\cancel{\delta} = p\cancel{\delta}b\cancel{a}\left(\frac{3}{16}\right) \rightarrow p = \frac{32m}{b^2}$$

6 (a)



all $\times 10^{-6}$

(b)



$$R^2 = 60^2 + 80^2$$

$$R = 100$$

$$2\theta = \sin^{-1}\left(\frac{60}{100}\right) \rightarrow \theta = 18.4^\circ$$

$$\boxed{\begin{aligned} \varepsilon_1 &= 120 \times 10^{-6} \quad @ \quad \theta = -18.4^\circ \\ \varepsilon_2 &= -80 \times 10^{-6} \quad @ \quad \theta = 71.6^\circ \end{aligned}}$$

(c) Plane stress:

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\varepsilon_1 + \nu\,\varepsilon_2\right)$$

$$\sigma_2 = \frac{E}{1-\nu^2}\left(\varepsilon_2 + \nu\,\varepsilon_1\right)$$

$$\varepsilon_3 = \frac{1}{E}\left(\cancelto{0}{\sigma_3} - \nu\,\sigma_1 - \nu\,\sigma_2\right) = \frac{-\nu}{E}\left(\sigma_1 + \sigma_2\right)$$

$$= \frac{-\nu}{\cancel{E}}\left(\frac{\cancel{E}}{1-\nu^2}\right)\left(\varepsilon_1 + \nu\,\varepsilon_2 + \varepsilon_2 + \nu\,\varepsilon_1\right)$$

$$= \frac{-\nu}{\cancel{(1+\nu)}(1-\nu)}\cancel{(1+\nu)}\left(\varepsilon_1 + \varepsilon_2\right)$$

$$\varepsilon_3 = \frac{-\nu}{1-\nu}\left(\varepsilon_1 + \varepsilon_2\right) = \frac{-0.3}{0.7}\left(40 \times 10^{-6}\right) = \underline{\underline{-17.1 \times 10^{-6}}}$$

⑥ (d) Only ε_3 will change.

$$\Delta\varepsilon_1 = \frac{1}{E}\left(\Delta\sigma_1 - \nu(\Delta\sigma_2) - \nu\,(\cancelto{0}{\Delta\sigma_3})\right) + \alpha\Delta T = 0$$

$$\Delta\varepsilon_2 = \frac{1}{E}\left(\Delta\sigma_2 - \nu(\Delta\sigma_1)\right) + \alpha\Delta T = 0$$

$$\therefore \quad \Delta\sigma_1 - \nu(\Delta\sigma_2) = \Delta\sigma_2 - \nu(\Delta\sigma_1) \rightarrow \Delta\sigma_1 = \Delta\sigma_2$$

$$\frac{1}{E}\left(\Delta\sigma_1 - \nu\Delta\sigma_1\right) = -\alpha\Delta T$$

$$\Delta\sigma_1 = \frac{-E}{1-\nu}\alpha\Delta T$$

$$\Delta\varepsilon_3 = \frac{-\nu}{E}\left(\Delta\sigma_1 + \Delta\sigma_2\right) + \alpha\Delta T$$

$$= \frac{-\nu}{\cancel{E}}\left(\frac{-\cancel{E}}{1-\nu}\right)2\alpha\Delta T \quad + \alpha\Delta T$$

$$= \alpha\Delta T\left(\frac{2\nu}{1-\nu} + 1\right) = (11\times10^{-6})\,10\left(\frac{0.6}{0.7} + 1\right) = 204.3\times10^{-6}$$

$$\varepsilon_{3,\text{total}} = -17.1 + 204.3 = \underline{\underline{187.2\times10^{-6}}}$$

– M. DeJong, June 2015