Dr M P Juniper
Dr N Crilly
Dr A Johnson
Dr L Xu

Q1. (a) At both sections 1 and 2 the flow is far away from the jump, can be treated as quasi-steady, the streamlines are horizontal, the flow has no accelerations in vertical direction. [2]

(b) Force Momentum Equation in horizontal direction:

$$F_x + \int p dA_1 + \rho A_1 V_1^2 = \int p dA_2 + \rho A_2 V_2^2$$

$$F_x + \frac{1}{2}\rho g w_1 h_1^2 + \rho w_1 h_1 V_1^2 = \frac{1}{2}\rho g w_2 h_2^2 + \rho w_2 h_2 V_2^2$$

Assumptions: 1). Inviscid flow – no viscous friction force from the bed and sides on the flow (and water is treated as incompressible here by default);

2). Uniform channel width – no horizontal pressure force due to channel width change;

3). Steady flow at sections 1 and 2.

The assumptions lead to $F_x = 0$; ρ and w constant so can be divided throughout, the result is: $\frac{1}{2}gh_1^2 + h_1V_1^2 = \frac{1}{2}gh_2^2 + h_2V_2^2$ as required. [4]

(c) Similarity means across the jump h_2/h_1 will be the same for both the real and the model jumps. The expression of Froude number given indicated that for constant h_2/h_1 the non-dimensional group Froude number must stay the same for the real and the model jumps: $\left.\frac{V_1}{\sqrt{gh_1}}\right|_{model} = \left.\frac{V_1}{\sqrt{gh_1}}\right|_{real}$,

$$V_{1,model} = \sqrt{\frac{h_{1,model}}{h_{1,real}}}V_{1,real} = \sqrt{\tfrac{1}{4}} \cdot 10 = 5\ ms^{-1}$$ [4]

Q2. (a). Locations 1 and 2 are far away from the impingement point, streamlines are straight and parallel so pressure is the same as atmospheric pressure, no pressure gradient across the streamlines at these locations. The flow is steady, inviscid and incomressible, Apply Bernoulli's Equation on a streamline in the middle of the jet, ignore the potential energy change of the air jet due to small height change:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$, this leads to $V_2 = V_1 = 40\ \text{ms}^{-1}$

By continuity: $h_1V_1 = h_2V_2$, this leads to $h_2 = h_1 = 20\ \text{mm}$

[4]

Q2 continued

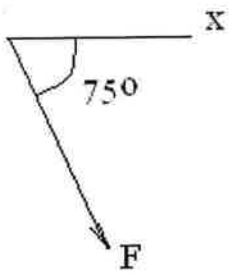
(b). Apply SFME on a control volume consisting inlet at 1, the jet itself and exit at 2: $F_x = \dot{m}(V_{x2} - V_{x1}) = 40(\frac{\sqrt{3}}{2} - 1)\dot{m} = -4.29\rho_{air}$;

$F_y = \dot{m}(V_{y2} - V_{y1}) = \dot{m} \cdot 40 \cdot \frac{1}{2} = 20\dot{m} = 19.2\rho_{air}$

Take $\rho_{air} = 1.225$ kgm^{-3} (databook P31), $F_x = -5.15$N; $F_y = 19.2$N

Total force: $F = \sqrt{F_x^2 + F_y^2} = 19.88$N; Direction: $\beta = arctg\frac{F_y}{F_x} = 105°$

The force acting on the plate due to the flow has the same amplitude and opposite direction: 19.88 N and pointing $-75°$ from x direction.



$$pV^n = k$$

3\. (a)(i) Isentropic process: Entropy is constant
From the perfect gas relationships, $pv^{\gamma} = \text{const}$ $\Rightarrow n = \gamma$

(ii) Isothermal process: Temperature is constant
From the ideal gas equation of state, $pv = RT = \text{const}$ $\Rightarrow n = 1$

(iii) Isobaric process: pressure is constant
$\Rightarrow$ no dependence on v, so $n = 0$

(iv) Isochoric process: volume is constant
$\Rightarrow$ no dependence on p, so $n \to \infty$ [4]

(b) displacement work $= \int_{V_1}^{V_2} p\,dv = \int_{V_1}^{V_2} k \frac{dv}{v^n} = \left[k \frac{v^{1-n}}{1-n} \right]_{V_1}^{V_2}$

$$= \frac{p_2 V_2^n V_2^{1-n} - p_1 V_1^n V_1^{1-n}}{1-n} = \frac{p_2 V_2 - p_1 V_1}{1-n}$$ [4]

(c) $n = 1$ corresponds to an isothermal process. As the gas in the cylinder expands, heat is added such that pv remains the same. This means that the numerator and denominator of the expression in (b) are both zero and therefore that the displacement work is undefined by this expression. (Note, it does not mean that the displacement work is zero or infinite.) A new solution to the integral is required: $k \ln(V_2/V_1)$. [2]

M. Juniper 2011

4.

adiabatic & no work

$A_1 = 0.05 m^2$
$p_1 = 1$ MPa
$T_1 = 850 + 273 = 1123 K$ $\qquad$ $T_2 = 1119 K$
$\dot{m}_1 = 30 kg\,s^{-1}$

(a) $\rho_1 = \dfrac{p_1}{RT_1} = \dfrac{1 \times 10^6}{287 \times 1123} = 3.10268\ kg\,m^{-3}$

$V_1 = \dfrac{\dot{m}_1}{\rho A_1} = \dfrac{30}{3.10268 \times 0.05} = \underline{193.4\ ms^{-1}}$ [2]

(b) SFEE: $\quad Q - W_x = \dot{m}\, C_p (T_2 - T_1) + \dfrac{\dot{m}}{2}(V_2^2 - V_1^2) = 0$

$\Rightarrow V_1^2 - 2C_p(T_2 - T_1) = V_2^2$

$\Rightarrow V_2 = \left(193.4^2 - 2 * 1005 * (-4)\right)^{1/2} = \underline{213.2\ ms^{-1}}$

$p_2 = \rho_2 R T_2 \quad$ where $\quad \rho_2 = \dfrac{\dot{m}}{AV_2}$

$\Rightarrow p_2 = \dfrac{\dot{m}}{AV_2} RT_2 = \dfrac{30 * 287 * 1119}{0.05 * 213.2} = \underline{0.9039 \times 10^6\ Pa}$ [4]

(c) perfect gas $\Rightarrow \quad S_2 - S_1 = C_p \ln\left(\dfrac{T_2}{T_1}\right) - R \ln\left(\dfrac{p_2}{p_1}\right)$

$= 1005 \ln\left(\dfrac{1119}{1123}\right) - 287 \ln\left(\dfrac{0.9039}{1}\right)\ J\,kg^{-1}K^{-1}$

$= 25.4\ J\,kg^{-1}K^{-1}$

The entropy has increased due to irreversible processes within the mesh. The main process is viscous dissipation of mechanical energy to thermal energy within the fluid. [4]

M. Juniper 2011

[6]

Q5. (a). Take a coordinate s along the gate BA starting from hinge B, at any point s, the local gauge pressure p is: $\frac{1}{4}L+\frac{\sqrt{2}}{2}s$. Total hydrostatic force

$$F=\int_0^L pdA=\int_0^L(\frac{1}{4}L+\frac{\sqrt{2}}{2}s)\rho gwds=\rho gwL^2(\frac{1}{4}+\frac{\sqrt{2}}{4})\ (=0.604\rho gwL^2),$$

as the pressure force acts perpendicular on the surface, the direction of the force is 45° south-east on Fig.3.

Alternatively, horizontal force

$$F_x=\int pdA=\int_{L/4}^{h_0}\rho gwhdh=\frac{1}{2}\rho gw(h_0^2-\left(\frac{L}{4}\right)^2)=\frac{1}{2}\rho gwL^2(\left(\frac{1}{4}+\frac{\sqrt{2}}{2}\right)^2-\left(\frac{1}{4}\right)^2)=(\frac{1}{4}+\frac{\sqrt{2}}{8})\rho gwL^2$$

$(=0.427\rho gwL^2)$

Vertical force equals to the weight over the gate:

$$F_y=\rho gw(\frac{\sqrt{2}}{2}L\cdot\frac{L}{4}+\frac{1}{2}\left(\frac{\sqrt{2}L}{2}\right)^2)=\rho gwL^2(\frac{\sqrt{2}}{8}+\frac{2}{8})=\rho gwL^2(\frac{\sqrt{2}}{8}+\frac{1}{4})\quad(=0.427\rho gwL^2)$$

Since $F_x=F_{y;}$ $F=\sqrt{F_x^2+F_y^2}=F_x\cdot\sqrt{2}=F_y\cdot\sqrt{2}=\rho gwL^2(\frac{\sqrt{2}}{4}+\frac{1}{4})$, and the force acting at 45° from vertical (or horizontal) direction. [6]

(b). Appy Bernoulli's equation on an imaginary streamline from the top of the water to the top of the flow at the downstream location 2 where h_2 is measured and where streamlines are horizontal (being away from point A) and local pressure is atmospheric (gauge pressure = 0): $gh_0=\frac{1}{2}V_2^2;\ V_2=\sqrt{2gh_0}$

Q5 continued

Continuity requires: $\dot{Q} = h_2 w V_2$; $\quad h_2 = \dfrac{\dot{Q}}{w\sqrt{2gh_0}} = \dfrac{\dot{Q}}{w\sqrt{g(\frac{1}{2}+\sqrt{2})L}}$

$\because h_2/h_1 = 0.8; \quad \therefore h_1 = \dfrac{5}{4}h_2 = \dfrac{5\dot{Q}}{4w\sqrt{(\frac{1}{2}+\sqrt{2})gL}} (= 0.9\overset{0}{\wedge}35\dfrac{\dot{Q}}{w\sqrt{gL}})$ [9]

(c) The moment of an element hydrostatic force acting on point B is $dM = \rho gwpsds$, integrate along the gate,

$$M = \int_0^L \rho gw(\frac{1}{4}Ls + \frac{\sqrt{2}}{2}s^2)ds = \rho gw(\frac{L}{4}\cdot\frac{L^2}{2} + \frac{\sqrt{2}}{2}\frac{1}{3}L^3) = \rho gwL^3(\frac{1}{8}+\frac{\sqrt{2}}{6})$$

To activate the gate the moment due to T should be larger than that due to hydrostatic force:

$$T\frac{\sqrt{2}}{2}L > M = \rho gwL^3(\frac{1}{8}+\frac{\sqrt{2}}{6}); \quad T > M/\frac{\sqrt{2}}{2}L = \rho gwL^2(\frac{\sqrt{2}}{8}+\frac{1}{3})$$

$\therefore T > \rho gwL^2(\dfrac{1}{3}+\dfrac{\sqrt{2}}{8}) \quad (= \overset{0.5101}{\cancel{0.519}}\rho gwL^2)$ [9]

(d) The horizontal hydrostatic force does not change, but the vertical force will reduce as the volume of the water on the top of the gate is smaller for the quarter-circular cylindrical gate. The moment to hinge B due to the horizon force does not change. The difference in the moment due to the elementary vertical hydrostatic forces is $d(\Delta M) = \rho gw(\Delta p)sds = \rho gw(p_{flat} - p_{cylind.})sds$, because $\Delta p = (p_{flat} - p_{cylind.})$ is always positive over the gates, $d(\Delta M)$ is always positive therefore ΔM, the difference between the moments of the flat gate and of the cylindrical gate is positive so $M_{flat} > M_{cylind.}$, therefore $T_{flat} > T_{cylind.}$, i.e. the cylindrical gate requires less force to activate as compared to the flat gate. [6]

6 (a) Assume isentropic compression $\Rightarrow \frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$

with $p_3/p_2 = 20$, $T_2 = (25+273) = 298\,K$, $\gamma = 1.4$

$$\Rightarrow T_3 = 298\,(20)^{2/7} = 701\,K \qquad (701.357\,K)$$

Assuming that K.E. terms may be neglected, the static pressure/temperature equal the stagnation pressure/temperature.

S.F.E.E. $\underbrace{\cancel{Q}}_{=0 \text{ because adiabatic}} - W_x = \dot{m}\,C_p(T_3 - T_2) = 50 * 1005 * (701.357 - 298)$

$= 2.026869 \times 10^7$ Watts

$-W_x$ = power input to compressor $= 20.3$ MW [5]

(b) $T_4 = (950 + 273) = 1223\,K$

S.F.E.E. $Q - \underbrace{\cancel{W_x}}_{0} = \dot{m}\,C_p(T_4 - T_3) = 50 * 1005 * (1223 - 701.357)$

$= 2.621256 \times 10^7$ Watts

$= 26.2$ MW [2]

(c) The compressor work = the turbine work = 20.3 MW

S.F.E.E.: $\underbrace{\cancel{Q}}_{0} - W_x = \dot{m}\,C_p(T_5 - T_4)$

$$\Rightarrow T_5 = T_4 - \frac{W_x}{\dot{m}C_p} = 1223 - \frac{2.026869\times10^7}{50 * 1005} = 819.643\,K$$

$= 819\,K$

Assume isentropic expansion in the turbine $\Rightarrow \frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \underbrace{20}_{\text{in bar}}\left(\frac{819.643}{1223}\right)^{7/2} = 4.9286 \text{ bar}$$

$= 4.93$ bar [5]

(d) Assume isentropic expansion in the nozzle $\Rightarrow \frac{T_6}{T_5} = \left(\frac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow T_6 = T_5\left(\frac{2.60}{4.9286}\right)^{2/7} = 682.7590\,K$$

$= 683\,K$

S.F.E.E. in nozzle: $\underbrace{\cancel{Q}}_{0} - \underbrace{\cancel{W_x}}_{0} = \dot{m}\left(C_p(T_6 - T_5) + \tfrac{1}{2}V_6^2\right) = 0$

$$\Rightarrow V_6 = \left(2C_p(T_5 - T_6)\right)^{1/2}$$

$$= \left(2 * 1005 * (819.643 - 682.759)\right)^{1/2} = 524.5350\ ms^{-1}$$

$= 525\ ms^{-1}$

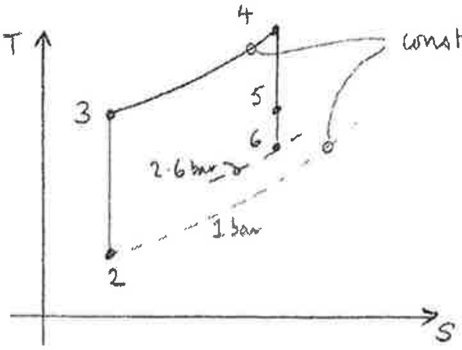
The speed of sound at location 6 is $\sqrt{\gamma R T}$

$$= (1.4 * 287 * 682.759)^{1/2} = 523.768 \text{ ms}^{-1}$$

$$= 524 \text{ ms}^{-1}$$

The gas is moving at the speed of sound (within the error in these calculations). This is known as a "choked" nozzle. [8]

(e) T-S diagram



[5]

(f) Let us consider the temperatures at points 1 and 2. They are linked via the S.F.E.E. :

$$\cancel{\dot{Q}}^{0} - \cancel{\dot{W}_x}^{0} = \dot{m}\left(c_p(T_2 - T_1) + \tfrac{1}{2}V_2^2 - V_1^2 \right)$$

Point 1 is far from the gas turbine so we take $V_1 = 0$.
We calculate V_2 from $\dot{m} = \rho_2 A_2 V_2$ where $\rho_2 = p_2 / RT_2$

$$\Rightarrow V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{\dot{m} R T_2}{p_2 A_2} = \frac{50 * 287 * 298}{10^5 * 0.75} = 57.02 \text{ ms}^{-1}$$

If $T_2 = 298\text{K}$ then $T_1 = T_2 + \frac{1}{c_p}\frac{1}{2}V_2^2 = 298 + \frac{57.02^2}{1005 * 2} = 299.62 \text{ K}$

(The temperature drop between 1 and 2 is around 1.62 Kelvin, which is 0.5 %)

The pressure drop is around 1.9%. These are sufficiently small that, to a good approximation, we can assume that $p_2 = p_1$ and $T_2 = T_1$. [5]

M. Juniper 2011

7) AREA, $A = \dfrac{\pi h v}{4}$ MASS, $m = \rho A = \dfrac{\rho \pi h v}{4}$

a) $\perp$ AXIS:

$$J_G = \frac{\rho\pi h v}{4}\left[\frac{\left(\frac{v}{2}\right)^2}{4} + \frac{\left(\frac{h}{2}\right)^2}{4}\right]$$

$$= \frac{\rho\pi h v}{64}\left(v^2 + h^2\right)$$

b) // AXIS:

$$J_O = J_G + \text{"}mr^2\text{"}$$

$$= \frac{\rho\pi h v}{64}\left(v^2 + h^2\right) + \frac{\rho\pi h v}{4}\left(\frac{h}{2}\right)^2$$

$$= \frac{\rho\pi h v}{64}\left(v^2 + 5h^2\right)$$

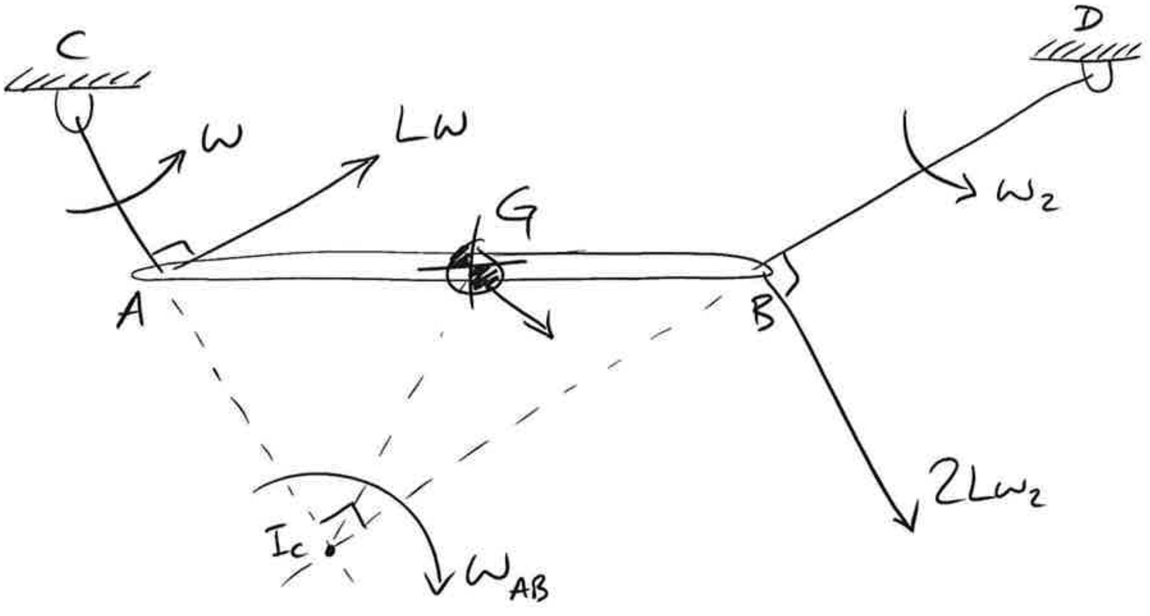
c) "$T = Fr$"

$$\therefore T = \frac{\rho\pi h v g}{4}\cdot\frac{h}{2} = \frac{\rho\pi h^2 v g}{8}$$

"$T = J\alpha$"

$$\therefore \alpha = \frac{T}{J_O} = \frac{8hg}{v^2 + 5h^2}$$

8)



a) RESOLVING HORIZONTAL VELOCITY COMPONENTS

$$L\omega \cos 30 = 2L\omega_2 \cos 60$$

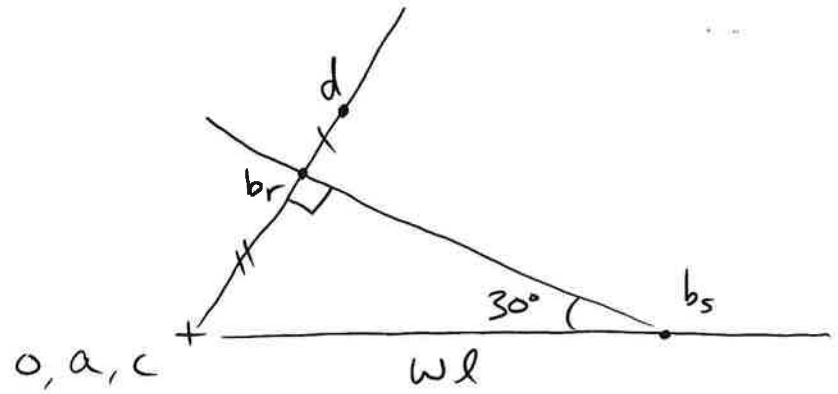
$$\therefore \omega_2 = \frac{\omega\sqrt{3}}{2}$$

b) "$v = r\omega$" $\quad v = L\omega$; $\omega_{AB} = \frac{\omega}{2}$; $r = I_c A = x \sin 30$

$$\therefore x \sin 30 = \frac{2L\omega}{\omega} \quad \therefore x = 4L$$

c) BY INSTANT CENTRE, V_G HAS A DOWNWARD COMPONENT AND IS THEREFORE MOVING TOWARD EQUILIBRIUM.

9a)



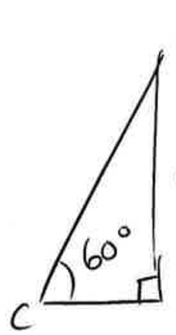
$$cb_r = \omega l \sin 30 = \frac{\omega l}{2}$$

$$cd = \frac{3}{2} cb_r = \frac{3\omega l}{4} \quad \angle 60°$$

$$\omega_{CD} = \frac{cd}{CD} = \frac{\omega}{4} \; (\curvearrowright)$$

b) "POWER IN = POWER OUT"

$$T\omega = F(cd_{VERT}) + M \underbrace{(\omega_{AB} + \omega_{CD})}_{\text{ADDITIVE BECAUSE IN OPPOSITE DIRECTIONS}}$$



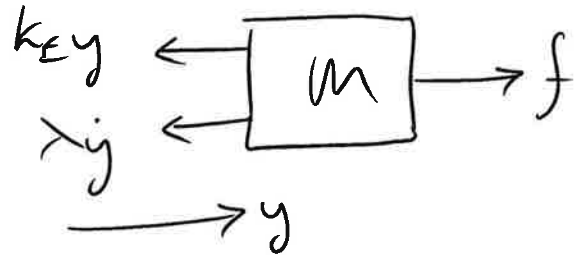
$$cd_{VERT} = \frac{3\omega l}{4} \sin 60 = \frac{3\omega l\sqrt{3}}{8}$$

$$\therefore \; T = \frac{3\sqrt{3}}{8} Fl + \frac{5}{4} M$$

10) FROM p8 OF THE DATA BOOK

LET k_E BE THE EQUIV. SPRING STIFFNESS $= 4k$

"$\Sigma F = ma$"



$$f - k_E y - \lambda \dot{y} = m\ddot{y}$$

$$\therefore \ddot{y}\frac{m}{k_E} + \dot{y}\frac{\lambda}{k_E} + y = \frac{f}{k_E}$$

$$\therefore \omega_n = \sqrt{k_E/m} \quad \text{AND} \quad \zeta = \frac{\lambda}{2\sqrt{k_E m}}$$

$$\therefore \omega_n = \sqrt{4k/m} \quad \text{AND} \quad \zeta = \frac{\lambda}{4\sqrt{km}}$$

b) $k_E = 1000 \text{ N/m}$

$$\omega_n = \sqrt{\frac{1000}{250}} = 2 \text{ RAD/S} \quad ; \quad \zeta = \frac{50}{2\sqrt{1000 \times 250}} = 0.05$$

FROM GRAPH ON p9 $\left(\omega/\omega_n = 1/2 \;;\; \zeta = 0.05\right)$

$\frac{Y}{X} \approx 1.33$; PHASE $\approx 4°$ LAG

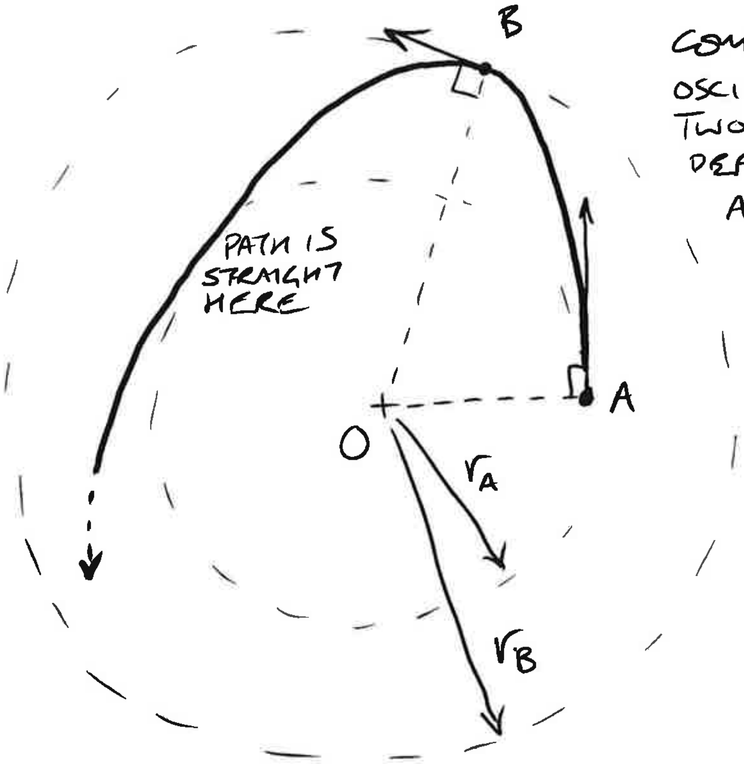
$X = F/k_E \quad \therefore Y = 1.33\left(\frac{50}{1000}\right) \approx 66 \text{ mm}$

c) $\omega/\omega_n = 1$; $\zeta = 0.1$: SYSTEM HAS A VERY LARGE AMPLITUDE OF VIBRATION $\left(\left|\frac{Y}{X}\right| = 5\right)$. IT IS LIGHTLY DAMPED AND NEAR THE RESONANT PEAK. OUTPUT LAGS THE INPUT BY 90°

11(a) CONSERVATION OF MOMENTUM:

$$(1)(5) = (1+0.5)V_A \quad \therefore \quad V_A = \frac{10}{3} \text{ m/s} \uparrow$$

b)



COMBINED PARTICLE OSCILLATES BETWEEN TWO CIRCLES DEFINED BY r_A (0.5m) AND r_B (UNKNOWN)

c) BY CONSERVATION OF MOMENT OF MOMENTUM:

$$V_B m r_B = V_A m r_A$$

$$\therefore V_B = \frac{V_A r_A}{r_B} \quad \text{——} \quad (1)$$

BY CONSERVATION OF ENERGY:

$$\frac{1}{2} m V_B^2 + \frac{1}{2} k (r_B - r_A)^2 = \frac{1}{2} m V_A^2 \quad \text{——} \quad (2)$$

SUBSTITUTE (1) INTO (2):

$$\frac{1}{2} m \left(\frac{V_A r_A}{r_B}\right)^2 + \frac{1}{2} k (r_B - r_A)^2 = \frac{1}{2} m V_A^2$$

IF $r_B = 1\text{m}$, THEN $\dfrac{V_A r_A}{r_B} = \dfrac{10(0.5)}{3(1)} = \dfrac{5}{3}\ \text{m/s}$

$$\therefore \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)^2 + \frac{50}{2}(0.5)^2 = \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{10}{3}\right)^2$$

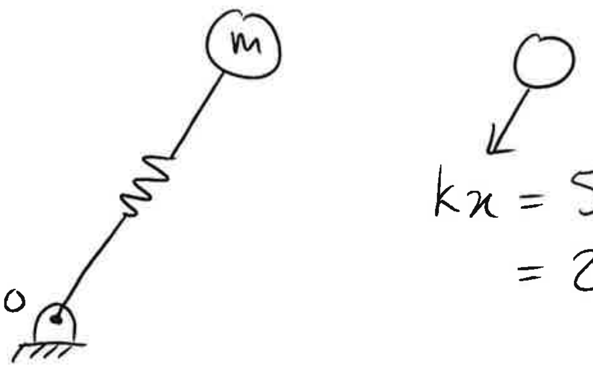
$$\frac{300}{36} = \frac{300}{36}$$

THIS SHOWS THAT THE SUGGESTED VALUE FOR r_B IS VALID.

d) $V_A = V_{MAX} = 10/3$ m/s

$V_B = V_{MIN} = 5/3$ m/s

e) AT B:



$kx = 50(0.5)$
$= 25$ N

"F=ma"

$$\therefore a_B = \frac{25}{1.5} = \underline{\underline{16^2/_3\ \text{m/s}^2}}$$ (NORMAL TO VELOCITY VECTOR)

FROM p1 OF THE DATABOOK:

"$a_{NORMAL} = \dot{s}^2/R$"

$$\therefore R = \frac{V_B^2}{a_B} = \frac{(5/3)^2}{(25/1.5)} = \underline{\underline{\frac{1}{6}\ \text{m}}}$$

12a) USING 'CASE b' FROM p10-11 OF DATABOOK

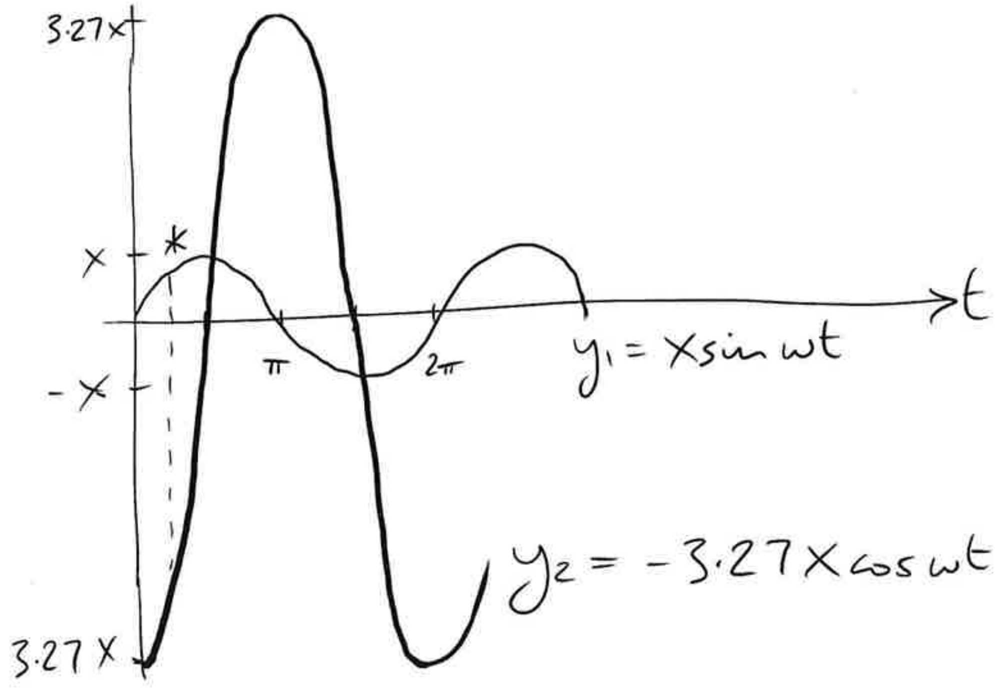
$\omega_{n_1} = \sqrt{k_1/m_1} = 2$ RAD/S $\therefore \frac{\omega}{\omega_n} = 1$

$\zeta = \frac{\lambda}{2\sqrt{k_1 m}} = \frac{1}{2} \therefore |Y_1| = |X|$; $\phi = 90°$ LAG

b) IF $x = X\cos\omega t$ THEN $y_1 = X\sin\omega t$

$\omega_{n_2} = \sqrt{k_2/m_2} = \sqrt{25/9} = 5/3 \therefore \frac{\omega}{\omega_n} = 1.2$

$\zeta = 0 \therefore \left|\frac{Y_2}{X}\right| = 3.27$; $\phi = 180°$ LAG



c) COLLISION IF $|y_1 - y_2| = 10$mm (AT ANY TIME)

$\therefore |X\sin\omega t + 3.27X\cos\omega t| = 10$

" $|a\sin x + b\cos x| = \sqrt{a^2+b^2}$ "

$\therefore X\sqrt{1+3.27^2} = 10 \quad \therefore X_{MAX} = 2.98$mm